The Capacitated Vehicle Routing Problem with Evidential Demands: models and perspectives

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The Capacitated Vehicle Routing Problem (CVRP)

Finding the least cost routes to serve customers with known demands while respecting problem constraints, in particular capacity constraints.

Uncertainty on client demands

- Random demands $\Rightarrow$ CVRP with Stochastic Demands (CVRPSD)
  - Chance Constrained Programming (CCP);
  - Stochastic Programming with Recourse (SPR).
- Interval demands $\Rightarrow$ Robust CVRP
  - Robust solutions against all realisations of customer demands deemed possible.
Intermediary situation between probabilistic and set-valued approach

each customer demand is known to belong to one or more sets with a given probability associated to each set.

⇒ the Capacitated Vehicle Routing Problem with Evidential Demands (CVRPED).

The CVRPED [Helal et al., 2016], [Helal et al., 2017]

Extending stochastic programming approaches into evidence theory framework:

- extending CCP ⇒ Belief Constrained Programming (BCP);
- extending SPR ⇒ recourse approach.

The resulting models are connected to the robust CVRP.
## Outline

1. CVRPSD
   - CVRPSD modelled by CCP
   - CVRPSD modelled by SPR

2. CVRPED
   - CVRPED modelled by BCP
   - CVRPED modelled by a recourse approach

3. Perspectives
   - Extensions
   - Advanced analysis
   - The metaheuristic
Capacitated Vehicle Routing Problem (CVRP)

Given:

- \( n \) = number of customers
- \( m \) = number of vehicles
- \( Q \) = vehicle capacity
- \( d_i \) = (known) demand of client \( i \)
- \( c_{i,j} \) = cost of travelling edge \((i, j)\)
- \( w_{i,j,k} = \begin{cases} 1 & \text{if } k \text{ travels } (i, j) \\ 0 & \text{otherwise} \end{cases} \)
- \( R_k \) = route associated to vehicle \( k \)

Objective function:

\[
\text{Minimize} \sum_{k=1}^{m} C(R_k)
\]

where

\[
C(R_k) = \sum_{i=1}^{n} \sum_{j=1}^{n} c_{i,j} w_{i,j,k}
\]

(Travel cost of route \( R_k \))

S.t : \[ \sum_{i=1}^{n} \sum_{j=1}^{n} d_i w_{i,j,k} \leq Q, \quad k = 1, \ldots, m \]
CVRPSD

$d_i$ represents the stochastic demand of $i$.

CVRPSD via CCP

Same formal model as the CVRP except that capacity constraints are replaced by

$$P \left( \sum_{i=1}^{n} d_i \sum_{j=1}^{n} w_{i,j,k} \leq Q \right) \geq 1 - \beta, \quad k = 1, \ldots, m,$$

where $1 - \beta$ is the minimum allowable probability that the capacity constraint is fulfilled.
Recall in the CVRP

\[
\text{Minimize} \sum_{k=1}^{m} C(R_k)
\]

Such that: \(\sum_{i=1}^{n} \sum_{j=1}^{n} d_{ij}w_{i,j,k} \leq Q, \quad k = 1, \ldots, m\)

CVRPSD via SPR (incorporates constraints into the objective)

\[
\text{Minimize} \sum_{k=1}^{m} C_E(R_k),
\]

where \(C_E(R_k)\) the expected cost of \(R_k\) defined by

\[
C_E(R_k) = C(R_k) + C_P(R_k),
\]

with \(C_P(R_k)\) the expected penalty cost on \(R_k\) induced by violating capacity constraints of vehicle \(k\).
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Evidence theory

A variable $x$ taking values in a finite domain $X$.

- A MF $m^X : 2^X \rightarrow [0, 1]$ s.t. $\sum_{A \subseteq X} m^X(A) = 1$.

- Belief and Plausibility functions:
  
  $$Bel(x \in A) = \sum_{\emptyset \neq B \subseteq A} m(B), \ \forall A \subseteq \Omega,$$
  
  $$Pl(x \in A) = \sum_{B \cap A \neq \emptyset} m(B), \ \forall A \subseteq \Omega.$$  

- Given a MF $m^X$ and a function $h : X \rightarrow \mathbb{R}^+$, then the upper expected value of $h$ relative to $m^X$ is:
  
  $$E^*(h, m^X) = \sum_{A \subseteq X} m^X(A) \max_{x \in A} h(x).$$
CVRPED

\(d_i\) represents the evidential demand of client \(i\) (represented by MF).

CVRPED via BCP

Same formal model as the CVRP except that capacity constraints become

\[
Bel \left( \sum_{i=1}^{n} d_i \sum_{j=1}^{n} w_{i,j,k} \leq Q \right) \geq 1 - \beta, \quad k = 1, \ldots, m, \tag{1}
\]

\[
Pl \left( \sum_{i=1}^{n} d_i \sum_{j=1}^{n} w_{i,j,k} \leq Q \right) \geq 1 - \overline{\beta}, \quad k = 1, \ldots, m, \tag{2}
\]

s.t \(\beta \geq \overline{\beta}\)

and \(1 - \beta\) (resp \(1 - \overline{\beta}\)) the minimum allowable degree of belief (resp plausibility) that the capacity constraint is respected on each route.
Particular cases

- Bayesian MF $\Rightarrow$ CVRPSD via CCP with $\beta = \overline{\beta}$.
- Categorical MF (and $\beta < 1$) $\Rightarrow$ robust CVRP (minimax optimization procedure).

Properties

- The optimal solution cost is non increasing in $Q$, $\beta$ and $\overline{\beta}$.

Properties based on change in knowledge specificity

$\forall i : d_i$ independent and known in the form $m_i^{\Theta_i}$:

- $m_i^{\Theta_i}$ built from $m_i^{\Theta_i}$: for each $A = [A; \overline{A}]$ s.t $m_i^{\Theta_i}(A) > 0$, transfer the mass $m_i^{\Theta_i}(A)$ to $A^+ = [A; \overline{A} + a^+]$, with $a^+ \in [0; Q - \overline{A}]$.

$\Rightarrow$ the more pessimistic knowledge is about customer demands, the greater the cost of the optimal solution.
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- Bayesian MF $\Rightarrow$ CVRPSD via CCP with $\beta = \bar{\beta}$.
- Categorical MF (and $\underline{\beta} < 1$) $\Rightarrow$ robust CVRP (minimax optimization procedure).

Properties

- The optimal solution cost is non-increasing in $Q$, $\underline{\beta}$ and $\bar{\beta}$;

Properties based on change in knowledge specificity

$d_i$ independent and known in the form $m_i^{\Theta_i}$:

- $m_i^{\Theta_i}$ built from $m_i^{\Theta_i}$: for each $A = [A; \bar{A}]$ s.t. $m_i^{\Theta_i}(A) > 0$, transfer the mass $m_i^{\Theta_i}(A)$ to $A^+ = [A; \bar{A} + a^+]$, with $a^+ \in [0; Q - \bar{A}]$.

$\Rightarrow$ the more pessimistic knowledge is about customer demands, the greater the cost of the optimal solution.
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CVRPED via recourse (incorporates constraints into the objective)

\[
\text{Minimize } \sum_{k=1}^{m} C^*_E(R_k),
\]

where \( C^*_E(R_k) \) the upper expected cost of \( R_k \) defined by

\[
C^*_E(R_k) = C(R_k) + C^*_p(R_k),
\]

with \( C^*_p(R_k) \) the upper expected penalty cost on \( R_k \) induced by violating capacity constraints of vehicle \( k \).

\[
C^*_p(R_k) = E^*(g, m^\Omega), \text{ where } g \text{ a penalty cost function and } m^\Omega \text{ uncertainty on failing the capacity constraint on } R_k.
\]
Particular cases

- Categorical MF $\Rightarrow$ similarities with Robust CVRP;
- Bayesian MF $\Rightarrow$ CVRPSD via SPR.

Properties based on change in knowledge specificity

Evidential demands $d_i$ are independent and known in the form $m_i^{\Theta_i}$:

- $m_i^{\Theta_i}$ built from $m_i^{\Theta_i}$: for each $A \subseteq \Theta_i$ s.t $m_i^{\Theta_i}(A) > 0$, transfer the mass $m_i^{\Theta_i}(A) > 0$ to a subset $A'$ such $A \subseteq A' \subseteq \Theta_i$.

$\Rightarrow$ the less specific knowledge is about customer demands, the greater the cost of the optimal solution.

Solution method for recourse and BCP models

Both models solved using a Simulated annealing metaheuristic.
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Extensions

- **Extend** to the evidential framework, other variations of stochastic CVRP:
  - stochastic travel times;
  - presence of clients is stochastic;
  - other combinatorial optimisation problems.

- **Extend/adapt** our models to other uncertainty frameworks, which extend probabilistic and set-valued approaches.
  - Imprecise probability;
  - possibility theory.
Advanced analysis

For the BCP and the recourse model, we would like to

- **perform a sensitivity analysis**: identifying clients, whom more knowledge about their demands leads to better solutions;

- identify from the set of non-dominated solutions, some parts of routes more preferred to be included in a solution:
  - previous work on label ranking [Destercke et al., 2015].

- extend our models, to the case of incomplete knowledge about dependency between evidential demands:
  - previous work on idempotent conjunctive combination of belief functions [Destercke and Dubois, 2011].
The metaheuristic

- Improve the metaheuristic:
  - improving the operators and some strategies adopted by the metaheuristic.

- Qualify the metaheuristic quality, by comparing optimal solutions to the metaheuristic solutions:
  - create small instances, so they can be solved by a brut force method in a reasonable time;
  - up until which instance size the metaheuristic is still able to find optimal solutions (how far from optimal).


Thank you for your attention.