EXPLOITING PRIOR KNOWLEDGE FOR EVALUATING CBR SYSTEMS USING A COVERAGE & CONSTRAINED EVIDENTIAL CLUSTERING BASED MODEL

Safa BEN AYED [1,2], Zied ELOUEDI [1], Eric LEFEVRE [2]

[1] *Université de Tunis, Institut Supérieur de Gestion de Tunis, LARODEC, Tunis, Tunisia*
safa.ben.ayed@hotmail.fr; zied.elouedi@gmx.fr

[2] *Univ. Artois, EA 3926, Laboratoire de Génie Informatique et d'Automatique de l'Artois (LGI2A), 62400 Béthune, France*
eric.lefevre@univ-artois.fr

ABSTRACT

Knowledge resource evaluation presents a concern of widespread interest in intelligent and knowledge management systems. For instance, the competence of Case Based Reasoning (CBR) systems in solving new problems depends mainly on the concept of cases coverage of the problem space. In this paper, we propose a new model for Case Base competence estimation that is based on cases coverage and partitioning, and enables the exploitation of available prior knowledge. This kind of background, which is handled in form of pairwise constraints, may be offered by domain-experts to aid the automated learning process. Actually, it is also important to manage the uncertainty involved by CBR systems' knowledge, since they reflect real-world situations. Our proposal tackles this latter problem under the belief function framework.

KEYWORDS

Case-Based Reasoning Systems; Evaluation; Competence Model; Prior Knowledge; Machine Learning; Uncertainty;

1. INTRODUCTION

Among the most important aspirations of Knowledge Engineering field is to conceive methods and techniques able to understand, imitate, and reproduce human reasoning. Case-Based Reasoning presents the artificial intelligence methodology that, similarly to human being, is based on reusing past experiences to solve new problems by assuming that "similar problems have similar solutions", where some adaptations can be done. After 3solving problems, new cases are retained within the CB at the aim to improve systems learning capabilities (Aamodt and Plaza, 1994). Since CBR systems have known a widespread utility in several domains, and since their knowledge containers present their success key factors, a wide range of research intend to assess their performance and competence (Smyth and Keane, 1995; Smyth and McKenna, 1998; Smiti and Elouedi, 2013; Ben Ayed et al, 2018a). Due to its distinguished reasoning, different concepts related to CBR systems evaluation have been proposed, such that *(1)* the *Coverage* which defines the space of problems that one case or a group of cases are able to successfully solve, and *(2)* the *Reachability* which presents the space of cases that is able to solve some target problem(s). Rather than the evaluation concerns, preserving CBs competence presents the main purpose of Case Base Maintenance (CBM) research field (Ben Ayed et al, 2017; Ben Ayed et al, 2018b). However, estimating the real global CB competence is a very complex task since it depends on various factors such as the CB size, relation between cases, and other statistical factors like the distribution and the density of cases. Moreover, the competence is relied to the problem-solving properties such that CB's vocabulary, similarity measures, and adaptation knowledge. Actually, all these factors may be taken into account either in part or in totality. However, it is essential, on the one hand, to handle uncertainty and imperfection attached to cases since they reflect real-world situations. In fact, we note, in the literature, a high interest in evaluating the competence of CBR systems' CBs (Smyth and McKenna, 1998; Smiti and Elouedi, 2013), where some of them prove the necessity of managing uncertainty within such systems (Ben Ayed et al, 2018a; Weber, 2006). On the other hand, there is no research, to the best of our knowledge, that provide the ability to available prior knowledge to aid learning and to lead a better evaluation. Yet, CBR systems are generally applied to solve problems for some specific domain, where experts may easily provide some highly useful information for the automated modeling process. To offer a high-quality estimation of the real CBs competence, we propose, in this paper, a new competence model, named CCEC-Model for "Coverage & Constrained Evidential Clustering based Model", that takes into account different factors that influence the ability of CBR systems to solve problems, such as size, density, and coverage. Besides, it is based on partitioning the CB into small ones while managing membership uncertainty using the belief function theory. One among the strengths of our current proposal is its ability to exploit prior knowledge in form of pairwise instance level constraints (Must-link and Cannot-link) that may notably increase the machine learning accuracy.

The rest of this paper is organized as follows. The next Section reviews the four most known CBR competence models in the literature. Section 3 is devoted to briefly present the belief function theory, on the one hand, and to define and model the used type of constraints, on the other hand. The principle and the different steps of our new proposed competence for CBR systems evaluation are detailed during Section 4. Thereafter, we experimentally investigate, in Section 5, two modes for artificial constraints generation, as well as the experimental settings and results. Finally, Section 6 concludes the paper.

2. COMPETENCE MODELS FOR CBR SYSTEMS EVALUATION

To trust the solutions offered by a given CBR system in some application field, we need to ascertain about their competence for this matter. Actually, the competence is defined by the range of problems that a given CBR system can solve successfully. However, as mentioned in the introduction, there are several reasons that make the straight estimation of the competence faulty. Let us present, therefore, four existing competence models, in the literature, for CBR systems evaluation.

- **Case Competence Categories Model:** Since modeling the competence of case bases is very useful when building CBM policies, authors in (Smyth and Keane, 1995) built a model for competence estimation at the aim of deciding the set of cases that should be retained. Hence, four case categories are defined by this model according to the coverage and reachability concepts.This model has been proposed in the frame of CBM in order to fix a strategy to remove the less competent cases in solving problems. However, it is not able to tangibly and mathematically quantify the global competence of the entire CB.
- **CMDC: Coverage model based on Mahalanobis Distance and Clustering:** Similarly to the previous presented model, CMDC (Smiti and Elouedi, 2013) categorizes the set of cases into three classes: Noisy cases, Similar cases, and Isolated cases. Concisely, CMDC model uses a density-based clustering method in order to be able to flag every case to its fitted type. Then, it defines the global CB competence through aggregating cases coverage as follows:

$$Comp\%(CB) = \left| 1 - \frac{\sum_{j=1}^{K} \sum_{i=1}^{N} Cov(c_{ij})}{Size(CB)} \right|$$

where *K* presents the number of clusters, *N* is the number of instances belonging to cluster *j*. Obviously, the competence of CBR systems depends on cases composing the CB. However, we cannot be able to well estimate it without deeply studying the relation between these cases.

- **Smyth & McKenna Model:** The overall competence estimation for a CBR system is complex since various factors and relations between cases intervene. Authors in (Smyth and McKenna, 1998) believe that we cannot estimate the true nature of competence in CBR without underlying the sources of competence in such systems. Hence, S&M model estimates that competence through some basic influencing statistical factors such as CB size, case density, and case distribution. The principle of the current model can be summed up by some main ideas: Dividing the CB into some independent competence groups, measuring the density of each group based on the similarity between cases, and estimating their coverage according to the size of each group as well as their densities. Finally, the overall CB competence is estimated as the sum of all the generated groups' coverages. Clearly, all models are agree that cases present the primary factor to estimate the CB competence. However, these real world situations are undoubtedly offering imprecise and uncertain knowledge, and all of the previously presented models cannot handle this imperfection to conduct the best decisions.

- CEC-Model: Coverage & Evidential Clustering based Model: To avoid ignorance during learning caused by uncertainty and overlapped data regions, CEC-Model (Ben Ayed et al, 2018a) uses the belief function theory as a theoretical framework for uncertainty management (see Subsection 3.1). The added value of this model appears mainly on its ability to manage uncertainty of cases membership to clusters and when measuring distances. To estimate the overall CB competence rate, CEC-Model goes through four main steps. First, it clusters the entire case base using an evidential clustering method called Evidential C-Means (Masson and Denoeux, 2008). This technique provides a credal partition that reflects the belief degrees of membership, which will be used, on the one hand, to softly measure the similarities between cases and, on the other hand, to form groups. Both of the latter steps are then used to estimate coverage. Undoubtedly, this research work is able to take into account important factors for competence estimation along with the ability to manage uncertainty. However, it suffers from two main weaknesses. First, it is very sensitive to noisy cases, since it does not prevent a step to eliminate irrelevant cases and distortion of data values. Second, similarly to all the other competence models, they suffer from their disability to aid their automated process when prior knowledge are available. The proposal of this paper tackles the ensemble of these limitations, such as shown during the rest of the paper.

3. RELATED BACKGROUND

In this Section, we briefly present the basics of the belief function theory (Subsection 3.1), and define how prior knowledge are expressed in the current work (Subsection 3.2).

3.1 Belief Function Theory

The belief function theory (Dempster, 1967; Shafer, 1976), called also Evidence theory, is a mathematical framework for reasoning under uncertainty. Its model is basically defined by a frame of discernment Ω that represents a finite elementary events set. The potency of this theory is that it models uncertainty on a power set 2^{Ω} that contains all the possible subsets of Ω (i.e. from the complete ignorance to the total certainty). The key point of this theory is the *basic belief assignment (bba) m* which is defined as a mapping function from 2^{Ω} to [0,1], and satisfies $\sum_{A\subseteq\Omega} m(\mathrm{A}) = 1$, in order to allocate to every set $\mathrm{A} \in 2^{\Omega}$ a degree of belief to represent the uncertain knowledge about the actual value of *y* defined on Ω. A mass function is said to be normalized if $m(\emptyset) = 0$. Otherwise, the assigned amount of belief to the empty set reflects the flexibility to consider that *y* may not belong to Ω. In fact, the latter situation has been used during the evidential clustering to identify noisy instances (Masson and Denoeux, 2008; Antoine et al, 2012), where the frame of discernment Ω defines the set of clusters. Furthermore, *m* can be presented by the plausibility (*pl*) which is the maximum belief's amount that can be assigned to A, and defined in such a way that $pl(A) = \sum_{A\cap B\neq\emptyset} m(\mathrm{B})$ for all $A \subseteq \Omega$. To calculate the distance between two mass functions defined in the same frame of discernment, we may use the Jousselme Distance (Jousselme, 2001), which is the most used distance between two pieces of evidence. It is defined such that $d(m_1, m_2) = \sqrt{\frac{1}{2}(\overrightarrow{m_1} - \overrightarrow{m_2})^T \,\overline{\overline{D}}\, (\overrightarrow{m_1} - \overrightarrow{m_2})}$, where $\overline{\overline{D}}$ is a square matrix of size 2^K ($K = |\Omega|$), and its elements are calculated such that:

$$D(A,B) = \begin{cases} 1 & if\ A = B = \emptyset \\ \dfrac{|A \cap B|}{|A \cup B|} & \forall A, B \in 2^{\Omega} \end{cases}$$

3.2 Prior Knowledge Modeling: Constraints Expression

In real application domains, the experimenter often possesses data background knowledge that could be greatly beneficial for learning. Actually, there are different ways to integrate this additional information. In this paper, we are interested to model this prior knowledge in form of instance-level constraints and integrate them during the learning of the CB, more precisely, inside the evidential clustering process. This kind of background gives knowledge regarding which cases that should or should not belong to the same group. For example, a doctor can provide, in the medical field, the knowledge if two patients should or should not follow the same treatment, by knowing their symptoms description. By this way, two kinds of constraints arise:

- **Must-link** constraints $\mathcal{M}$ to indicate that some pairs of cases should belong to the same cluster.
- **Cannot-link** constraints C to mention that some pairs of cases must not belong to the same cluster.

In order to express constraints under the evidential clustering framework, we handle every pair of cases through their mass functions m_i and m_j. Their joint class membership $m_{i\times j}$ is computed in the Cartesian product. Let $\theta = \{(y_1, y_1), (y_2, y_2), \ldots, (y_K, y_K)\}$, the subset in Ω^2, represents the event *"The pair of cases i and j are placed in the same cluster"*. Therefore, we calculate from $m_{i\times j}$ the plausibility $pl_{i\times j}$. The Cannot-link constraint (C) between cases i and j corresponds to $pl_{i\times j}(\theta) = 0$, and the Must-link constraint ($\mathcal{M}$) between i and j corresponds to $pl_{i\times j}(\bar{\theta}) = 0$.

4. COMPETENCE ESTIMATION UNDER PRIOR KNOWLEDGE

Evaluating knowledge-based systems presents a research direction with worth continued investigation. In particular, an extra effort must be assigned to CBR systems so as to measure their competence in problem-solving due to the diversity of influencing factors (Smyth and McKenna, 1998). In this Section, we detail the steps our new Coverage & Constrained Evidential Clustering based Model (CCEC-Model) which leads to the estimation of the overall CB's competence. We propose to show, using Figure 1, a general depict of the different CCEC-model's steps, that will be detailed during the following Subsections.

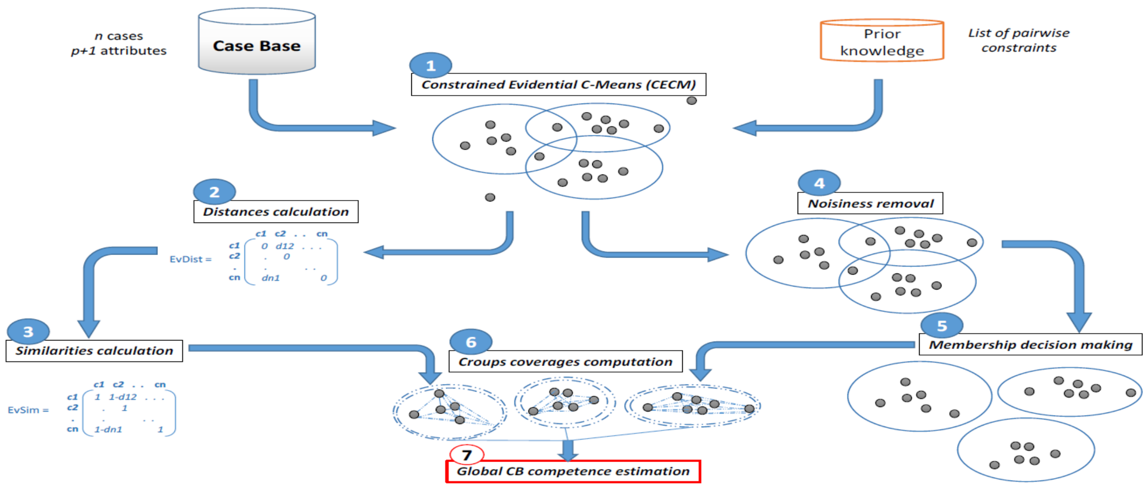

Figure 1: A general depict of the CCEC-Model's process

4.1 Step 1: Constrained evidential clustering of cases (CECM method)

Once we aim to measure CBR system's competence where some prior knowledge are available in form of constraints, our CCEC-Model starts by applying the Constrained Evidential C-Means (CECM) (Antoine et al, 2012) which is a constrained clustering technique under the evidence theory. It is a variant of ECM (Masson and Denoeux, 2008) and offers a credal partition in form of n-tuple mass functions. Each tuple describes the belief's membership degrees towards one case. To form clusters and generate the credal partition, CECM minimizes a cost function, in order to maximize distances between cases in different clusters, and inversely, with requiring that if the pair of cases $(i, j) \in \mathcal{C}$ (respectively $\in \mathcal{M}$), then $pl_{i\times j}(\theta)$ (respectively $pl_{i\times j}(\bar{\theta})$) have to be as low as possible. Due to lack of space, we invite you to see the entire work in (Antoine et al, 2012) for further details about the objective function and the optimization process of this machine learning method.

4.2 Step 2: Calculate distances between cases with uncertainty handling

Measuring distances between cases through their attributes values is equivalent to suppose that these values are totally certain and precise, which cannot be true. Hence, we measure the distance between every pair of cases through their generated mass functions. To do, we choose to use the Jousselme distance (Jousselme, 2001) as defined during Subsection 3.1. This distance offers normalized results in [0, 1].

4.3 Step 3: Similarity measurement between cases

Let suppose that all the generated evidential distances are stored in a $n \times n$ square matrix that we call *EvDist*. Because of requirements that our competence model impose, we move from a matrix of distances to a similarity matrix, that we call *EvSim*. A straightforward transformation has been therefore performed such that $EvSim = Ones - EvDist$ with $Ones$ presents a $n \times n$ matrix filled by *1*.

4.4 Step 4: Cleaning up the CB from noisiness

Logically, for a given CB, the more it contains distant cases, the more it covers the problem space. However, it is not the situation with noisiness. Actually, noisy cases are far to the rest of cases, but it is only because of the distortion of their data values. Consequently, before moving to the other steps, it is essential to clean up the CB from this undesirable type of data. Since Step 1 of our model assigns noisy cases to the empty set partition, we remove the set of cases where the empty set has a degree of belief higher than the sum of all those assigned to the other partitions of clusters. For instance, a case i is flagged as noisy if and only if $m_i(\emptyset) \geq \sum_{A_j \subseteq \Omega, A_j \neq \emptyset} m_i(A_j)$.

4.5 Step 5: Decision making about cases membership to clusters

Now, we need to move from the credal level of the belief function framework to the decision level in order to build independent groups of cases. The decision about cases membership to groups is done through the pignistic probability transformation of every mass function *m*, such that:

$$BetP(A) = \sum_{B \subseteq \Omega} \frac{|A \cap B|}{|B|} \frac{m(B)}{1 - m(\emptyset)} \qquad \forall A \in \Omega$$

Ultimately, the case will be assigned to the cluster having the highest *BetP*.

4.6 Step 6: Groups coverage estimation

The more a group coverage is large, the more it is competent in problem-solving (Smyth and McKenna, 1998). Based on this idea, we estimate, during this step, the coverage value of every group of cases. The principle consists at modeling the relationship between 'individual case' and 'global case' competence contributions, using the most influencing factors such as *the size*, *the distribution*, and *the density*.

First, we assume that our space is regular i.e., a high local case density implies a high mutual similarity degree, and vice versa. Hence, CCEC-Model measures the local density of one case *c* that belongs to group *G*, using similarities generated during Step 3, as follows:

$$CaseDensity(c, G) = \frac{\sum_{c' \in G-\{c\}} EvSim(c, c')}{|G| - 1}$$

We may then calculate the density of the group G by averaging all its local densities, such that:

$$GroupDensity(c, G) = \frac{\sum_{c \in G} CaseDensity(c, G)}{|G|}$$

where both of the latter two Equations give normalized values in [0,1].

Last but not least, we aim to estimate the coverage of each group. Similarly to (Smyth and McKenna, 1998) and (Ben Ayed et al, 2018a), we are interested to estimate them according to some characteristics, as follows:

- Directly proportional to the size of the group, since the larger the number of case is, the more probable to cover problem space.
- Inversely proportional to the density of the group, because the more cases are far apart, the more they are able to cover the problem space.
- If the group of cases is perfectly dense i.e., all its contained cases are projected in the same case, and then it is able to solve only one problem.

By this way, we estimate a group coverage (values in $[1, 1 + |G|]$) such that: $GroupCoverage(G) = 1 + [|G|(1 - GroupDensity)]$. If we want a normalized coverage in [0, 1], the following formula may be applied:

$$NormGroupCov(G) = \frac{GroupCoverage(G) - 1}{|G|}$$

4.7 Step 7: Stating the global CB competence rate

Finally, after constructing independent groups and measuring their coverage, we are now in a position to state a measure of global competence. Since we aim to measure this competence in form of rate, we may simply sum the normalized coverage estimates of the different groups to deduce the overall CB competence value. Consequently, if our frame of discernment Ω is composed by K clusters, i.e., K groups G_K, then the global competence rate is defined as follows:

$$Comp(CB)\% = \frac{\sum_{k=1}^{|\Omega|} NormGroupCov(G_k)}{|\Omega|} \times 100$$

5. EXPERIMENTAL ANALYSIS USING ARTIFICIAL CONSTRAINTS

During our experimental study, two ways for an artificial generation of pairwise constraints has been proposed (Subsection 5.1) to validate our proposed competence model (Subsections 5.2 and 5.3).

5.1 Artificial Constraints Generation strategies

Actually, the list of pairwise constraints is originated from available prior knowledge that may be offered by experts of the domain, on which the CBR system is applied, in order to aid the automatic leaning. Since we will perform our tests on CBs extracted from different domains, we propose two ways for constraints generation while managing uncertainty, which are *Batch* and *Alternate* modes.

Let us describe these two modes by two activity diagrams shown in Figures 2 and 3.

Figure 2: Batch mode constraints generation

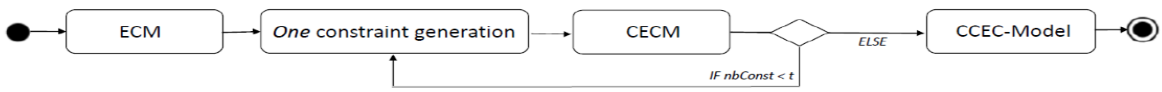

Figure 3: Alternate mode constraints generation

The first one indicates that a number t of constraints are generated simultaneously before performing our model, while the second consists at alternating between a constraint generation and learning, until reaching the desired number of constraints t (t is set equal to *10%* of the size of every tested CB).
For a single constraint generation, we handle uncertainty by using mass functions. The idea consists at randomly picking two cases: *If* both of them are classified with high certainty ($m(A) \geq 0.5$), where A is a partition of clusters, *then* we generate the appropriate constraint between them (Must-link if they have the same solution and Cannot-link otherwise). *Else*, we retake at random two other cases.

5.2 Experimental Settings and Evaluation Criteria

To validate our new competence model, we think, similarly to authors in (Smyth and McKenna, 1998), to make of the Percentage of Correct Classification criterion (using the 1-Nearest Neighbor as the most used classifier in CBR) our baseline for evaluation, since it may present substantive competence rate. Obviously, it is more reasonable to be interested on the correlation between PCCs and estimated competence values than focusing on the matching between them. Our model, with its two modes for constraints generation ($CCEC_{bat}$ and $CCEC_{alt}$), was tested on different datasets from UCI repository. Due to lack of space, we share results of only two datasets in this paper, where similar results with other CBs have been obtained. The number of clusters is set equal to the number of CBs solutions (classes). By carrying on two evaluation criteria, our study will be divided into two main parts. On the one hand, we are interested on randomly increment the size of every tested CB, generate the competence predictions and the PCC values, and measure the correlation between them. To do, we use the Pearson's Correlation Coefficient (Pearson, 1896), which is defined such that:

$$r = \frac{\sum_{i=1}^{n}(a_i - \bar{a})(b_i - \bar{b})}{\sqrt{\sum_{i=1}^{n}\ (a_i - \bar{a})^2}\ \sqrt{\sum_{i=1}^{n}\ (b_i - \bar{b})^2}}$$

where a_i (respectively b_i) are the PCC values (respectively the predicted CB competence by our model), and $\bar{a}$ (respectively $\bar{b}$) presents the mean value of a_i (respectively b_i) measurements. By this way, the more this correlation is close to 1, the more our model is supported by this correlation. On the other hand, we opt, like in (Smiti and Elouedi, 2013), to compute the error rate between existing competence models values and the PCC predictions such that $Error(\%) = \frac{|EstimatedComp - PCC|}{PCC} \times 100$.

5.3 Results and Discussion

In the first part, we plot, in Figure 4, the accuracy and competence values given by the two modes of constraints generation, against a randomly increasing size of Mammographic Mass and Ionosphere CBs. With these values, we can easily see the high correlation between them, which supports our CCEC-Model. Using the first criterion (correlation r), we opt to measure with precision this correlation, with reminding that the closer it is to one, the more it is interesting. Indeed, we noted a high matching between the predicted and the "actual" competence, where we found for Mammographic Mass (respectively Ionosphere) dataset correlation values equal to 0.869 and 0.811 (respectively 0.779 and 0.913) using the batch and alternate modes for constraints generation, respectively.

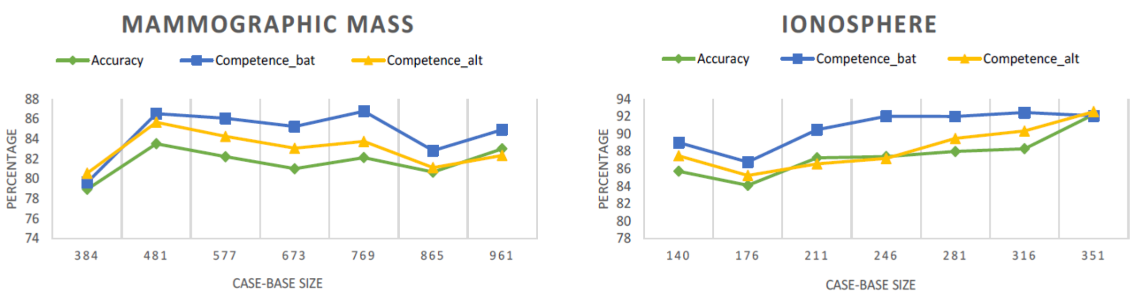

Figure 4: Comparing the predicted competence of CCEC-Model (with the two modes for constraints generation) to the accuracy for Mammographic Mass and Ionosphere CBs.

For the second part, Table 1 shows that our CEC-Model offers, according to the second criterion (Error rate), a competence estimation close to the accuracy offered by datasets using 1-NN. In fact, we note that the two modes for constraints generation do not greatly influence the error rate criterion. Comparing to S&M (Smyth and McKenna, 1998), CMDC (Smiti and Elouedi, 2013), and CEC-Model (Ben Ayed et al, 2018a), our model has been able to provide the lowest error rates. For instance, for Mammographic Mass dataset, our CCEC-Model provides values equal to 2.313% and 0.807%, where S&M estimates it to 21.1%, to 13.82% by CMDC, and to 5.928% by CEC-Model.

Table 1: Results in term of Error rate (%)

Case Base	S&M	CMDC	CEC	$CCEC_{bat}$	$CCEC_{alt}$
Mammographic Mass	21.10	13.820	5.928	2.313	0.807
Ionosphere	3.544	0.287	1.779	0.173	0.347

6. CONCLUSION

Aiming at the CB competence evaluation issue that CBR systems have, we have developped, in this paper, a new competence model that offers a high-quality estimation of their capability of problem-solving. This model, called CCEC-Model, is able to exploit prior available knowledge in form of constraints while managing uncertainty within knowledge. To validate our model, we have proposed two modes for constraints generation aiming at substituting knowledge coming from experts using two experiments. As future work, we aim to seek prior knowledge from an ensemble of experts within different domains on which the CBR systems are applied.

REFERENCES

Aamodt, A and Plaza, E., 1994, Case-based reasoning: Foundational issues, methodological variations, and system approaches. *In Artificial Intelligence Communications*, pp. 39-52.

Antoine, V. et al, 2012, CECM: constrained evidential c-means algorithm. *Comput. Stat. Data Anal. 56*, pp. 894-914.

Ben Ayed, S. et al, 2017,.: ECTD: Evidential Clustering and case Types Detection for case base maintenance. *In IEEE/ACS 14th International Conference on Computer Systems and Applications (AICCSA)*, pp. 1462-1469.

Ben Ayed, S. et al, 2018a, CEC-Model: A New Competence Model for CBR Systems Based on the Belief Function Theory. *In International Conference on Case-Based Reasoning*, pp. 28-44.

Ben Ayed, S. et al, 2018b, Exploiting Domain-Experts Knowledge Within an Evidential Process for Case Base Maintenance. *In International Conference on Belief Functions*, pp. 22-30.

Dempster, A.P., 1967, Upper and lower probabilities induced by a multivalued mapping. *Ann. Math. Stat. 38*, pp. 325-339.

Jousselme, A.L. et al, 2001, A new distance between two bodies of evidence. *Inf. Fusion 2(2)*, pp. 91-101.

Masson, M.H. and Denoeux, T., 2008, ECM: an evidential version of the fuzzy c-means algorithm. *Pattern Recogn. 41,* pp. 1384-1397.

Pearson, K, 1896, Mathematical contributions to the theory of evolution. III. Regression, heredity, and panmixia. *In Philosophical Transactions of the Royal Society of London*, pp. 253-318.

Shafer. G., 1976, A Mathematical Theory of Evidence. *Princeton University Press*, Princeton.

Smiti, A and Elouedi, Z., 2013, Modeling competence for case based reasoning systems using clustering. *The 26th International FLAIRS Conference, the Florida Artificial Intelligence Research Society*, pp. 399-404.

Smyth, B and Keane, M.T., 1995, Remembering to forget: a competence-perserving deletion policy for CBR systems. *The Thirteenth International Joint Conference on Artificial Intelligence*, pp. 377-382.

Smyth, B and McKenna, E., 1998, Modelling the competence of case-bases. *In: Smyth, B., Cunningham, P. (eds.) EWCBR. LNCS, vol. 1488*, pp. 208-220.

Weber, R., 2006, Fuzzy set theory and uncertainty in case-based reasoning. *Engineering intelligent systems for electrical engineering and communications*, pp. 121-136.