Handling Partial Preferences in the Belief AHP Method: Application to Life Cycle Assessment

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Abstract. This paper proposes a novel multi-criteria decision making method under uncertainty that combines the Analytic Hierarchy Process (AHP) with the belief function theory. Our method, named belief AHP, allows the expert to express incomplete and imprecise information about groups of alternatives instead of single ones. On the other hand and in order to judge the importance of criteria, he can also present his opinions on groups of criteria. Then, the uncertainty will be taken into account in the final decision. Finally, another purpose of this paper is also to solve a real application problem which deals with the PVC life cycle assessment.

1 Introduction

Within the framework of Multi-Criteria Decision Making (MCDM) problems, a decision maker often needs to make judgments on decision alternatives that are evaluated on the basis of its preferences (criteria) [11]. Amongst the most well known methods is the Analytic Hierarchy Process (AHP) [5] [6]. In fact, the strength of this method is that it is easier to understand and it can effectively handle both qualitative and quantitative data. In spite of its popularity, this method is often criticized [3] because, in real-life decision making situation, the decision maker may encounter several difficulties when building the pair-wise comparison. These difficulties arise due to different situations: the lack of data for making decisions, the inability to compare separate alternatives and/or criteria between each other, etc. As a result, several extensions of AHP method were proposed such as the Fuzzy AHP [4], the probabilistic AHP [1]. In particular in the belief function framework, the DS/AHP method [2] was proposed.

The objective of this paper is to develop what we call a belief AHP, a MCDM method adapted to imprecise and incomplete preferences, where the uncertainty is represented by the belief function theory. Our aim through this work is to allow the decision maker to give subjective judgments in two levels: the criterion and alternative levels. On the one hand, our method offers a formalism allowing the expert to express his ranking even over subgroups of alternatives. On the other hand, to judge the importance of criteria, the belief AHP method will be able to compare on groups of criteria instead of single criterion. Finally, to illustrate the feasibility of our approach, we have applied our proposed method on a real application problem.

This paper is organized as follows: we start by introducing the AHP method, then we give an overview of the basic concepts of the belief function theory. In the main body of the paper, we present our new approach: the belief AHP which is based on the belief function theory. Finally, our method will be illustrated on a real application problem in order to understand its real unfolding.

2 Analytic Hierarchy Process

The AHP approach is a decision-making technique developed by Saaty [5] [6] to solve complex problems of choice and prioritization. The basic idea of the approach is to convert subjective assessments of relative importance to a set of overall scores or weights. Its first step is to set up a hierarchy consisting of the final goal of the problem or the decision to be made, a number of criteria, and a number of alternatives to select. Once the hierarchy is built, the decision maker starts the prioritization procedure. Elements of a problem on each level are paired (with respect to their upper level decision elements) and then compared using a nine-point scale [5] [6]. This semantic scale is used to translate the preferences of a decision maker into crisp numbers. After filling the pair-wise comparison matrices, the relative importance (priority) of the elements on each level of the hierarchy are determined by using the eigenvalue method. Finally, AHP aggregates all local priorities from the decision table by a simple weighted sum. The global priorities thus obtained are used for final ranking of the alternatives and selection of the best one.

3 Belief Function Theory

In this section, we briefly review the main concepts underlying the belief function theory as interpreted by the TBM. Details can be found in [8], [10].

Let Θ be the frame of discernment representing a finite set of elementary hypotheses related to a problem domain. We denote by 2^{Θ} the set of all the subsets of Θ.

The impact of a piece of evidence on the different subsets of the frame of discernment Θ is represented by the so-called basic belief assignment (bba) (denoted by m). It quantifies the impact of a piece of evidence on the different subsets of the frame of discernment [10].

The belief function theory offers many interesting tools. To combine beliefs induced by distinct pieces of evidence, we can use the conjunctive rule of combination [8]. Also, the discounting technique allows to take in consideration the reliability of the information source that generates the bba m [9].

It is necessary when making a decision, to select the most likely hypothesis. One of the most used solutions within the belief function theory is the pignistic probability [7].

4 Belief AHP Approach

Belief AHP aims at performing a similar purpose as AHP. In fact, its main purpose is to find the preferences' rankings of the decision alternatives in an uncertain environment. Within this context, a first work has been tackled by Beynon et al. [2], they developed a method, called DS/AHP. Despite all the advantages of this method, which allows different comparisons to be made for certain group of alternatives, they do not take into account the uncertainty in the criterion level. Thus, we propose a more general method for solving complex problems under the condition that it tolerates imprecision and uncertainty when the expert expresses his preferences between criteria and also alternatives. In other words, our approach will be able to compare groups of criteria and also groups of alternatives. Hence, the computational procedure of our proposed approach is summarized in the following steps.

1. **Identification of the candidate criteria:** By nature, the importance of criteria is relative to each other. Therefore, a decision maker may encounter some difficulties to compare separate ones. In our work, a new method for judging the importance of these criteria is proposed. In fact, we suggest to extend the AHP method to an imprecise representation rather than forcing the decision maker to provide precise representations of imprecise perceptions. We suppose that there is a set of criteria $\Omega = \{c_1, ..., c_m\}$ consisting of m elements. Denote the set of all subsets of C by 2^{Ω}, and let C_k be the short notation of a subset of C, i.e., $C_k \subseteq C$ and $C_k \in 2^{\Omega}$. An expert chooses a subset $C_k \subseteq C$ of criteria from the set C and compares this subset with another subset $C_j \subseteq C$. Thus, criteria that belong to the same group have the same degree of preferences. Since we are not performing pair-wise comparisons of criterion but relating groups of criteria, these sets of criteria should not consider a criterion in common, because if one criterion is included in two groups, then each group will give a different level of favorability. By generalization, the subsets of criteria can be defined as:
$$C_k \succ C_j, \forall \ k, j | C_k, C_j \in 2^{\Omega}, C_k \cap C_j = \emptyset \ . \quad (1)$$
2. **Identification of the candidate alternatives:** As mentioned, under this approach we suggest to compare groups of alternatives instead of single one. The decision maker has to identify the subsets of favorable alternatives from all the set of the possible ones. One of the possible solutions of this task is to use the DS/AHP method [2]. Similarly to the criterion level, we assume that there is a set of alternatives $\Theta = \{a_1, ..., a_n\}$ consisting of n elements. Denote the set of all subsets of A by 2^{Θ}, and let A_k be the short notation of a subset of A, i.e., $A_k \subseteq A$ and $A_k \in 2^{\Theta}$. The main aim behind this method was explained in [2].
3. **Computing the weight of considered criteria and the alternative priorities:** After constructing the hierarchical structure of the problem, what is left is setting priorities of the subsets of alternatives and criteria. At this point, standard pair-wise comparison procedure is made to obtain these priorities.

4. **Updating the alternatives priorities:** Once the priorities of decision alternatives and criteria are computed, we have to define a rule for combining them. The problem here is that we have priorities concerning criteria and groups of criteria instead of single ones, whereas the sets of decision alternatives are generally compared pair-wise with respect to a specific single criterion. In order to overcome this difficulty, we choose to apply the belief function theory because it provides a convenient framework for dealing with individual elements of the hypothesis set as well as their subsets.
   At the decision alternative level, we propose to follow the main idea of the DS/AHP method. In fact, we have the priority vector corresponding to each comparison matrix sums to one. So, we can assume that $m(A_k) = w_k$, where w_k is the eigen value of the k^{th} sets of alternatives.
   The next step is to update the obtained bba with the importance of their respective criteria. In this context, our approach proposes to regard each priority value of a specific set of criteria as a measure of reliability. In fact, the idea is to measure most heavily the bba evaluated according to the most importance criteria and conversely for the less important ones. If we have C_k a subset of criteria, then we get β_k its corresponding measure of reliability, given by the division of the importance of criteria by the maximum of priorities. As a result, two cases will be presented: First, if the reliability factor represents a single criterion, then the corresponding bba will be directly discounted, and we get:

$$m^{\alpha_k}_{C_k}(A_j) = \beta_k.m_{C_k}(A_j),\ \forall A_j \subset \Theta\ . \quad (2)$$

$$m^{\alpha_k}_{C_k}(\Theta) = (1 - \beta_k) + \beta_k.m_{C_k}(\Theta)\ . \quad (3)$$

   where $m_{C_k}(A_j)$ the relative bba for the subset A_j, and we denote $\alpha_k = 1-\beta_k$. Second, if this factor represents a group of criteria, their corresponding bba's must be combined. Based on the belief function framework, our proposed approach assumes that each pair-wise comparison matrix is considered as a distinct source of evidence, which provides information on opinions towards the preferences of particular decision alternatives. Then, we apply the conjunctive rule of combination and we get:

$$m_{C_k} = \textcircled{\cap} m_{c_i},\quad i = \{1, ..., h\}\ . \quad (4)$$

   where h is the number of element of a specific group of criteria C_k and $c_i \in C_k$ (c_i a singleton criterion). Finally, these obtained bba's (m_{C_k}) will be discounted by their corresponding measure of reliability (the same idea used in Equation 2 and 3).
5. **Synthetic utility and decision making:** After updating the alternatives priorities', we must compute the overall bba. An intuitive definition of the strategy to calculate these bba's will be the conjunctive rule of combination ($m_{final} = \textcircled{\cap} m^{\alpha_k}_{C_k}$).
   To this end, the final step is to choose the best alternative. In this context, we choose to use the pignistic transformation to help the expert to make his final choice.

5 Application

The problem in this application is not to use or not the Polyvinyl chloride (PVC) in general, but to know in which country the environmental impact is less important for the destruction of a kilogram of PVC?

1. **Identification of the candidate criteria and alternatives:** In this application problem, the environmental criteria are playing the role of multiple criteria, and it was decided to restrict them to four areas: $\Omega = \{$abiotic depletion (C1), eutrophication (C2), toxicity infinite (C3), Fresh water aquatic ecotoxicity infinite (C4)$\}$. Apart from the four criteria, the initial interview also identified three selected countries on the set of alternatives: $\Theta = \{\text{France}(FR), \text{USA}(US), \text{England}(EN)\}$.
2. **Computing the weights of considered criteria:** Now the expert is asked to express the intensity of the preference for one criterion versus another. By using the eigenvector method, the final priorities values can be obtained as shown in Table 1, and a normalized vector is given.

Table 1. The weights assigned to the criteria according to the expert's opinion

Criteria	$\{C1\}$	$\{C4\}$	$\{C2, C3\}$	Priority	Nomalized vector
$\{C1\}$	1	2	6	0.58	1
$\{C4\}$	$\frac{1}{2}$	1	4	0.32	0.55
$\{C2, C3\}$	$\frac{1}{6}$	$\frac{1}{4}$	1	0.1	0.17

3. **Computing the alternatives priorities:** Similarly to the standard AHP, comparison matrices are constructed, and we suppose that each priority vector is considered as a bba (see Table 2).

Table 2. Priorities values

$C1$	m_{C1}	$C2$	m_{C2}	$C3$	m_{C3}	$C4$	m_{C4}
$\{EN, US\}$	0.896	$\{EN\}$	0.526	$\{EN\}$	0.595	$\{US\}$	0.833
$\{EN, US, FR\}$	0.104	$\{US, FR\}$	0.404	$\{FR\}$	0.277	$\{EN, US, FR\}$	0.167
		$\{EN, US, FR\}$	0.07	$\{EN, US, FR\}$	0.128		

4. **Updating the alternatives priorities:** Firstly, this step concerns the groups of criteria $\{C2, C3\}$. Our aim is to combine the bba relative to the criteria $C2$ and $C3$. Then, the obtained bba's is discounted by their measure of reliability $\beta_{C2,C3} = 0.17$. After that, this step concerns the single criterion $\{C1\}$ and $\{C4\}$. The relative bba's are directly discounted by their reliability measure $\beta_{C1} = 1$ and $\beta_{C4} = 0.55$ and we get the following Table 3.
5. **Synthetic utility and decision making:** After updating the sets of priority value, the conjunctive rule of combination can be applied, this leads us

Table 3. The adjusted Priority values

	$\emptyset$	$\{EN\}$	$\{FR\}$	$\{US, FR\}$	Θ		$\{EN, US\}$	Θ		$\{US\}$	Θ
$m_{C2,C3}^{\alpha_{C2,C3}}$	0.066	0.0718	0.0224	0.0088	0.831	$m_{C1}^{\alpha_{C1}}$	0.896	0.104	$m_{C4}^{\alpha_{C4}}$	0.458	0.542

to get a single bba denoted by $m_{final} = m_{C1}^{\alpha_{C1}} \bigcirc\!\!\!\!\!\cap\, m_{C2,C3}^{\alpha_{C2,C3}} \bigcirc\!\!\!\!\!\cap\, m_{C4}^{\alpha_{C4}}$.
Then, these obtained bba is transformed into pignistic probabilities, we get $BetP_{final}(EN) = 0.2911$, $BetP_{final}(US) = 0.6894$ and $BetP_{final}(FR) = 0.0195$. Consequently, USA is the recommended country since it has the highest values.

6 Conclusion

Our objective through this work is to develop a new MCDM method providing a formal way to handle uncertainty in AHP method within the belief function framework. Moreover, our approach has reduced the number of comparisons because instead of using single elements, we have used subsets. At the end, we have shown the flexibility and feasibility of our proposed approach by applying it on a real application problem related to "the end of life phase" of PVC product.

This work calls for several perspectives. One of them consists in comparing our proposed approach with other MCDM methods. In addition, the proposed method will be more flexible, if it will be able to handle uncertainty in the Saaty's scale.

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