Multicriteria Decision Making Based on Qualitative Assessments and Relational Belief

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Abstract. This paper investigates a multi-criteria decision making method in an uncertain environment, where the uncertainty is represented using the belief function framework. Indeed, we suggest a novel methodology that tackles the challenge of introducing uncertainty in both the criterion and the alternative levels. On the one hand and in order to judge the criteria weights, our proposed approach suggests to use preference relations to elicitate the decision maker assessments. Therefore, the Analytic Hierarchy Process with qualitative belief function framework is adopted to get adequate numeric representation. On the other hand, our model assumes that the evaluation of each alternative with respect to each criterion may be imperfect and it can be represented by a basic belief assignment. That is why, a new aggregation procedure that is able to rank alternatives is introduced.

1 Introduction

In real life decision making, the decision maker is faced with many situations in which he has to make a decision between different alternatives. However, the most preferable one is not always easily selected. Thus, he often needs to make judgments about alternatives that are evaluated on the basis of different criteria [16]. In this context, the problem is called a multi-criteria decision making (MCDM) problem.

Within this MCDM framework, a large number of methods has been proposed. On the one hand, the outranking approach introduced by Roy, where some methods like Electre and Promethee are developed [4]. On the other hand, the value and utility theory approaches mainly started by Keeney and Raiffa [8], and then implemented in a number of methods. However, classical methods applying both multi-attribute utility theory and outranking model do not take into account imperfection in their parameters. To cope with this problem, several approaches have been developed. The idea was to combine theories managing uncertainty or imprecision, such as probability theory, belief function theory and fuzzy set theory, with MCDM methods [2], [9].

In this context, belief function theory has shown its efficiency. In fact, there are several MCDM approaches which have been developed such as DS/AHP approach [2], belief Analytic Hierarchy Process (AHP) method [5], [7], etc.

In spite of many advantages of the presented approaches, they have a crucial limitation. First, they still treat criteria weights and alternatives performances in the same way. By nature, the criteria weights are relative to each other. Therefore, it is reasonable to elicit their importance by pair-wise comparison. However, the evaluation of an alternative on each criterion should be independent of each other. Second, the relative importance of criteria can be provided by the expert based on some previous experience or obtained by some elaborated approach. As a result, uncertain, imprecise and subjective data are usually present which make the decision-making process complex and challenging. Therefore, we consider MCDM problem for which expert estimates the evaluation of alternatives according to ordinal criteria. The information provided can be uncertain and/or imprecise.

Based on these reasons, we suggest a new methodology for MCDM problems based on belief function theory. In fact, our model integrates one of the most important weight calculation procedures with a new ranking process of alternatives in an uncertain environment. In the proposed methodology, AHP with its qualitative belief functions extension is applied to evaluate criteria and weighting them in the presence of uncertainty. Then, in the second step, a fusion procedure is proposed to aggregate the alternatives priorities and the criteria weights.

In this paper, section 2 and 3 describe an overview of the basic concepts of respectively the belief function theory and the qualitative belief function methods. Then, in the main body of the paper, we present our new approach namely belief MCDM method. Finally, our method will be illustrated by an example.

2 Belief Function Theory

2.1 Basic Concepts

Let Θ be the frame of discernment representing a finite set of elementary hypotheses related to a problem domain. We denote by 2^{Θ} the set of all the subsets of Θ [12].

The impact of a piece of evidence on the different subsets of the frame of discernment Θ is represented by the so-called basic belief assignment (bba), denoted by m [12]:

$$\sum_{A \subseteq \Theta} m(A) = 1. \tag{1}$$

For each $A \subseteq \Theta$, the value $m(A)$, named a basic belief mass (bbm), represents the portion of belief committed exactly to the event A. The events having positive bbm's are called focal elements. Let $\mathcal{F}(m) \subseteq 2^{\Theta}$ be the set of focal elements of the bba m.

Associated with m is the belief function (bel) is defined for $A \subseteq \Theta$ and $A \neq \emptyset$ as:

$$bel(A) = \sum_{\emptyset \neq B \subseteq A} m(B) \text{ and } bel(\emptyset) = 0. \tag{2}$$

The degree of belief $bel(A)$ given to a subset A of the frame Θ is defined as the sum of all the basic belief masses given to subsets that support A without supporting its negation.

2.2 Combination

In the Transferable Belief Model (TBM), one interpretation of the belief function theory [13], the basic belief assignments induced from distinct pieces of evidence can be combined using the conjunctive rule [13]:

$$(m_1 \textcircled{\scriptsize$\cap$} m_2)(A) = \sum_{B,C \subseteq \Theta, B \cap C = A} m_1(B) m_2(C), \quad \forall A \subseteq \Theta. \tag{3}$$

$m_1 \textcircled{\scriptsize$\cap$} m_2$ is the bba representing the combined impact of two pieces of evidence.

2.3 Discounting

The technique of discounting allows to take into consideration the reliability of the information source that generates the bba m. Let $\beta = 1 - \alpha$ be the degree of reliability ($\alpha \in [0, 1]$) assigned to a particular belief function. If the source is not fully reliable, the bba it generates is "discounted" into a new less informative bba denoted m^α:

$$m^\alpha(A) = (1 - \alpha) m(A), \forall A \subset \Theta \tag{4}$$

$$m^\alpha(\Theta) = \alpha + (1 - \alpha) m(\Theta) \tag{5}$$

2.4 Uncertainty Measures

In the case of the belief function framework, the bba is defined on an extension of the powerset: 2^Θ and not only on Θ. In the powerset, each element is not equivalent in terms of precision. Indeed, $\theta_i \subset \Theta$ ($i \in \{1, 2\}$) is more precise than $\theta_1 \cup \theta_2 \subseteq \Theta$. In order to try to quantify this imprecision, different uncertainty measures (UM) have been defined, such as the composite measures introduced by Pal et al. [10] such as:

$$H(m) = \sum_{A \in \mathcal{F}(m)} m(A) \log_2\left(\frac{|A|}{m(A)}\right). \tag{6}$$

The interesting feature of $H(m)$ is that it has a unique maximum.

2.5 Decision Making

The TBM considers that holding beliefs and making decision are distinct processes. Hence, it proposes a two level model: (1) The credal level where beliefs are entertained and represented by belief functions. (2) The pignistic level where beliefs are used to make decisions and represented by probability functions called the pignistic probabilities, denoted $BetP$ [14]:

$$BetP(A) = \sum_{B \subseteq \Theta} \frac{|A \cap B|}{|B|} \frac{m(B)}{(1 - m(\emptyset))}, \forall A \in \Theta \tag{7}$$

3 Qualitative Belief Function method

The problem of eliciting qualitatively expert opinions and generating basic belief assignments have been addressed by many researchers [1] [15] [6]. In this subsection, we present the approach of Ben Yaghlane et al. [1], since in next section we will use this method to elicitate the expert preferences. This method is chosen since it handles the issue of inconsistency in the pair-wise comparisons. Also, the originality of this method is its ability to generate quantitative information from qualitative preferences only.

Giving two alternatives, an expert can usually express which of the propositions is more likely to be true, thus they used two binary preference relations: the preference and the indifference relations, defined as follows:

$$a \succ b \Leftrightarrow bel(a) - bel(b) \geq \varepsilon \tag{8}$$

$$a \sim b \Leftrightarrow |bel(a) - bel(b)| \leq \varepsilon \tag{9}$$

ε is considered to be the smallest gap that the expert may discern between the degrees of belief in two propositions a and b. Note that ε is a constant specified by the expert before beginning the optimization process.

Then, a mono-objective technique was used to solve such constrained optimization problem:

$$\begin{array}{c} Max_m UM(m) \\ s.t. \\ bel(a) - bel(b) \geq \varepsilon \quad (a \text{ is prefered to } b) \\ bel(a) - bel(b) \leq \varepsilon \quad (a \text{ is indifferent to } b) \\ bel(a) - bel(b) \geq -\varepsilon \quad (a \text{ is indifferent to } b) \\ \sum_{a \in \mathcal{F}(m)} m(a) = 1, m(a) \geq 0, \forall a \subseteq \Theta; m(\emptyset) = 0, \end{array} \tag{10}$$

where the first, second and third constraints are derived from the previous equations. The last constraint ensures that the total amount of masses allocated to the focal elements of the bba is equal to one, also it specifies that masses are non negative and imposes that the bba to be generated must be normalized.

A crucial step that is needed before beginning the task of generating belief functions, is the identification of the candidate alternatives.

4 MCDM Method Based on Qualitative Assessments and Relational Belief

The new framework called Belief MCDM method mixes a multi-criteria decision making method inspired by the Analytic Hierarchy Process (AHP) and belief function theory. The originality of our approach is to apply the qualitative AHP to compute the importance of criteria and to replace the aggregation step by a new fusion process. Its main aim is to take into account both imprecision and uncertain assessments. In other words, the assumption was made that the performances of alternatives are provided on the form of bba while the weights of criteria are introduced using qualitative assessment and preference relations.

4.1 Assigning Criteria Weight Via Qualitative AHP.

In most multi-criteria methods, a numerical value is assigned to each criterion expressing its relative importance. This reflects the corresponding criterion weight. In fact, there are many elicitation techniques, but the AHP has some advantages. One of the most important advantage of the AHP attributes to its pair-wise comparison scheme.

In fact, the AHP method is a decision-making technique developed by Saaty [11]. This method elicits preferences through pair-wise comparisons which are constructed from decision maker's answers. Indeed, the expert can use both objective information about the elements as well as subjective opinions about the elements' relative meaning and importance. The responses to the pair-wise comparison question use a nine-point scale [11], which translates the preferences of a decision maker into numbers. An eigenvector method is applied to solve the reciprocal matrix for determining the criteria importance and alternative performance. The simple additive weighting method is used to calculate the utility for each alternative across all criteria.

However, standard AHP do not handle the problem of uncertainty. Therefore, in the proposed methodology, AHP with its belief extension, namely qualitative AHP, is applied to obtain more decisive judgments by prioritizing the evaluation criteria and weighting them in the presence of imperfection.

Step 1: Let Ω be a set of criteria where $\Omega = \{c_1, \ldots, c_m\}$. In this first stage, qualitative AHP computations are used for forming a pair-wise comparison matrix in order to determine the criteria weights using preferences relations only. Thus to express his preferences, the decision maker has only to express his opinions qualitatively, based on knowledge and experience that he provides in response to a given question rather than direct quantitative information. Therefore, he only selects the related linguistic variable using preference modeling instead of using a nine-point scale. It is illustrated in Table 1.

Table 1. Preferences relation matrix

	c_1	c_2	$\ldots$	c_m
c_1	-	P_{12}	$\ldots$	P_{1m}
c_2	-	-	$\ldots$	P_{2m}
$\ldots$	-	-	-	$\ldots$
c_m	-	-	-	-

In this table, P_{ij} is a preference assessment. It may be:

1. a strict preference relation $\succ$ iff $(c_i \succ c_j) \wedge \neg(c_j \succ c_i)$
2. an indifference relation $\sim$ iff $(c_i \succ c_j) \wedge (c_j \succ c_i)$
3. an unknown relation (no relation is given).

Under this approach, the expert is not constrained to quantify the degree of preferences and to fill all pair-wise comparisons matrix. He is able to express his preferences freely.

Step 2: Once the pair-wise comparison matrix is complete, our objective is then to compute the importance of each criterion. In fact, within our model, we propose to transform these preference relations into numerical values using the belief function framework. By adopting our approach, we try to closely imitate the expert reasoning without adding any additional information. Therefore, we suggest to apply Ben Yaghlane et al. approach [1] to convert the preferences relations into constraints of an optimization problem whose resolution, according to some uncertainty measures (UM) such as H (Equation 6), allows the generation of the least informative or the most uncertain belief functions. Indeed, we assume that the criterion weight is then described by a basic belief assignment and it is denoted by m^{Ω}. It can then be determined by the resolution of an optimization problem as defined is the previous section (Equation 10).

Furthermore, the proposed method addresses the problem of inconsistency. In fact, if the preference relations are consistent, then the optimization problem is feasible. Otherwise, no solutions will be found.

Finally, to obtain a relative importance of each criterion, we propose to transform the obtained bba m^{Ω} into pignistic probabilities:

$$BetP^{\Omega}(c_i) = \omega_i, \forall i = 1, \ldots, m \tag{11}$$

4.2 Aggregation of the Assessments with Respect to all Criteria for each Alternative.

Step 3: In our proposed method the evaluation of each alternative to each criterion is introduced as a basic belief assignment (bba). This comes from the fact that in most cases the input data cannot be defined within a reasonable degree of accuracy. In Table 2, a belief decision matrix representing all the alternatives' performances with respect to each criterion is introduced, where m_j^i represents the belief assessment of each alternative a_j with respect to each criterion c_i.

Table 2. Belief decision matrix

	Criteria			
Alternatives	c_1	c_2	...	c_m
a_1	m_1^1	m_1^2	...	m_1^m
...				
a_n				

Under this approach, we consider MCDM problem, where alternatives are evaluated with regard to ordinal criteria. Since it is generally thought that ordinal criteria are difficult to assess directly and the decision maker is required to give an accurate evaluation of the performances of all the alternatives on the given criteria, which is usually inaccurate, unreliable or even unavailable, especially in an uncertain environment. For the sake of simplicity, we assume that all criteria have the same assessment grades. In line with this assumption, many methods were defined such as the evidential reasoning approach and the AHP method. Let X be the set of assessment grades: $X = \{x_1, \ldots, x_h\}$.

To summarize, alternative a_j is assessed on each criterion c_i, using the same set of the ordinal assessment grades x_k which are required to be mutually exclusive and exhaustive. The ordinal assessment grades constitute our frame of discernment in the belief function theory.

Step 4: By using standard MCDM method, the performance matrix is obtained by multiplying the weighting vector by the decision matrix. Therefore, in this stage, we must update the alternative evaluations (bbas) with the importance of their respective criteria. In this context, our approach proposes to regard the criteria weight as a measure of reliability [5]. In fact, the idea is to measure most heavily the bba evaluated according to the most importance criteria and conversely for the less important ones. If we have c_i an evaluation criterion, then we get β_i its corresponding measure of reliability.

$$\beta_i = \frac{\omega_i}{\max_k \omega_k} \qquad \forall i, k = \{1, \ldots, m\} \tag{12}$$

Then, we get [5]:

$$m_j^{i,\alpha_i}(x_k) = \beta_i m_j^i(x_k), \ \forall x_k \subset X. \tag{13}$$

$$m_j^{i,\alpha_i}(X) = (1 - \beta_i) + \beta_i m_j^i(X). \tag{14}$$

where m_j^i is the relative bba for the alternative a_j, and we denote $\alpha_i = 1 - \beta_i$.

Step 5: Using Equation 13 and 14, an overall performance matrix is calculated. In this step, the main difficulty of our approach is how to combine and to compare different bbas. An intuitive definition of the strategy is to combine them using the conjunctive rule of combination, since we can assume that for each alternative performance is considered as a distinct source of evidence, and provides opinions towards the preferences of particular alternative. In addition, the obtained bba is defined on a common frame of discernment. So, the conjunctive rule may then be applied. This rule is used in order to aggregate the bbas induced by the expert for every alternative on all the criteria. The objective is to yield a combined bba that represents the performance of every alternative on the overall objective:

$$m_{a_j} = \textcircled{\cap} m_j^{i,\alpha_i}, \quad i = \{1, ..., m\} \tag{15}$$

Step 6: The results of the calculations are in the form of a bba. So each alternative is characterized by a single bba. As defined above, the main problem arises in comparing these bbas. In the present work, we consider first belief dominance (FBD) [3] in order to compare evaluations expressed by belief functions. By using this concept, we obtain a preference relation between each pair of alternatives. First, we start by computing the ascending belief function noted $\overrightarrow{bel}_i$ induced by m_{a_i} and associating to the evaluation of alternative a_i which is defined such as: $\overrightarrow{bel}_i(A_k) = \sum_{C \subseteq A_k} m_{a_i}(C) \quad$ for all $A_k \in \overrightarrow{S}(X)$, where k is the number of assessments grades, $A_k = \{x_1, \ldots, x_k\}$, and $\overrightarrow{S}(X)$ denote the set of $\{A_1, \ldots, A_l\}$.

Then, the descending belief function noted $\overleftarrow{bel}_i$ induced by m_{a_i} and associating to the evaluation of alternative a_i is defined such as: $\overleftarrow{bel}_i(B_k) = \sum_{C \subseteq B_k} m_{a_i}(C) \quad$ for all $B_k \in \overleftarrow{S}(X)$, where $B_k = \{x_k, \ldots, x_1\}$.

Thus, a bba m_{a_i} is said to dominate a bba m_{a_j} if and only if the following conditions are satisfied simultaneously:

- For all $A_k \in \overrightarrow{S}(X) \quad \overrightarrow{bel}_i(A_k) \leq \overrightarrow{bel}_j(A_k)$
- For all $B_k \in \overleftarrow{S}(X) \quad \overleftarrow{bel}_i(B_k) \geq \overrightarrow{bel}_j(B_k)$

The first condition means that there is greater belief mass of $\overrightarrow{bel}_j(A_k)$ than that of $\overrightarrow{bel}_i(A_k)$. On the contrary, the second condition means that there is greater belief mass of $\overleftarrow{bel}_i(B_k)$ than that of $\overleftarrow{bel}_j(B_k)$. In the case where the two conditions are not verified simultaneously, m_i does not dominate m_j ($\overline{FBD}$). Furthermore, it permits establishing four partial preference situations:

- if m_i FBD m_j and m_j FBD m_i, then m_i is indifferent from m_j.
- if m_i FBD m_j and m_j $\overline{FBD}$ m_i, then m_i is strictly preferred to m_j.
- if m_i $\overline{FBD}$ m_j and m_j FBD m_i, then m_j is strictly preferred to m_i.
- if m_i $\overline{FBD}$ m_j and m_j $\overline{FBD}$ m_i, then m_i and m_j are incomparable.

Once these relations are determined, the decision maker can then identify the subsets of best alternatives.

To summarize, Figure 1 shows the decision maker's process of the proposed approach.

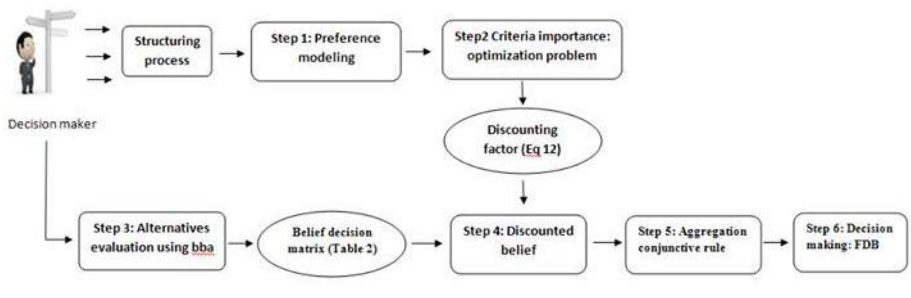

Fig. 1. Decision making process

5 Illustrative Example

In this section, we apply our proposed MCDM method to deal with a relatively simple decision problem of purchasing a car. The problem involves three criteria: $\Omega = \{\text{Comfort } (c_1), \text{ Style } (c_2), \text{ Fuel efficiency } (c_3)\}$, and three selected alternatives: $\Theta = \{\text{Peugeot } (a_1), \text{Renault } (a_2), \text{Ford } (a_3)\}$.

Step 1: The determination of the criteria importance: Along with our new MCDM method, a judgment matrix based on the pair-wise comparison process using preference modeling is defined in Table 3. As mentioned above, the decision maker was asked to indicate his level of preference between the selected criteria.

Table 3. Preference relation matrix for criterion level

Criteria	c_1	c_2	c_3
c_1	-	$\succ$	$\succ$
c_2	-	-	$\sim$
c_3	-	-	-

From Table 3, we remark that the decision maker has identified his preferences qualitatively. He identifies that $\{c_1\}$ is evaluated to be more important than $\{c_3\}$ and $\{c_1\}$ is evaluated to be more preferred than $\{c_2\}$.

Now, for deriving the weights of criteria, we apply our presented model. Therefore, we must transform these qualitative assessments into an optimization problem (Equation 10), then we solve the obtained system to compute the criteria importance. We assume that $\varepsilon = 0.01$ and the uncertainty measures is H since it has a unique maximum as defined in Equation 6. We obtain then the following optimization problem example:

$$\begin{aligned}
Max_m H(m) = -m(\{c_1\}) * log_2(1/m(\{c_1\})) - m(\{c_2\}) log_2(1/m(\{c_2\})) \\
-m(\{c_3\}) * log_2(1/m(\{c_3\})) - m(\Omega) * log_2(3/m(\Omega)); \\
s.t. \\
bel(\{c_1\}) - bel(\{c_2\}) \geq \varepsilon \\
bel(\{c_1\}) - bel(\{c_3\}) \geq \varepsilon \\
bel(\{c_2\}) - bel(\{c_3\}) \leq \varepsilon \\
bel(\{c_2\}) - bel(\{c_3\}) \geq -\varepsilon \\
\sum_{c_i \in \mathcal{F}(m)} m(c_i) = 1, m(A) \geq 0, \forall A \subseteq \Omega; m(\emptyset) = 0,
\end{aligned} \tag{16}$$

Finally, the obtained results (weighting vector) are represented in Table 4.

Step 2: The aggregation of the assessments with respect to all criteria for each alternative: In order to evaluate each alternative with respect to each criterion, our first step is to choose the set of evaluation grades as:

Table 4. The weights assigned to the subset of criteria

Criteria	c_1	c_2	c_3	Ω
m^{Ω}	0.238	0.208	0.208	0.346
$BetP^{\Omega}$	0.352	0.324	0.324	
β_i	1	0.92	0.92	

$$X = \{\text{poor, indifferent, average, good, excellent}\}.$$

In fact, the same set of evaluation grades is used for the three qualitative criteria. We propose to consider three alternatives. The evaluation of each alternative with respect to each criterion is given by a bba (see Table 5). For instance, to evaluate the alternative a_1, the expert hesitates between the fourth and the fifth assessment grades. He is sure that a_1 is either good or excellent without being able to refine his judgment.

Table 5. The bbas assigned to the alternatives performances.

Criteria	c_1	c_2	c_3
a_1	$m_1^1(\{x_4\}) = 0.3$ $m_1^1(\{x_4, x_5\}) = 0.7$	$m_1^2(\{x_1\}) = 0.4$ $m_1^2(X)$=0.6	$m_1^3(\{x_4\}) = 1$
a_2	$m_2^1(\{x_2, x_3\}) = 0.6$ $m_2^1(\{x_3, x_4\}) = 0.4$	$m_2^2(\{x_5\}) = 0.7$ $m_2^2(X) = 0.3$	$m_2^3(\{x_4, x_3\}) = 0.4$ $m_2^3(\{x_3\}) = 0.3$ $m_2^3(X) = 0.3$
a_3	$m_3^1(\{x_3\}) = 1$	$m_3^2(\{x_1, x_2\}) = 0.2$ $m_3^2(\{x_2, x_3\}) = 0.8$	$m_3^3(\{x_4, x_5\}) = 0.5$ $m_3^3(X) = 0.5$

By applying equation 13 and 14 using the discounting technique, the overall performance matrix was calculated and the decision matrix was determined in Table 6.

Table 6. The bbas assigned to the alternatives performances after discounting.

Criteria	c_1	c_2	c_3
a_1	$m_1^{\alpha 1}(\{x_4\}) = 0.3$ $m_1^{\alpha 1}(\{x_4, x_5\}) = 0.7$	$m_1^{\alpha 2}(\{x_1\}) = 0.368$ $m_1^{\alpha 2}(X)$=0.632	$m_1^{\alpha 3}(\{x_4\}) = 0.92$ $m_1^{\alpha 3}(X) = 0.08$
a_2	$m_2^{\alpha 1}(\{x_2, x_3\}) = 0.6$ $m_2^{\alpha 1}(\{x_3, x_4\}) = 0.4$	$m_2^{\alpha 2}(\{x_5\}) = 0.644$ $m_2^{\alpha 2}(X) = 0.356$	$m_2^{\alpha 3}(\{x_4, x_3\}) = 0.368$ $m_2^{\alpha 3}(\{x_3\}) = 0.276$ $m_2^{\alpha 3}(X) = 0.356$
a_3	$m_3^{\alpha 1}(\{x_3\}) = 1$	$m_3^{\alpha 2}(\{x_1, x_2\}) = 0.184$ $m_3^{\alpha 2}(\{x_2, x_3\}) = 0.736$ $m_3^{\alpha 2}(X) = 0.08$	$m_3^{\alpha 3}(\{x_4, x_5\}) = 0.46$ $m_3^{\alpha 3}(X) = 0.54$

In order to determine the overall performance of each alternative, the conjunctive rule of combination is used to combine the obtained bbas. Therefore, for each alternative, we propose to combine its corresponding bbas. Table 7 shows the results.

Table 7. The combined bbas

	$\emptyset$	$\{x_4\}$	$\{x_4, x_5\}$		
m_{a_1}	0.368	0.5966	0.0354		
	$\emptyset$	$\{x_3\}$	$\{x_2, x_3\}$	$\{x_3, x_4\}$	
m_{a_2}	0.6441	0.1769	0.076	0.103	
	$\emptyset$	$\{x_1, x_2\}$	$\{x_2, x_3\}$	$\{x_4, x_5\}$	X
m_{a_3}	0.5126	0.01	0.3974	0.0368	0.0432

Now and after getting a single bba for each alternative, we suggest to apply the FBD concept to compare and to rank the obtained bbas. Therefore, the ascending belief function and the descending belief function are calculated.

For instance, to calculate the ascending belief function and the descending belief function for the alternative a_1, we get:

$$a_1 = \begin{cases} \overrightarrow{bel}_1^1(\{x_1\}) = 0 \\ \overrightarrow{bel}_1^1(\{x_1, x_2\}) = 0 \\ \overrightarrow{bel}_1^1(\{x_1, x_2, x_3\}) = 0 \\ \overrightarrow{bel}_1^1(\{x_1, x_2, x_3, x_4\}) = 0.5966 \\ \overrightarrow{bel}_1^1(\{x_1, x_2, x_3, x_4, x_5\}) = 1 \end{cases} \quad a_1 = \begin{cases} \overleftarrow{bel}_1^1(\{x_5\}) = 0 \\ \overleftarrow{bel}_1^1(\{x_4, x_5\}) = 0.632 \\ \overleftarrow{bel}_1^1(\{x_3, x_4, x_5\}) = 0.632 \\ \overleftarrow{bel}_1^1(\{x_2, x_3, x_4, x_5\}) = 0.632 \\ \overleftarrow{bel}_1^1(\{x_1, x_2, x_3, x_4, x_5\}) = 1 \end{cases}$$

Similarly, the FDB is computed for the alternatives a_2 and a_3, and finally, the preference situations between the alternatives are established. The results are given in the Table 8.

Table 8. Observed belief dominances between the alternatives

	a_1	a_2	a_3
a_1	$-$	FBD	$\overline{FBD}$
a_2	$\overline{FBD}$	$-$	$\overline{FBD}$
a_3	$\overline{FBD}$	$\overline{FBD}$	$-$

From this table, we have the alternative a_2 is outranked by alternative a_1, so it cannot be chosen. Moreover, alternative a_3 is incomparable to a_1. Then, a_1 and a_3 are the set of best alternatives according to our expert.

6 Conclusion

In this paper, a new MCDM approach in an uncertain environment was developed. Our proposed method, named belief MCDM, is based on the belief function framework. Indeed, to compute the weight of criteria, we apply a qualitative AHP approach then a new aggregating procedure is applied in order to update the alternatives performances. The approach developed is simple and comprehensible in concept, efficient in modeling human evaluation processes which makes it of general use for solving practical qualitative MCDM problems.

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