Analytical Network Process Method Under the Belief Function Framework

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Abstract. This paper describes a belief extension of the analytic network process (ANP), a multi-criteria prioritization method to model decision making under uncertain context. The approach accommodates the use of qualitative preference relations as input information in the pairwise comparison matrices. Instead of applying the Saaty?s scale in the prioritization process, a new method, based on the belief function theory, is applied. The proposed approach is illustrated by examples.

1 Introduction

The Analytic Hierarchy Process (AHP) [13, 14] is one of the most widely used multi-criteria decision making method. This approach assumes that the decision making problem can be structured hierarchically, where each element is independent from all the others.

However, in some situations, strong dependencies between inter-level or intra-level elements may exist. To solve this problem, a supermatrix approach [15] was proposed by Saaty, named the Analytic Network Process (ANP). Its main aim is to extend the AHP method to model interactions and feedback.

Like AHP method, the pairwise comparison technique is used to compute ANP method priorities. Each element is then paired and compared using the Saaty's scale. However, due to the uncertainty on the expert assessments, the crisp pairwise comparison technique in the standard AHP and ANP approaches seems insufficient and imprecise to capture the right assessments of decision makers.

As a way to handle this uncertainty, use of fuzzy set theory has been largely suggested in the literature. Accordingly, several fuzzy ANP methods [1, 9, 10] and fuzzy AHP [8] have been introduced.

In the same way, under the belief function framework, the AHP method has been extended to handle imperfection. Many AHP extensions were introduced [3, 4, 7]. Therefore, we propose in this research to combine the ANP method and the belief function theory. A new multicriteria decision making technique that is able to represent decision making under uncertain context.

This paper is aimed at presenting a Qualitative ANP approach. The proposed method suggests the use of belief preference relations in the supermatrix [4, 5]. We propose to derive meaningful priorities from qualitative decision structures. Instead of applying the Eigenvector method in the prioritization process, a new model, which obtains crisp priorities from qualitative assessments is applied.

In what follows, we first present some definitions needed for belief function theory. Next, we describe the ANP method in section 3. Then, section 4 details our new multicriteria method, and gives an example to show its application. Finally, section 5 concludes the paper.

2 Belief Function Theory

In this section, we briefly introduce the belief function theory as interpreted by the Transferable Belief Model (TBM). More Details can be found in [16–18].

2.1 Basic concepts

Let Θ be the frame of discernment representing a finite set of elementary hypotheses related to a problem domain, where 2^{Θ} is the set of all the subsets of Θ [16].

The belief assignment (bba), denoted by m, represent the impact of a piece of evidence on the different subsets of Θ.

$$\sum_{A \subseteq \Theta} m(A) = 1. \tag{1}$$

The basic belief mass (bbm), denoted $m(A)$, models the part of belief committed exactly to A. The events having positive bbm's are called focal elements, where $\mathcal{F}(m) \subseteq 2^{\Theta}$ is the set of all focal elements of the bba m.

Accordantly, a belief function is defined for $A \subseteq \Theta$ and $A \neq \emptyset$ as:

$$bel(A) = \sum_{\emptyset \neq B \subseteq A} m(B) \text{ and } bel(\emptyset) = 0. \tag{2}$$

2.2 Decision making

The TBM proposes two level models. First, the credal level where beliefs are entertained and represented by belief functions. Second, the pignistic level where beliefs are used to make decisions and represented by the so called pignistic probabilities, $BetP$ [17]:

$$BetP(A) = \sum_{B \subseteq \Theta} \frac{|A \cap B|}{|B|} \frac{m(B)}{(1 - m(\emptyset))}, \forall A \in \Theta. \tag{3}$$

2.3 Uncertainty Measures

In the case of the belief function framework, different uncertainty measures (UM) have been defined, such as [11] [12]:

$$H(m) = \sum_{A \in \mathcal{F}(m)} m(A) \log_2(\frac{|A|}{m(A)}). \quad (4)$$

The measure H is aimed at assessing the total uncertainty arising in a body of evidence due to both randomness (ignorance and inconsistency) and nonspecificity associated with a bba.

The measure H attains its global maximum when the bba distributes both randomness and nonspecificity uniformly over the largest possible set of focal elements.

3 Analytical Network Process method

One of the most known multi-criteria decision making approach is the Analytical Network Process (ANP). The originality of ANP lies in its ability to represent complex decision making problems and to involve dependencies and feedbacks between decision elements.

Unlike AHP, ANP makes no assumptions about the independence of higher level elements from lower level elements and about the independence of the elements within a level (see Figure 1). An AHP hierarchy is based on a main objective, a selected criteria, sub-criteria and decision alternatives, but a network model has cycles connecting its clusters of elements (outer dependence) and loops that connect a cluster to itself (inner dependence).

The main steps of ANP are described in what follows [15].

3.1 Construction of the network structure

The ANP process starts by constructing the group of elements and clusters that would best model the problem. Indeed, the elements in terms of criteria, sub-criteria and alternatives are defined and clusters of these elements are respectively formed. So, a network is developed based on the relationships between and within these clusters.

3.2 Building pairwise comparaison

Like AHP, the pairwise comparison process is used to model the expert preferences and to estimate the local priorities of the selected criteria, sub-criteria and alternatives.

To represent his assessments, the decision maker has to respond to the following question: Given an element (in the same cluster or in another cluster) or a cluster, how much more does a given element (cluster) of a pair influence that element (cluster) with respect to a criterion?

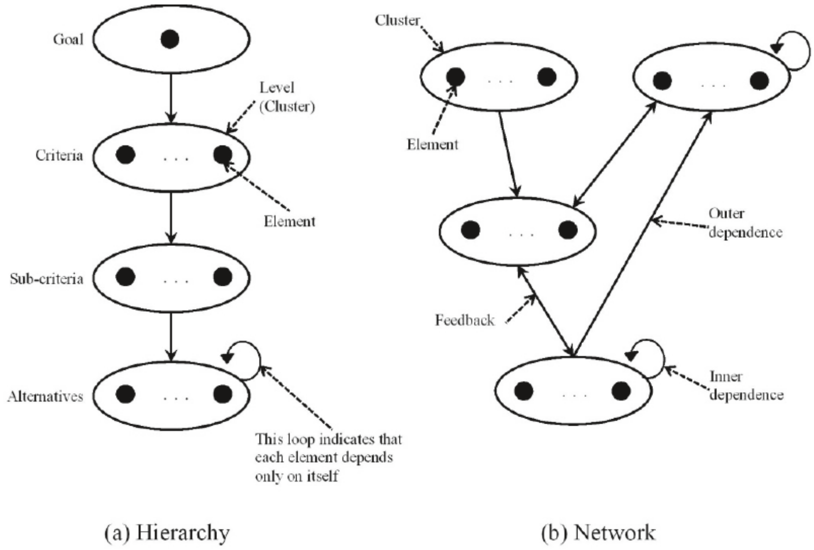

Fig. 1. The difference between a hierarchy and a network models [15]

The responses to these questions, the expert use the Saaty's scale, where 1 indicates indifference between the two elements and 9 indicates a strong preference of the element under consideration over the comparison element (see Table 1).

Table 1. The Saaty rating scale

Saaty's scale	Definition
1	Equal importance
3	Somewhat more important
5	Much more important
7	Very much more important
9	Absolutely more important
2,4,6,8	Intermediate values

Once the pairwise comparisons are completed, a local priority vector is derived for all comparison matrices.

To validate the expert judgments', a consistency index is calculated to check the consistency of the pairwise comparisons matrices.

3.3 Construction the supermatrix

When all the local priorities are calculated, these are used to form the supermatrix. In this method, each local priority vectors is entered as a part of some column of a matrix (see Figure 2),

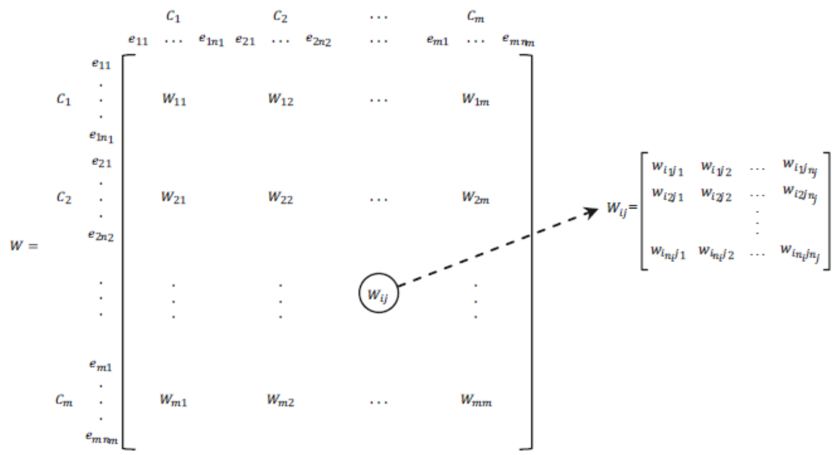

Fig. 2. An example of one of the supermatrix block matrices [15]

where C_m denotes the m^{th} cluster with n_m elements $(e_{m1}, e_{m2}, ..., e_{mn}$, where e_{mn} represents the n^{th} element in the m^{th} cluster).

W_{ij} is a priority vector representing the impact of the elements in the i_{th} cluster on the elements in the j_{th} cluster ($W_{ij} = 0$ if there is no influence between the clusters). Figure 3 illustrates a sample example [20].

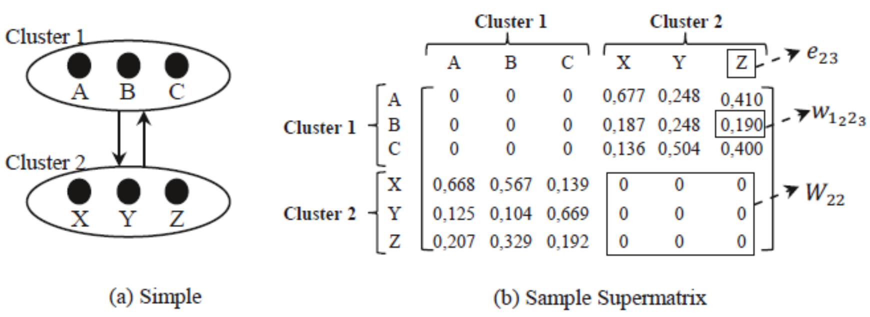

Fig. 3. An illustrative example of a network

The next step, according to Saaty methodology, is to construct the weighted supermatrix using the component matrix to weight the supermatrix previously assembled.

To calculate the weighted supermatrix is a need to make the matrix stochastic. This condition is necessary for obtaining the resulting limit supermatrix. This matrix is calculated elevating the weighted supermatrix to the n^{th} power. This procedure allows capturing the transmission of influences from all the paths of the network. For example, to obtain indirect influences through a third element, the weighted supermatrix must be powered to the square.

3.4 Ranking alternatives

The global priorities can be found in the relevant rows of the normalized limit supermatrix and those elements receiving high priorities deserve more attention.

4 A qualitative ANP approach

ANP is a well-known MCDM method. that has some drawbacks. Like AHP method, ANP uses a predefined scale. This one cannot handle the problem of uncertainty in the evaluation process. Therefore, the expert needs more than $1 - 9$ scale to express this imperfection.

As already noted, many ANP extensions were proposed such as Fuzzy ANP. It utilizes interval and fuzzy prioritization methods to represent the pair-wise comparisons matrix and to derive interval or fuzzy local priorities [1, 9, 10].

However, our proposed method, which derives belief priorities from qualitative assessments, can be easily applied to increase the capabilities of the ANP for dealing with inconsistent and uncertain judgments.

In the following, we present a new way of comparisons under the ANP approach and we introduce our suggested solution.

4.1 Step 1: Network model

Select and define the hierarchy or network structure including clusters, the candidate criteria, sub-criteria and the selected alternatives. Detailed discussions on every criterion, sub-criterion and alternative have been conducted. These data must be carefully collected to assure the reliably of the model.

4.2 Step 2: Dependency and feedback

Identify the dependences among all components of the network and list them in a table in order to define the impacts between each.

4.3 Step 3: Pairwise comparison matrices

Construct pairwise comparison matrices of the elements with preference relations.

Like standard ANP, the comparison of elements connected to others follows the same principal and method as in AHP. So, each two elements are compared in terms of dominance with respect to a common aspect. Therefore, instead of using Saaty's scale, the responses to the pairwise comparison questions are scaled on the basis of a flexible way using binary preference relations [5].

For example, to compare criteria to the main objective, the corresponding matrix is shown in Table 2 [7].

Table 2. An example of the pairwise comparison matrix

	e_{11}	e_{12}	$\dots$	e_{mn}
e_{11}	-	P_{12}	$\dots$	P_{1n}
e_{12}	-	-	$\dots$	P_{2n}
$\dots$	-	-	-	$\dots$
e_{mn}	-	-	-	-

In this Table, P_{ij} is a binary relation. It may be:

1. a strong preference relation $\succ$
2. a weak preference relation $\succeq$
3. an indifference relation $\sim$
4. an unknown relation.

4.4 Step 4: Local priorities

Derive relative importance weights (local priorities) from the constructed pairwise comparison matrix using the belief prioritizing method.

Transforming belief functions from qualitative assessments and generating quantitative beliefs have been handled by many works [19, 2, 6]. In this study, we consider approaches presented in [2] and [6].

As we have mentioned, to express his assessments, an expert may use the strict preference ($\succ$) or the indifference ($\sim$) or the weak-preference ($\succeq$) or unknown relations.

Based on [6], these relations are transformed into constraints as follows:

$$e_{11} \succ e_{12} \Leftrightarrow bel(e_{11}) - bel(e_{12}) \geq \gamma \tag{5}$$

$$e_{11} \succeq e_{12} \Leftrightarrow \varepsilon \leq bel(e_{11}) - bel(e_{12}) \leq \gamma \tag{6}$$

$$e_{11} \sim e_{12} \Leftrightarrow |bel(e_{11}) - bel(e_{12})| \leq \varepsilon \tag{7}$$

where ε is the smallest gap between two degrees of belief. Its value is a constant defined by the decision maker. Similarly to the preference index ε, γ is the indifference threshold.

After modeling the different preference relations, we introduce the preference relations as an optimization problem whose resolution, according to some uncertainty measures (nonspecificity measures, conflict measures, composite measures), allows the generation of the least informative or the most uncertain belief functions. Therefore, each preference relation is transformed into its corresponding constraint as follows.

$$
\begin{array}{c}
Max_m UM(m) \\
s.t. \\
bel(e_{11}) - bel(e_{12}) \geq \gamma \quad \forall (e_{11}, e_{12}) \text{ for which } \; e_{11} \succ e_{12} \\
bel(e_{11}) - bel(e_{12}) \leq \gamma \quad \forall (e_{11}, b) \text{ for which } \; e_{11} \succeq e_{12} \\
bel(e_{11}) - bel(e_{12}) \geq \varepsilon \quad \forall (e_{11}, e_{12}) \text{ for which } \; e_{11} \succeq e_{12} \\
bel(e_{11}) - bel(e_{12}) \leq \varepsilon \quad \forall (e_{11}, e_{12}) \text{ for which } \; e_{11} \sim e_{12} \\
bel(e_{11}) - bel(e_{12}) \geq -\varepsilon \quad \forall (e_{11}, e_{12}) \text{ for which } \; e_{11} \sim e_{12} \\
\sum_{e_{1i} \in \mathcal{F}(m)} m(e_{1i}) = 1; m(a) \geq 0; \forall e_{1i} \subseteq \Theta; m(\emptyset) = 0.
\end{array}
\tag{8}
$$

where UM is a measure of uncertainty. In this case, we take the measure of uncertainty H as defined equation 4.

The first constraint models the strict preference. The second and third ones represent the weak preference relation. The fourth and fifth constraints are derived from the indifference relation. ε and γ are a constant specified by the decision maker.

The choice of ε and γ affects whether a binary relationship holds. While selecting an appropriate value is not an easy task, because in most cases there are good reasons for choosing non-zero.

Consequently, the obtained *bba* provide an estimate of the local priorities for the decision elements being compared. Then, the obtained vector has to be transformed into pignistic probabilities (see Equation 3).

4.5 Step 5: Consistency

The quality of the estimation of local priorities highly depends on the consistency of judgments that the decision makers performed throughout the pairwise comparisons. At this level, expert has to evaluate the obtained bba. Thus, the proposed method addresses this problem. In fact, if the preference relations are consistent, then the optimization problem is feasible. Otherwise no solutions will be found. Thus, the expert may be guided to reformulate his assessments.

4.6 Step 6: Build Supermatrix

Construct the supermatrix with the obtained priorities in order to form an unweighted supermatrix. In this step, each pignistic probabilities (local priority vectors) computed in the step 4 is entered as a part of a relevant column of the supermatrix.

Then, we follow the same steps as standard ANP. We have to normalize the supermatrix to column stochastic to get the sum of the elements in each column is equal to one.

After that, we limit the weighted supermatrix by raising it to a sufficiently large power (where is an arbitrarily large number) until it converges into a stable supermatrix.

Finally, we aggregate the weights of criteria and the scores of alternatives into final priorities by multiplying the scores by the weights of the control criteria.

5 Conclusion

This paper presents a new method which combines both the AHP approach and the belief function theory to deal with complex decision making problems. Our solution proposes to handle uncertain pairwise comparison judgments. Thus, using preference relations in all pair-wise comparison matrices has facilitated the elicitation process. In fact, our approach suggests an easier elicitation of expert assessments through the reduction of necessary information and using qualitative information rather than exact numbers. At the same time, our approach handles uncertainty by adopting each obtained priority as a basic belief assignment.

As a further work, we propose to consider uncertainty in the supermatrix calculations to represent the uncertainty associated with the cumulative influence of each element on every other element with which it interacts in the network.

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