On Modelling and Solving the Shortest Path Problem with Evidential Weights

Tuan-Anh Vu[1], Sohaib Afifi[1], Éric Lefèvre[1], and Frédéric Pichon[1]

Univ. Artois, UR 3926, Laboratoire de Genie Informatique et d'Automatique de l'Artois (LGI2A), F-62400 Bethune, France
{tanh.vu, firstname.lastname}@univ-artois.fr

Abstract. We study the single source single destination shortest path problem in a graph where information about arc weights is modelled by a belief function. We consider three common criteria to compare paths with respect to their weights in this setting: generalized Hurwicz, strong dominance and weak dominance. We show that in the particular case where the focal sets of the belief function are Cartesian products of intervals, finding best, *i.e.*, non-dominated, paths according to these criteria amounts to solving known variants of the deterministic shortest path problem, for which exact resolution algorithms exist.

Keywords: Shortest path · Belief function · Exact method.

1 Introduction

The Shortest Path Problem (SPP) is one of the most studied problems in combinatorial optimization with a wide range of applications in, *e.g.*, transportation and telecommunications. In many realistic situations, uncertainty on arc weights is encountered; for instance, the travel times between cities can be affected by external factors such as weather conditions or traffic jams. Many approaches have been proposed to model the uncertainty on arc weights. In particular, robust optimization frameworks have represented uncertainty by discrete scenario sets [15,3] and by intervals [3,10].

In this paper, we investigate the case where the uncertainty on arc weights is *evidential*, *i.e.*, modelled by a belief function [12]. More specifically, we assume that each focal set of the considered belief function is a Cartesian product of intervals with each interval describing possible values of each arc weight. Such a belief function is a direct and natural generalization of the above-mentioned interval-based uncertainty representation, which arises when considering probabilities that the intervals hold. It can be illustrated as follows: in a network with three cities A, B, and C, under good weather conditions, it may take 20 to 30 minutes to travel from A to B, and 10 to 20 minutes to travel from B to C; however under bad weather conditions, the travel times from A to B (resp. B to C) takes 30 to 40 minutes (resp. 15 to 25 minutes) and the forecast tells us that the probability of good weather (resp. bad weather) is 0.8 (resp. 0.2).

In the presence of evidential uncertainty on arc weights, the notion of best, *i.e.*, shortest, paths becomes ill-defined. In a similar vein as [3] and using decision theory under evidential uncertainty [2], best paths are defined in this paper as the non-dominated ones with respect to a preference relation over paths, built from some criterion relying on the notions of upper and lower expected weights of paths. We consider in particular three common criteria, called generalized Hurwicz, strong dominance and weak dominance; the first one induces a complete preference relation while the latter two induce partial relations leading, as will be seen, to more challenging optimisation problems.

Combinatorial optimization problems under evidential uncertainty have received some attention recently. Notably, in [8,14], the authors studied different variants of the Vehicle Routing Problem (VRP) with different uncertainty factors and with similar particular focal sets as in this paper. They proposed approximate resolution methods based on metaheuristics to find non-dominated solutions with respect to a complete relation built from a particular case of the generalized Hurwicz criterion. Guillaume *et al.* [6] studied a general optimization problem in which the coefficients in the objective function are subject to uncertainty. They considered also the generalized Hurwicz criterion and provided results about the complexity of finding a non-dominated solution.

In contrast to [8,14], we provide in this paper *exact* methods to find non-dominated solutions with respect to both complete *and partial* relations, owing to the fact that the SPP is much simpler than the VRP. Furthermore, although Guillaume *et al.* [6] showed that in general it is intractable to find best solutions, our results indicate that it can nonetheless be done when focal sets are of a particular kind. Finally, we may note that the particular optimization problems that we consider allow us to take advantage of specialized (SPP-related) algorithms, in contrast to [11] which also provides means to find best elements according to some criteria, such as strong dominance, but which cannot benefit from such specialized algorithms as it is framed in a more general setting.

The rest of this paper is organized as follows. Section 2 presents necessary background material. Section 3 is devoted to the formalization and resolution of the SPP with evidential weights. The paper ends with a conclusion in Section 4.

2 Preliminaries

In this section, we present basic elements necessary for the rest of the paper.

2.1 Deterministic shortest path problem

Let $G = (V, A)$ be a directed graph with set of vertices V, set of arcs A and weight $c_{ij} > 0$ for each arc (i, j) in A. Let s and t be two vertices in V called the source and the destination, respectively. Let $\mathcal{X}$ be the set of all s-t paths in G with the assumption that $\mathcal{X} \neq \emptyset$. If all arc weights c_{ij} are known then finding a s-t shortest path, *i.e.*, a s-t path of lowest weight, can be modelled as the following optimization problem

$$\min \sum_{(i,j)\in A} c_{ij} p_{ij} \tag{1}$$

$$\sum_{(s,i)\in A} p_{si} - \sum_{(j,s)\in A} p_{js} = 1 \tag{2}$$

$$\sum_{(t,i)\in A} p_{ti} - \sum_{(j,t)\in A} p_{jt} = -1 \tag{3}$$

$$\sum_{(k,i)\in A} p_{ki} - \sum_{(j,k)\in A} p_{jk} = 0, \quad \forall k \in V \backslash \{s,t\} \tag{4}$$

$$p_{ij} \in \{0,1\}, \quad \forall (i,j) \in A \tag{5}$$

where each path in $\mathcal{X}$ is identified with a set $p = \{p_{ij} | (i,j) \in A\}$ of which element $p_{ij} = 1$ if arc (i,j) is in the path and $p_{ij} = 0$ otherwise.

Example 1. Considering the directed graph depicted in Figure 1, the set of all s-t paths is $\mathcal{X} = \{s\text{-}a\text{-}t, s\text{-}b\text{-}t, s\text{-}t\}$ and s-a-t is the shortest s-t path with weight 2.

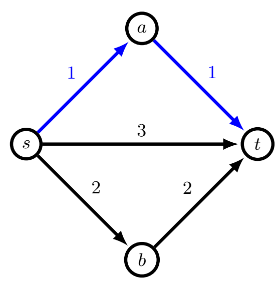


Fig. 1. Shortest path s-a-t between vertices s and t.

2.2 Belief function theory

Let $\Theta = \{\theta_1, \ldots, \theta_n\}$ be the set, called frame of discernment, of all possible values of a variable θ. In belief function theory [12], partial knowledge about the true (unknown) value of θ is represented by a mapping $m : 2^\Theta \mapsto [0,1]$ called mass function and such that $\sum_{A\subseteq\Theta} m(A) = 1$ and $m(\emptyset) = 0$, where mass $m(A)$ quantifies the amount of belief allocated to the fact of knowing only that $\theta \in A$. A subset $A \subseteq \Theta$ is called a focal set of m if $m(A) > 0$.

Assume θ represents the state of nature and its true value is known in the form of some mass function m. Assume further that a decision maker (DM) needs to choose an act (decision) f from a finite set $\mathcal{Q}$, where each act $f \in \mathcal{Q}$ induces a cost $l(f, \theta_i)$ for each possible state of nature $\theta_i \in \Theta$. In this context, the DM's preference over acts is denoted by $\preceq$, where $f \preceq g$ means that act f is preferred to act g. Relation $\preceq$ is complete if for any two acts f and g, $f \preceq g$ or $g \preceq f$, otherwise, it is partial. Furthermore, f is strictly (resp. equally) preferred to g, which is denoted by $f \prec g$ (resp. $f \sim g$), if $f \preceq g$ but not $g \preceq f$ (resp. if $f \preceq g$ and $g \preceq f$).

Typically, the DM seeks elements in the set Opt of non-dominated acts:

$$Opt = \{f \in \mathcal{Q} : \nexists g \text{ such that } g \prec f\}. \tag{6}$$

If relation $\preceq$ is complete, finding one element in Opt is enough since elements in Opt are preferred equally between each other and strictly preferred to the rest $\mathcal{Q} \setminus Opt$. In this case, elements in Opt are also called optimal acts. On the other hand, if relation $\preceq$ is partial, the DM may need to identify all elements in Opt.

Usually, the DM constructs his preference over acts based on some criterion. We denote by $\preceq_{cr}$ his preference according to some criterion cr and by Opt_{cr} its associated set of non-dominated (or best) acts. In this paper, we consider three common criteria defined as follows for any two acts f and g [2]:

1. Generalized Hurwicz criterion: $f \preceq_{hu} g$ if

$$\alpha \overline{E}_m(f) + (1-\alpha)\underline{E}_m(f) \leq \alpha \overline{E}_m(g) + (1-\alpha)\underline{E}_m(g) \tag{7}$$

   for some fixed parameter $\alpha \in [0,1]$, and where $\overline{E}_m(f)$ and $\underline{E}_m(f)$ denote, respectively, the upper and lower expected costs of act f with respect to mass function m defined as

$$\overline{E}_m(f) = \sum_{A \subseteq \Theta} m(A) \max_{\theta_i \in A} l(f, \theta_i), \tag{8}$$

$$\underline{E}_m(f) = \sum_{A \subseteq \Theta} m(A) \min_{\theta_i \in A} l(f, \theta_i). \tag{9}$$

   Relation $\preceq_{hu}$ is complete and we have $f \prec_{hu} g$ if (7) is strict.
2. Strong dominance criterion: $f \preceq_{str} g$ if

$$\overline{E}_m(f) \leq \underline{E}_m(g). \tag{10}$$

   Relation $\preceq_{str}$ is partial and we have $f \prec_{str} g$ if (10) is strict.
3. Weak dominance criterion: $f \preceq_{weak} g$ if

$$\overline{E}_m(f) \leq \overline{E}_m(g) \text{ and } \underline{E}_m(f) \leq \underline{E}_m(g). \tag{11}$$

   Relation $\preceq_{weak}$ is partial and we have $f \prec_{weak} g$ if at least one inequality in (11) is strict.

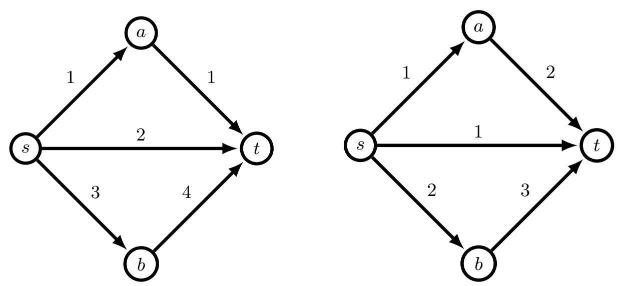
3 Shortest path problem with evidential weights

In this section, we formalize what we mean by best paths in a graph with evidential weights and provide methods for finding such paths.

3.1 Modelling

Let us assume that the arc weights c_{ij}, for all $(i,j) \in A$, of the graph introduced in Section 2.1 are only partially known. More specifically, we consider the case where information about arc weights is modelled by a mass function. Formally, let Ω_{ij} denote the frame of discernment for the variable c_{ij}, *i.e.*, the set of possible values for the weight c_{ij}. We assume that $\Omega_{ij} \subset \mathbb{N}_{>0}$. Let $\Omega := \times_{(i,j)\in A}\Omega_{ij}$. Any $\boldsymbol{c} \in \Omega$ will be called a scenario: it represents a possible assignment of values for all the weights in the graph. A mass function m on Ω, with set of focals sets denoted by $\mathcal{F} = \{F_1, \ldots, F_K\}$, represents then uncertainty about arc weights.

Example 2. Let $\boldsymbol{c^1}$ and $\boldsymbol{c^2}$ be the two scenarios represented by Figures 2a and 2b, respectively. The mass function m such that $m(F_1) = 0.4$ and $m(F_2) = 0.6$, with $F_1 = \{\boldsymbol{c^1}, \boldsymbol{c^2}\}$ and $F_2 = \{\boldsymbol{c^1}\}$, represents partial knowledge about arc weights.



(a) Scenario $\boldsymbol{c^1}$ (b) Scenario $\boldsymbol{c^2}$

Fig. 2. Two possible assignments of values, *i.e.,* two scenarios, for the arc weights.

As will be seen, making a particular assumption about the nature of the focal sets of m is useful. This assumption, denoted CI for short, is the following: each focal set of m can be expressed as a Cartesian product of intervals, *i.e.*, $F_r = \times_{(i,j)\in A}[l_{ij}^r, u_{ij}^r]$ for all $r \in \{1,\ldots,K\}$. Such a focal set is illustrated by Example 3.

Example 3. Let F be the Cartesian product of intervals (depicted by Figure 3):

$$\begin{aligned} F &= [l_{sa}, u_{sa}] \times [l_{sb}, u_{sb}] \times [l_{st}, u_{st}] \times [l_{at}, u_{at}] \times [l_{bt}, u_{bt}] \\ &= [1,5] \times [2,4] \times [2,4] \times [1,3] \times [2,5]. \end{aligned}$$

F is a subset of Ω: it includes, for instance, the scenario $\boldsymbol{c} = \{c_{sa}, c_{sb}, c_{st}, c_{at}, c_{bt}\}$ with $c_{sa} = 1, c_{sb} = 3, c_{st} = 2, c_{at} = 1$, and $c_{bt} = 3$.

When arc weights are evidential, *i.e.*, there is some uncertainty about them in the form of a mass function m on Ω, the preference over s-t paths with

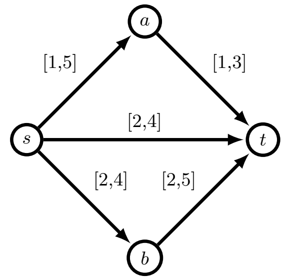


Fig. 3. Focal set as a Cartesian product of intervals.

respect to their (uncertain) weights can be established using the decision-making framework recalled in Section 2.2. Specifically, the set Ω of scenarios represents the possible states of nature. The set $\mathcal{X}$ of s-t paths represents the possible acts. The weight $\sum_{(i,j)\in A} c_{ij}p_{ij}$ of path $p = \{p_{ij}|(i,j) \in A\} \in \mathcal{X}$ under scenario $\boldsymbol{c} = \{c_{ij}|(i,j) \in A\} \in \Omega$ represents the cost $l(p, \boldsymbol{c})$ of path (act) p for the scenario (state of nature) $\boldsymbol{c}$. The preference over s-t paths, and the associated best s-t paths, can then be defined using any of the three criteria recalled in Section 2.2. In the next section, we provide the main results of this paper, which concern best s-t paths according to these three criteria and under assumption CI.

3.2 Solving

In this section, methods for finding best paths according to, in turn, the generalized Hurwicz, strong dominance, and weak dominance criteria, are provided. We can remark that these criteria rely on the notions of upper and lower expected costs of acts, acts being here paths. These costs $\overline{E}_m(p)$ and $\underline{E}_m(p)$ of a path p can be computed easily under assumption CI:

Proposition 1. *Under assumption CI, we have*

$$\overline{E}_m(p) = \sum_{(i,j)\in A} \bar{u}_{ij}p_{ij} \tag{12}$$

$$\underline{E}_m(p) = \sum_{(i,j)\in A} \bar{l}_{ij}p_{ij} \tag{13}$$

with $\bar{u}_{ij} := \sum_{r=1}^{K} m(F_r)u^r_{ij}$ *and* $\bar{l}_{ij} := \sum_{r=1}^{K} m(F_r)l^r_{ij}$ *for all* $(i,j) \in A$.

Proof. By definition, the upper and lower expected costs of path p are

$$\overline{E}_m(p) = \sum_{r=1}^{K} m(F_r) \max_{\boldsymbol{c}^r\in F_r} (\sum_{(i,j)\in A} c^r_{ij}p_{ij}), \tag{14}$$

$$\underline{E}_m(p) = \sum_{r=1}^{K} m(F_r) \min_{\boldsymbol{c}^r\in F_r} (\sum_{(i,j)\in A} c^r_{ij}p_{ij}). \tag{15}$$

The inner maximum and minimum in (14) and (15) are obtained when each arc weight c_{ij}^r in scenario $\boldsymbol{c}^r$ equals u_{ij}^r and l_{ij}^r, respectively. By regrouping terms we get the desired result. □

Proposition 1 is instrumental to uncover exacts methods finding best s-t paths.

Generalized Hurwicz criterion Since relation $\preceq_{hu}$ is complete, it is sufficient to find one element of the set Opt_{hu}, as explained in Section 2.2. To find one such element, *i.e.*, best path according to the generalized Hurwicz criterion, we need to solve the optimization problem

$$\min \ \alpha\overline{E}_m(p) + (1-\alpha)\underline{E}_m(p) \tag{16}$$

$$p \in \mathcal{X}. \tag{17}$$

The complexity of the problem (16-17), in the case of general focal sets, has been studied in the literature. If $\alpha = 1$, the problem is weakly NP-hard already in the case when mass function m has a single focal set containing two elements [15]. If $\alpha = 0$, the problem is even harder: it is strongly NP-hard and not approximable [6, Theorem 1]. However, under assumption CI, the problem (16-17) becomes much easier to solve:

Proposition 2. *Under assumption CI, solving the problem (16-17) amounts to solving the SPP in graph $G = (V, A)$ with arc weights $c_{ij} = \alpha\bar{u}_{ij} + (1-\alpha)\bar{l}_{ij}$.*

Proof. Using Proposition 1, the problem (16-17) becomes

$$\min \sum_{(i,j)\in A} (\alpha\bar{u}_{ij} + (1-\alpha)\bar{l}_{ij})p_{ij} \tag{18}$$

$$p_{ij} \text{ satisfies (2-5) } \forall (i,j) \in A \tag{19}$$

□

According to Proposition 2, to find one element in Opt_{hu}, we can use a fast algorithm for the SPP such as [4].

Strong dominance criterion Since relation $\preceq_{str}$ is partial, it may be necessary to find all elements of the set Opt_{str}, *i.e.*, all best paths according to the strong dominance criterion.

Proposition 3. *Under assumption CI, finding all elements in Opt_{str} amounts to finding all paths, in graph $G = (V, A)$ with arc weights $c_{ij} = \bar{l}_{ij}$, whose weights are lower than or equal to $\bar{d}_\star := \min_{q\in\mathcal{X}} \overline{E}_m(q)$.*

Proof. By definition,

$$p \in Opt_{str} \Leftrightarrow \nexists q \in \mathcal{X} \text{ such that } \overline{E}_m(q) < \underline{E}_m(p) \tag{20}$$

$$\Leftrightarrow \forall q \in \mathcal{X} \text{ then } \overline{E}_m(q) \geq \underline{E}_m(p) \tag{21}$$

$$\Leftrightarrow \min_{q\in\mathcal{X}} \overline{E}_m(q) \geq \underline{E}_m(p) \tag{22}$$

As a special case of Proposition 2, when $\alpha = 1$, $\min_{q\in\mathcal{X}} \overline{E}_m(q)$ is obtained by solving the deterministic SPP in G with arc weights $c_{ij} = \bar{u}_{ij}$. From Proposition 1, we have $\underline{E}_m(p) = \sum_{(i,j)\in A} \bar{l}_{ij} p_{ij}$. Hence, to find p such that $\underline{E}_m(p) \leq \bar{d}_\star$, we set arc weights c_{ij} of G to $\bar{l}_{ij}$ and finding all elements in Opt_{str} amounts to finding all s-t paths in G whose weights are lower or equal than $\bar{d}_\star$. □

To find all elements in Opt_{str}, we can use efficient algorithms such as the one in [1], where the authors studied a problem of determining near optimal paths; for example, they wished to find all s-t paths in a directed graph whose weights do not exceed more than 10% the lowest weight, which is basically finding all paths whose weights are lower than or equal to a given value.

Weak dominance criterion Similarly as for the strong dominance criterion, all elements of the set Opt_{weak} may need to be found since $\preceq_{weak}$ is partial.

There is a strong connection between the weak dominance criterion and bi-objective optimization. A bi-objective optimization problem can be expressed as

$$\min \ f_1(x) \tag{23}$$

$$\min \ f_2(x) \tag{24}$$

$$x \in X \tag{25}$$

As the objectives (23-24) are typically conflicting, there is usually no solution x that minimizes simultaneously $f_1(x)$ and $f_2(x)$. Instead, we seek to find all so-called efficient solutions of (23-25): a solution x is efficient if there is no feasible solution $y \in X$ such that $f_1(y) \leq f_1(x)$ and $f_2(y) \leq f_2(x)$ where at least one of the inequalities is strict.

The bi-objective SPP is a particular bi-objective optimization problem. Assume that each arc (i, j) in G has two deterministic attributes c_{ij} and t_{ij} that describes, *e.g.*, the distance and the travel time from i to j, respectively. The goal is to find all efficient solutions, *i.e.*, s-t paths of the following problem

$$\min \sum_{(i,j)\in A} c_{ij} p_{ij} \tag{26}$$

$$\min \sum_{(i,j)\in A} t_{ij} p_{ij} \tag{27}$$

$$p_{ij} \text{ satisfies (2-5) } \forall (i, j) \in A. \tag{28}$$

Proposition 4. *Under assumption CI, finding all elements in Opt_{weak} amounts to finding all efficient solutions of a bi-objective SPP in graph G where each arc $(i, j) \in A$ has two attributes $\bar{u}_{ij}$ and $\bar{l}_{ij}$.*

Proof. Finding all elements in Opt_{weak} is equivalent to finding all efficient solutions $p \in \mathcal{X}$ of a bi-objective optimization problem with objectives $f_1(p) := \overline{E}_m(p)$ and $f_2(p) := \underline{E}_m(p)$, which, using Proposition 1, comes down to a bi-objective SPP in graph G where each arc (i, j) has two attributes $\bar{u}_{ij}, \bar{l}_{ij}$. □

To find all elements in Opt_{weak}, we can apply fast algorithms developed for the bi-objective SPP such as the one in [5].

We note that any generalized Hurwicz optimal solution for $0 < \alpha < 1$ is also an element in Opt_{weak}, hence finding such solutions for various α will provide an inner approximation of Opt_{weak}. This stems from bi-objective optimization theory, where they are known as the *supported* efficient solutions, which are the solutions of $\min\{\lambda_1 f_1(x) + \lambda_2 f_2(x) : x \in X\}$ for some $\lambda_1, \lambda_2 > 0$.

Example 4. Assume that mass function m has a single focal set, which is the one in Figure 3. There are three s-t paths with their lower and upper expected costs indicated between parentheses: s-a-t : $(2, 8)$; s-t : $(2, 4)$; s-b-t : $(4, 9)$. If $\alpha = 0$, the two optimal paths according to generalized Hurwicz criterion are s-a-t and s-t with expected cost 2. If $\alpha = 0.5$, s-t is the unique optimal path with expected cost 3. We also have $Opt_{str} = \{s\text{-}a\text{-}t, s\text{-}t, s\text{-}b\text{-}t\}$ and $Opt_{weak} = \{s\text{-}t\}$.

3.3 Sizes of Opt_{weak} and Opt_{str}

It is clear that if $p \prec_{str} q$ then $p \prec_{weak} q$, and that the converse does not hold, hence $Opt_{weak} \subseteq Opt_{str}$. Example 4 showed that $Opt_{weak} \subset Opt_{str}$ in general.

The sets Opt_{weak} and Opt_{str} can be huge so enumerating their elements can be time-consuming. In fact, it is shown in [7, Theorem 1] that in the worst case, the size of the set of efficient paths grows exponentially with $|V|$. Therefore, it is useful to be able to know the size of these sets in advance, without enumerating their elements explicitly. Proposition 5 is a first result in this direction:

Proposition 5. *If $\bar{d}_\star$ and $\bar{l}_{ij}$ in Proposition 3 are rational numbers, $|Opt_{str}|$ (and thus an upper-bound of $|Opt_{weak}|$) can be computed in $O(|V|^2 \times W)$, with $W = \bar{d}_\star \times D$ where D is a common denominator of $\bar{d}_\star$ and of $\bar{l}_{ij}$, for all $(i, j) \in A$.*

Proof. Hereafter, consider graph G with integer arc weights $c_{ij} = \bar{l}_{ij} \times D$. It is easy to show that $|Opt_{str}|$ is equal to the number of s-t paths in G whose weights are lower than or equal to integer value W. Furthermore, let $|V| = n$ and assume, without loss of generality, that vertices are indexed by $0, \ldots, n-1$, with 0 and $n-1$ the source and destination vertices, respectively. Denoting by $N_w(i)$ the number of paths in G from i to $n-1$ whose weights are lower than or equal to w, then we need to calculate $N_W(0)$ since it is equal to $|Opt_{str}|$. We have clearly, for all $i \in \{0, \ldots, n-2\}$ and all $w \in \{1, \ldots, W\}$,

$$N_w(i) = \sum_{j \text{ such that } (i,j) \in A \text{ and } c_{ij} \leq w} N_{w-c_{ij}}(j), \tag{29}$$

with $N_0(i) = 0$ for all $i \in \{0, \ldots, n-2\}$ and $N_w(n-1) = 1$ for all $w \in \{0, \ldots, W\}$.

Consider a $(W+1) \times n$ 2-dimensional array M with each cell $M[w][i]$, $w \in \{0, \ldots, W\}$, $i \in \{0, \ldots, n-1\}$, storing $N_w(i)$; by filling this array row-wise starting with row $w = 0$, computing each row costs $O(|V|^2)$. This leads to the desired complexity. □

We note that given [9, Theorem 1] and the above proof, computing $|Opt_{str}|$ is actually NP-hard. Nonetheless, in practice, W in Proposition 5 may not be too big, so that computing $|Opt_{str}|$ may be quite fast.

4 Conclusion

In this paper, we have considered the case where uncertainty about arc weights in a graph is represented by a mass function. We have proposed extensions of the notion of shortest path to this context, as the sets of non-dominated paths according to the generalized Hurwicz, strong dominance, and weak dominance criteria. We have shown that if the focal elements of the mass function are Cartesian products of intervals, these sets can be found by applying algorithms developed for variants of the deterministic SPP. Future works include considering other criteria, such as maximality [2] or minimization of expected costs according to Shenoy's expectation operator [13].

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