Application of the belief function theory to validate multi-agent based simulations

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Abstract—In this article, an architecture to validate agent based simulations is presented. The proposed pyramidal architecture is based on the belief function theory to represent and handle imperfect information at three different levels. We are then able to provide, at many steps during the simulation, the validity state compared to the real system. First results of this architecture are presented within the framework of an application in forensic entomology.

Keywords: belief functions, multi-agent based simulation, validation.

I. INTRODUCTION

Modeling biological systems, often considered as complex systems, with a large number of heterogeneous individuals interacting, is not an obvious exercise. Various paradigms can be used, but intuitively, the multi-agent based paradigm [1] seems to be an ideal alternative, particularly to enable property emergence and self-organization from individual interactions. While the computational cost of running a multi-agent based simulation can be exorbitant, it increases significantly when a reasoning or treatment is desired from these simulations [2]. Indeed, it is often necessary to validate the simulation through observations ; these observations are usually made from local agent properties, the global simulation properties are not accessible or available.

Simulation validation consists in determining if the simulation is "reasonably" similar to a given reality [3]. The idea is then to compare the data obtained from the simulation to their counterparts in reality. The problem is that these data are, by definition in complex systems, numerous, and their units and types (*e.g.* qualitative, quantitative) are different. In this context, the proximity determination of a simulation from a certain reality can be difficult, particularly because the quality of their information may be very imperfect (*i.e.* uncertain, imprecise). It seems interesting to use and develop a specific formalism to represent and manage such information. Among the existing theories, the theory of belief functions [4], [5] can be well adapted to this kind of problem.

In the framework of developing a decision support system dedicated to forensic medicine, we face the problem of the validation of multi-agent based simulations. In the aim of comparing the proximity of a simulation to a reality provided by experts (*i.e.* a set of biological data measured at a crime scene), we develop a pyramidal observation system of multi-agent simulations based on the belief function theory.

In the first part of this document (*cf.* sub-section II-A), the belief function theory is presented. In the next sub-section II-B, foundations of multi-agent systems are briefly presented. The sequel of the article is dedicated to the validation of agent based simulations (*cf.* sub-section II-C) and to the proposed pyramidal architecture (section III). First results are then presented in the forensic application (section IV) before developing discussions and conclusions (section V).

II. BACKGROUND

The belief function theory, also called *evidence theory*, was introduced by Dempster [6] during his work on the lower and upper bounds of a distribution probability family. The initial theory was modified and ameliorated on several occasions, for instance through the work of Shafer [4] then later thanks to the work of Philippe Smets on the transferable belief model (the *TBM*), a non-probabilistic interpretation of the evidence theory. We present in this part, the main concepts of the belief function theory. For more details, the interested reader may refer to [5].

A. Belief function theory

1) Basic concepts: Let Ω be the exclusive set of N hypotheses, solution of a given problem. Ω is called the *frame of discernment* and is defined as follows:

$$\Omega = \{\omega_1, \omega_2, \ldots, \omega_N\} \tag{1}$$

From this frame of discernment Ω, the power set 2^Ω can be built, including the 2^N proposals A of Ω:

$$2^\Omega = \{A | A \subseteq \Omega\} \tag{2}$$

A mass function (or allocation[1]), noted m, is defined by 2^Ω in $[0, 1]$, and verifies:

$$\sum_{A \subseteq \Omega} m(A) = 1 \tag{3}$$

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[1]The term *BBA* for *basic belief assignment* is also found in literature.

Each subset $A \subseteq \Omega$ such that $m(A) \neq 0$ is called a focal element of m. Thus, mass $m(A)$ represents the degree of belief allocated to the proposal A and that cannot, in the present state of knowledge, be attributed to a more specific subset than A. A BBA m is said to be *dogmatic* if Ω is not a focal set and *normal* if $m(\emptyset) = 0$. As an example, in the transferable belief model of Philippe Smets, the condition $\sum_{\emptyset \neq A \subseteq \Omega} m(A) = 1$ is not imposed and $m(\emptyset) \neq 0$ can exist. This can introduce the notion of *open world* while assuming that the belief cannot be attributed to a subset of Ω. In this case, $\emptyset$ can be interpreted as a proposal which is not in the frame of discernment Ω and that it is likely to be the solution to the problem as opposed to the *closed world* where the set Ω is assumed to be exhaustive.

From the mass function, other belief functions such as plausibility (noted pl), credibility (bel), implicability (b) and communality (q) functions can be defined. These functions are dual measures and represent the same information expressed in different ways. Moreover, they can be translated from one to the other thanks to the Möbius transform [7], as follows between the mass function and the communality function:

$$ q(B) = \sum_{A \supseteq B} m(A) \qquad \forall B \subseteq \Omega \qquad (4) $$

Based on Shafer's work [4] on simple basic belief assignments (*SBBA*), Smets proposed the notion of generalized simple *bba* (*GSBBA*) [8]. A *GSBBA* can be noted $A^{\rho(A)}$ and Smets showed that these weights $\rho(A)$ for all $A \in 2^{\Omega} \setminus \{\Omega\}$ can be obtained by the following formula:

$$ \rho(A) = \prod_{B \supseteq A} q(B)^{(-1)^{|B|-|A|+1}} \qquad (5) $$

2) Combination rules: When many sources share beliefs in relation to the validity of a hypothesis of Ω, the different points of view can be fused using combination rules. Historically, within the framework of the belief function theory, the *Dempster's conjonctive combination rule* and the *TBM conjunctive combination rule* (also called the *unnormalized conjunctive combination rule*) have played an important part, especially thanks to their axiomatic justifications [9]. The merge of two distinct sources m_i and m_j can be made using the TBM conjunctive rule of combination, denoted by $m_i \bigcirc m_j = m_{i \bigcirc j}$. This rule is commutative and associative and is defined by:

$$ m_{i \bigcirc j}(C) = \sum_{A \cap B = C} m_i(A).m_j(B), \forall C \subseteq \Omega \qquad (6) $$

Due to undesirable behaviors (*e.g.* too important conflict $m(\emptyset)$) after the combination or need of source independance), and based on the works on SBBA and GSBBA, Thierry Denoeux proposed in [10] a new rule of combination, the *cautious rule*. So, the combination of m_1 and m_2, two non dogmatic BBAs, using the cautious rule depends on the weight function:

$$ \omega_{1 \bigwedge 2}(A) = \rho_1(A) \wedge \rho_2(A), \forall A \in 2^{\Omega} \setminus \Omega, \qquad (7) $$

and the final combination $m_i \bigwedge m_j = m_{i \bigwedge j}$ is obtained by:

$$ m_{1 \bigwedge 2}(A) = \bigcirc_{A \subsetneq \Omega} A^{\ \rho_1(A) \ \wedge \ \rho_2(A)}. \qquad (8) $$

Like the above rules, the cautious rule is commutative and associative. But the interested property is the idempotent property ($m \bigwedge m = m$) which allows to combine two non distinct mass functions given by two non independant sources. Other combination rules have been proposed, *e.g.* Yager's combination or Dubois and Prade's rules (*cf.* [9] for a good preview of these rules).

3) Discounting: When the resulting information in the belief function is not totally reliable, it may be necessary to discount this belief. In order to do that, a coefficient α can be used, which represents the knowledge of the source reliability and allows to redistribute the beliefs to the set Ω proportionally to the source reliability. The discounted belief function m^{α} can be deduced from m and α (*i.e.* $m^{\alpha} = Disc(m, \alpha)$) and can be obtained by the following expression:

$$ \begin{cases} m^{\alpha}(A) = \alpha m(A) \\ m^{\alpha}(\Omega) = 1 - \alpha + \alpha m(\Omega). \end{cases} \qquad (9) $$

In literature several methods have been developed to compute the discounting factor, *e.g.*, [11].

4) Decision making: Many solutions have been proposed to make decisions in the evidential framework (*e.g.* maximize the credibility or minimize the plausibility degrees), we present in this paper the retained solution for our application, the pignistic probability of Philippe Smets, defended within the transferable belief model [5], which is defined by the following equation:

$$ \forall \, \omega_n \in \Omega \qquad BetP(\omega_n) = \frac{1}{1-m(\emptyset)} \sum_{A \ni \omega_n} \frac{m(A)}{|A|} \qquad (10) $$

where $|A|$ represents the cardinality of $A \subseteq \Omega$. Once the *pignistic* probability obtained, it is possible to use classic tools of statistical decision theory. Readers could find justifications and details of this transformation in [12].

B. Multi-agent systems

The ever more increasing needs to understand real and complex systems have encouraged modelers to implement modeling paradigms from distributed artificial intelligence (*DAI*). Especially, the multi-agent paradigm has growed in the modeling of living systems. As in the previous section, essential concepts of multi-agent systems are presented and the interested reader may refer to [1], [13] for more details.

The term "agent" is a generic term for which no definition really comes to a consensus. A general definition[2] comes from [1]. Ferber defines the agent as an autonomous virtual (or physical) entity that:

- can act in an environment,
- can perceive its environment,
- can communicate directly (*e.g.*, by sending messages)or indirectly (*e.g.*, via the environment) with other agents,

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[2]A simplified version of Ferber's definition is presented on purpose.

- is governed by a set of trends (*e.g.* objective optimization) and is limited by a set of constraints (*e.g.* limited resources),
- has competences and offers services.

From this definition, a definition of multi-agent sytems (*MAS*) can be given. It is a system composed of:

- an environment En with a certain metric,
- a set of object Ob situated in En,
- a set of agents Ag that can perceive and manipulate the objects,
- a relation set R_{Ob} between the objects,
- a relation set R_{Ag} between the agent (*e.g.* communicate, share resources *etc.*),
- a relation set R between the agents and the objects (*e.g.* carry, move *etc.*),

Historically, two types of agents can be distinguished in MAS: reactive or cognitive agents. First ones has no explicit environment representation and they react reflexively to stimuli (*e.g.* an agent modelizing an insect). Cognitive agents have a more developed environment representation, explicit goals, memory abilities or capabilities of individual reasoning. There may also be hybrid agents and multi-agent systems composed of reactive and cognitive agents, as in our application presented thereafter.

C. *Validation of multi-agent simulations and associated problems*

Validation of any model is an important task [3]. This problem matters of course within the framework of multi-agent simulations, especially with their expanding importance and their implementations in various fields [14]. Validate a MAS usually requires expert interventions, expert typically compares the real system outputs to their modeled equivalents[3]. Comparing the model to reality is done using various tests, that can be objective, quantitative, subjective or qualitative. Because the information conveyed in such systems are generally numerous, very heterogeneous and largely inside the agents themselves, the validation of multi-agent simulations is directly done through observation of agents (and/or their communications [15]). Observation of emergent properties is more difficult, particularly because of the difficult characterization of such properties.

Automation of observation process has already been addressed and architectures (*e.g.*, [16]) have been presented to validate agent based simulations. In a recent publication [2], we have also shown that a statistical approach of observation could be interesting. The sample survey theory [17] seems to provide a non-negligible interest if the number of agents is too high (several thousands).

But the real system complexity inevitably leads to a difficult access to parameter values, *e.g.*, it seems rather difficult to observe and know status of each ant in a colony at a given

time. These difficulties are all the more important as the system dynamics lead to rapid, regular, agent-specific changes of these parameters. Real parameters can only be observed occasionally and usually at so called "simple" or "obvious" moments: at the initialization step, during downturns (*e.g.* when ants enter in diapause stage) or at the end of the experiment. Within the framework of the agent-based simulation validation, the lack of knowledge of these values has a direct impact on possible times of validation: it seems intuitively difficult to validate a model at a time t_v when the known values of the real system have been observed at a time $t_w \neq t_v$.

However, when the observation data are only available at t_w, and just because the simulation costs are important, it may be interesting to estimate the model state and predict the validation results. If the parameter dynamics can be known or estimated and that the imperfections can be managed, even if the system is complex, it seems that predictions may be possible. The usual definition of validation (*i.e.*, "compare the results of the model – in our case, outputs of the simulation – to those of the real system") can be extended into "check if the model is still in agreement with the real system". Thus, it seems interesting to develop a validation system based on recent work, the previous remarks, and based on the belief function theory to manage imperfect information. We present afterwards, the simulation validation process and the appropriate agent based architecture.

III. VALIDATION OF MULTI-AGENT SIMULATIONS BASED ON THE BELIEF FUNCTION THEORY

In this section, the proposed validation architecture is detailed: the sub-section III-A presents the general multi-agent organisation of validation system and the next one presents the interest of belief functions within such a framework.

A. *Pyramidal architecture*

We consider a real system and its multi-agent model, both evolving in time $t \in [t_0; t_{final}]$. An expert can give us the set of validity domains $Dom = \{dom_1, \ldots, dom_x, \ldots, dom_X\}$ of the parameters $P = \{p_1, \ldots, p_x, \ldots, p_X\}$ observed at the time $t_{obs} \in [t_0; t_{final}]$. These parameters concern properties of agents and groups of agents.

The adopted validation strategy is to "agentify[4]" each parameter of P. So, we consider the time $t_v \in [t_{val}^-; t_{val}^+]$ with $[t_{val}^-; t_{val}^+] \subseteq [t_0; t_{obs}]$. At t_v, each p_x is a source that can answer the next question:

Q: "At t_v, am I still in agreement with facts found by the expert at t_{obs}?".

Each parameter can answer the question Q whenever wanted between the minimum time constraint t_{val}^- and the maximum time constraint t_{val}^+. To answer the question Q, p_x is able to probe the set of appropriate agents $Ag' \subseteq Ag$ by transferring them the related question Q. The answer of the question Q belongs to the set $\Omega = \{yes, no\}$.

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[3]When the validation is a part of an iterative process to successively retroactive on simulation inputs, in order to obtain a model enough closed to the reality, this iterative process is called calibration [3].

[4]"Agentify" is an expression used to turn system actors or non-actors (*e.g.*, the parameters) into an agent in the simulated model.
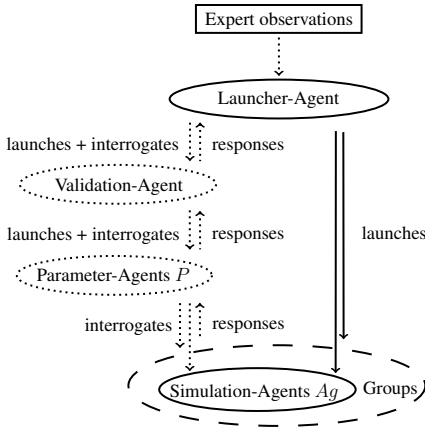
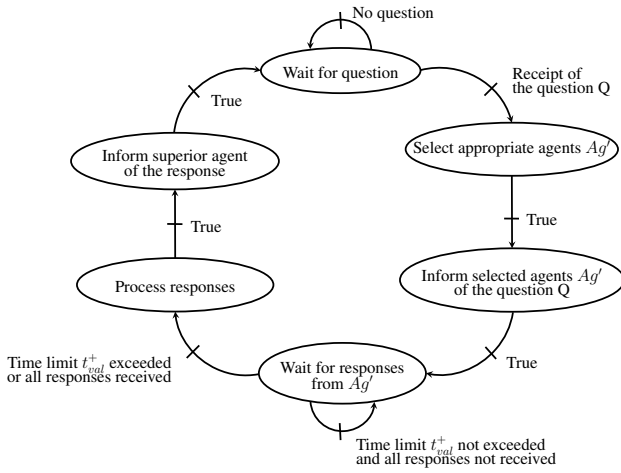Figure 1. Pyramidal architecture of validation strategy.

Figure 2. General behaviour of a specific *Parameter-agent* p_x

As in most multi-agent simulation platforms, simulation could be controlled (*i.e.*, launching, agent creation...) by an agent named *Launcher-Agent*[5]. In these platforms, this agent usually initializes the simulation and launches the simulation agents. In the proposed architecture, the *Launcher-Agent* also launches the *Validation-Agent* needed to validate simulations and responsible for the set of *Parameter-Agents P* (*cf.* figure 1). So, a validation system on two intermediate levels, between the *Launcher-Agent* and simulation agents Ag, has been developed. This architecture is called pyramidal because of the increasing number of involved agents from the *Validation-Agent* level to the last level.

1) General behaviour of validation agents: Whatever the validation level (*Validation-Agent* or *Parameter-Agents*), an agent has always the same general behaviour: it receives a question, it transfers this question to appropriate "subordinate" agents and waits for responses before informing its superior agent (*cf.* figure 2).

[5]Name used for example in the MadKit platform [18].

The first validation level, the *Validation-Agent* manages the validation process (*e.g.*, start, stop, time management...) after being created by the *Launcher-Agent*. Once launched, this agent launches *Parameter-Agents* (their number varies with expert observations). At a minimum time constraint t_{val}^-, it informs each *Parameter-Agent* of:

1 - information of all the potential simulation agents Ag capable of answering the question Q,
2 - the validity domain dom_x of the parameter p_x at t_{obs},
3 - the next time limit t_{val}^+ before which the *Parameter-Agents* should have answered.

As shown on figure 2, once launched, a *Parameter-Agent* p_x:

1 - selects an agent group $Ag' \subseteq Ag$ to probe,
2 - informs the Ag'-agents of the question Q and the validity domain dom_x of the parameter p_x at t_{obs},
3 - communicates the time limit t_{val}^+ given by the *Validation-Agent*.

2) Behaviour of the probed simulation agents: At each simulation step, before running its regular life cycle, a simulation agent ag_i verifies if it receives a message Q from a *Parameter-Agent* p_x. When it is contacted at t_{val}^-, it checks if it is concerned by this request. If so, it saves the question Q, the time constraint t_{val}^+ and the parameter validity domain dom_x. Depending on its activity, it can answer the question between the time steps t_{val}^- and t_{val}^+. At $t_v \in [t_{val}^-; t_{val}^+]$, to answer the question, ag_i takes into account the validity domain dom_x and compares it to the courant value of p_x and send its response to its p_x.

B. Interest of belief functions to validate multi-agent simulations

To handle imperfect information exchanged by agents, responses are expressed as belief functions. We present in this section the way of contruction and management of *basic belief assignments* in the validation system. An illustrative example is presented in figure 4.

1) Evidential response of the simulation agents Ag: At $t_v \in [t_{val}^-; t_{val}^+]$, to answer the question, $ag_i \in Ag'$ (with $Ag' \subseteq Ag$) takes into account the validity domain dom_x and compares it to the courant value of p_x. It sends an evidential response to its p_x. Various methods can be implemented to create the mass function $m_{x,ag_i}^{t_v} : 2^\Omega \mapsto [0,1]$, where $\Omega = \{yes, no\}$. For example, in the case of a quantitative parameter within the framework of our application (see section IV), we have defined the mass assignment method presented on figure 3.

2) Management of basic belief assignments by the Parameter-Agents: Between its own time limits $[t_{val}^-; t_{val}^+]$, a *Parameter-Agent* p_x can receive multiple responses at different times t_v from subordinate Ag'-agents. To get a *bba* related to the parameter validity, p_x has to combine the mass functions $m_{x,ag_i}^{t_v}$ but this can only be done at the time t_{val}^+ (*i.e.* when all the *Simulation-Agents* should have answered the question).
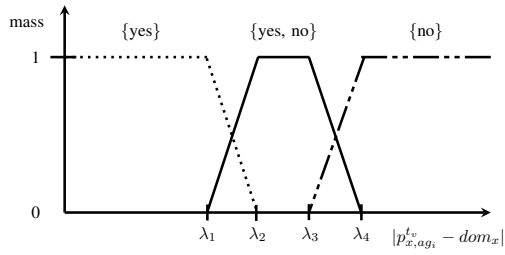
Figure 3. Method of mass assignment for a simulation agents ag_i and for a quantitative parameter p_x. The value $p_{x,ag_i}^{t_v}$ corresponds to the value of the p_x-parameter of the agent ag_i at the time t_v and dom_x is the limit value observed at t_{obs}. The expression $|p_{x,ag_i}^{t_v} - dom_x|$ is an absolute value and the values $\lambda_1, \lambda_2, \lambda_3$ and λ_4 correspond to thresholds currently defined empirically. So, for example, when the result of the expression $|p_{x,ag_i}^{t_v} - dom_x|$ is inferior to λ_1, the value $p_{x,ag_i}^{t_v}$ seems enough similar to the limit dom_x to be considered as a correct value and to assign all the belief to the singleton yes.

So, before combining the *bbas*, p_x discounts them by respecting the principle of memory decay presented by Philippe Smets in [9]. This principe states that every *bba* is discounted with time: the longer the time since the *bba* has been collected, the stronger the discounting. The *bba* is discounted by the reliability factor $\alpha(t_v)$ that is a decreasing function of time with $\alpha(t_{val}^-) = 0$ and $\lim_{t_v \to t_{val}^+} \alpha(t_v) \to 1$. At t_{val}^+, for all $ag_i \in Ag'$, p_x gets the masses $m_{x,ag_i}^{t_{val}^+}$ by discounting the $m_{x,ag_i}^{t_v}$ with time (*cf.* sub-section II-A3):

$$m_{x,ag_i}^{t_{val}^+} = Disc(m_{x,ag_i}^{t_v}, \alpha(t_v)), \quad \forall \, ag_i \in Ag' \quad (11)$$

Finally, to get the *bba* $m_x^{t_{val}^+}$ related to the validity of the parameter p_x at t_{val}^+, the *Parameter-Agent* p_x combines all the $m_{x,ag_i}^{t_{val}^+}$ with the cautious rule of Thierry Denoeux (*cf.* sub-section II-A2):

$$m_x^{t_{val}^+} = \mathbin{\bigwedge}_{ag_i \in Ag'} \quad m_{x,ag_i}^{t_{val}^+} \quad (12)$$

In this context, the cautious rule of combination is preferred for its idempotence property since the surveyed agents of Ag' may not be completely independent (*e.g.* they can interact together, evolve with the same behaviour models...).

3) Evidential validation at the Validation-Agent level: The last validation level is to combine the *bbas* $m_x^{t_{val}^+}$ given by the *Parameter-Agents* of P at t_{val}^+. Currently, because the *Parameter-Agents* probe different *Simulation-Agents* and attempt to estimate the validity of different parameters, they can be considered totally independent. The conjunctive rule of combination seems to be well adapted. So, the final basic belief assignment $m^{t_{val}^+}$ can be obtained with the next equation:

$$m^{t_{val}^+} = \mathbin{\bigcirc}_{p_x \in P} \quad m_x^{t_{val}^+} \quad (13)$$

Once this last *bba* $m^{t_{val}^+}$ obtained, it only remains to forward the information (processed or not, using for example the pignistic transformation) to its *Launcher-Agent* that can be able to take retrospective actions on the simulation (*e.g.* stop, reset...).
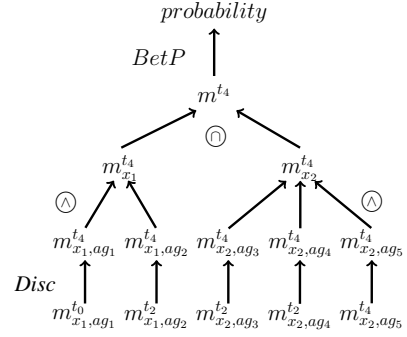

Figure 4. Example of a general architecture of belief functions with five *Simulation-Agents*, two parameters and between the time limits t_0 and t_4.

IV. First results within the framework of the forensic entomology project

During a criminal investigation, it is essential to obtain a maximum of information on the conditions of a manslaughter. Many methods to exploit the indications on the murder scene are known but, for large post-mortem intervals (PMI), only one of these techniques is useful in practice: forensic entomology. It consists in studying the insects found on a cadaver to estimate the time of its death. Modern PMI entomology estimation methods are based on insect development models but because of the important system complexity, results given by the experts are imperfect. To improve the decision-making and assist the forensic scientists, a decision support system (DSS) has been developed to take all the ecosystemic parameters and a significant quantity of biological models (*e.g.*, usually an expert can only use one or two single model). This project is based on a predictive multiagent model of insect development and cadaver decomposition in a complex ecosystem. It is used to determine if a hypothesis - a possible time of death - is coherent with the observations available on the ecosystem of the crime scene and the entomofauna found on the victim. The proposed pyramidal architecture has been implemented to compare the simulations to the reality given by experts at the cadaver discovery (*e.g.*, which species, numbers of insects by species, reached development rates *etc.*). The validation system has been integrated into a recursive process of calibration to calibrate the system and detect the most probable time of death. More information about this model, the real system and the DSS can be found in [19].

Figure 5 shows the final results of a real case in which the person had disappeared around June 17th and the cadaver was found on June 29th[6]. Experts have identified three species for which many development rates have been calculated or estimated (*i.e.*, it can have different laying moments for the same species and so at the cadaver's discovery, insects of same species can have different development rates). In this example, the development rate parameters are agentified and

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[6]For confidentiality reasons, some information, *e.g.*, the year and other details about crime circumstances, are omitted.
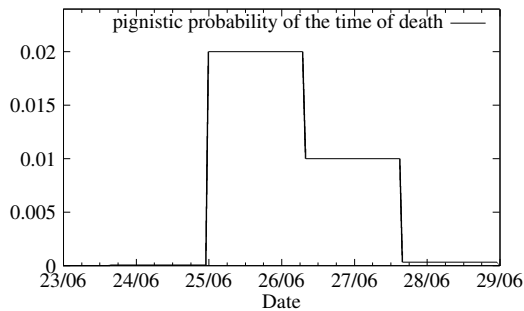
Figure 5. This figure shows the final pignistic probability generated after using the DSS on a real case.

have to answer question Q:

Q: "At t_v, am I still in agreement with the development rate given by the expert at t_{obs} (the date of cadaver discovery)?".

This example is only based on five estimated development rates and the *Parameter-Agents* interview the *Simulation-Agents* of the concerned species to answer the question. These first results are really interesting because experts had estimated the time of death around June 25th. The global system has still to be validated, but already, experts agree that the use of all ecosystemic information appears to be useful to estimate the time of death and make a more reliable and prudent decision.

V. DISCUSSIONS AND CONCLUSIONS

The proposed validation architecture of agent based simulation allows to take into account imperfect data while being faster and more efficient than other validation methods usually used in multi-agent simulation platforms. With slight changes in these platforms, this pyramidal architecture can be easily integrated.

At the moment, only the validation of agent and group properties are developed and implemented but possible extensions on validations of the states of environment, objects and their different relations (*cf.* section II-B) could be proposed. Moreover, even if the quality of the results can be dependent on several characteristics, such as the assignment method of beliefs of *Simulation-Agents*, various methods of assignment have been proposed in literature. Only the reasoning from quantitative parameters has been implemented but later work will concern qualitative ones (*e.g.*, reasoning on qualitative parameters could be very useful in our forensic application). For this, it is planned to switch to different works such as [20]. To go more thoroughly into the study of impacts on the final validation decision, different combination rules and forms of the discounting decreasing function will be analyzed.

Finally, and this is where this validation method seems very interesting, important work on learning the evolution of *bbas* at different levels will be developed. Indeed, to reduce the computation time related to the validation and to "cleverly" limit moments of validation, the issue of validation time choices and agents probed (*i.e.* type, number) remains important. At the

Validation-Agent level, interest in the reliability of *Parameter-Agents* may also be important: the analysis of the possible discounting or reinforcement of some of them could appear significant.

As a conclusion, this validation system is being integrated into a higher level system of evidential calibration dedicated agent based simulations. This work is ongoing but already promising.

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