Modeling Qualitative Assessments under the Belief Function Framework

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Abstract. This paper investigates the problem of preference modeling under the belief function framework. In this work, we introduce a new model that is able to generate quantitative information from qualitative assessments. Therefore, we suggest to represent the decision maker preferences in different levels where the indifference, strict preference, weak preference and incompleteness relations are considered. Introducing the weak preference relation separates the preference area from the indifference one by inserting an intermediate zone.

1 Introduction

Modeling the decision maker preferences is not an easy task because he usually prefers to express his opinions qualitatively, based on knowledge and experience that he provides in response to a given question rather than direct quantitative information. Therefore, preferences need to be implemented in an assessment, which reflects as accurate as possible the human mind.

In other words, solving a problem dealing with expert preferences is usually characterized by a high degree of uncertainty. Besides, in some cases, the decision maker may be unable to express his opinions due to his lack of knowledge. He is then forced to provide incomplete or even erroneous information. Obviously, rejecting this difficulty in eliciting the expert preference is not a good practice.

To tackle the problem, a numerical representation under the belief function framework is introduced. Our main aim is to propose numerical values that represent the portion of belief expressed by the decision maker. Some researchers have already dealt with this problem and generate associated quantitative belief functions like [1] [13]. However, these approaches introduced only two fundamental preference relations called indifference and strict preferences.

One can overcome these difficulties as follows: we propose a new model including the weak preference relation, that separates the preference area from the indifference area by inserting an intermediate zone called weak-preference area [9]. A possible interpretation is an hesitation between strict preference and indifference.

Formally, consider two discrimination threshold functions: the indifference threshold ε, and the preference threshold γ. So, including the weak preference relation allows for more flexibility and nuance to the decision maker while expressing his preferences.

This leads to define crisp binary relations called strict preference (P, $\succ$), indifference (I, $\sim$), and weak preference (Q, $\succeq$).

The originality of our model is to allow the expert to easily express his preferences and to provide a convenience framework for constructing quantitative belief functions from qualitative assessments by using different preference relations.

In this paper, section 2 and 3 describe an overview of the basic concepts of respectively the belief function theory and the qualitative belief function methods. Then, in the main body of the paper, we present our new contribution namely the preference modeling in the belief function framework. Finally, our method will be illustrated by an example.

2 Belief Function Theory

In this section, we briefly review the main concepts underlying the belief function theory as interpreted by the Transferable Belief Model (TBM). The latter is a useful model to represent quantified belief functions. Details can be found in [10] [12] [11].

2.1 Basic Concepts

The TBM is a model to represent quantified belief functions [12]. Let Θ be the frame of discernment representing a finite set of elementary hypotheses related to a problem domain. We denote by 2^{Θ} the set of all the subsets of Θ [10].

The impact of a piece of evidence on the different subsets of the frame of discernment Θ is represented by the so-called basic belief assignment (bba), denoted by m [10]:

$$\sum_{A \subseteq \Theta} m(A) = 1. \tag{1}$$

The value $m(A)$, named a basic belief mass (bbm), represents the portion of belief committed exactly to the event A. The events having positive bbm's are called focal elements. Let $\mathcal{F}(m) \subseteq 2^{\Theta}$ be the set of focal elements of the bba m.

Associated with m is the belief function is defined for $A \subseteq \Theta$ and $A \neq \emptyset$ as:

$$bel(A) = \sum_{\emptyset \neq B \subseteq A} m(B) \text{ and } bel(\emptyset) = 0. \tag{2}$$

The degree of belief $bel(A)$ given to a subset A of the frame Θ is defined as the sum of all the basic belief masses given to subsets that support A without supporting its negation.

2.2 Decision Making

The TBM considers that holding beliefs and making decision are distinct processes. Hence, it proposes a two level model:

- The credal level where beliefs are entertained and represented by belief functions.
- The pignistic level where beliefs are used to make decisions and represented by probability functions called the pignistic probabilities, denoted $BetP$ [11]:

$$BetP(A) = \sum_{B \subseteq \Theta} \frac{|A \cap B|}{|B|} \frac{m(B)}{(1 - m(\emptyset))}, \forall A \in \Theta. \tag{3}$$

2.3 Uncertainty Measures

In the case of the belief function framework, different uncertainty measures (UM) have been defined, such as [5] [6]:

$$H(m) = \sum_{A \in \mathcal{F}(m)} m(A) \log_2(\frac{|A|}{m(A)}). \tag{4}$$

The measure H is aimed at assessing the total uncertainty arising in a body of evidence due to both randomness (ignorance and inconsistency) and nonspecificity associated with a bba.

The measure H attains its global maximum when the bba distributes both randomness and nonspecificity uniformly over the largest possible set of focal elements.

3 Constructing Belief Functions from Qualitative Preferences

The problem of eliciting qualitatively expert opinions and generating basic belief assignments have been addressed by many researchers [1] [2] [3] [13].

In this section, we present Ben Yaghlane et al.'s method [1]. This approach is chosen since it handles the issue of inconsistency in the pair-wise comparisons.

So giving two alternatives, an expert can usually express which of the propositions is more likely to be true, thus they used two binary preference relations: the preference and the indifference relations.

The objective of this method is then to convert these preferences into constraints of an optimization problem whose resolution, according to some uncertainty measures (UM) (nonspecificity measures, conflict measures, composite measures), allows the generation of the least informative or the most uncertain belief functions defined as follows [1]:

$$a \succ b \Leftrightarrow bel(a) - bel(b) \geq \varepsilon. \tag{5}$$

$$a \sim b \Leftrightarrow |bel(a) - bel(b)| \leq \varepsilon \tag{6}$$

ε is considered to be the smallest gap that the expert may discern between the degrees of belief in two propositions a and b. Note that ε is a constant specified by the expert before beginning the optimization process.

Ben Yaghlane et al. proposed a method that requires that propositions are represented in terms of focal elements, and they assume that Θ (where Θ is the frame of discernment) should always be considered as a potential focal element. Then, a mono-objective technique was used to solve a constrained optimization problem.

The preference assessment is transformed into constraint according to the following relation:

$$bel(a) - bel(b) \geq \varepsilon \quad \forall(a,b) \text{ for which } \; a \succ b$$

Then, the indifference assessment is transformed into constraint according to this relation:

$$bel(a) - bel(b) \geq -\varepsilon \text{ and } bel(a) - bel(b) \leq \varepsilon \quad \forall(a,b) \text{ for which } \; a \sim b$$

Consequently, we obtain the following constrained optimization model:

$$\begin{array}{c} Max_m UM(m) \\ s.t. \\ bel(a) - bel(b) \geq \varepsilon \quad \forall(a,b) \text{ for which } \; a \succ b \\ bel(a) - bel(b) \geq -\varepsilon \quad \forall(a,b) \text{ for which } \; a \sim b \\ bel(a) - bel(b) \leq \varepsilon \quad \forall(a,b) \text{ for which } \; a \sim b \\ \sum_{a \in \mathcal{F}(m)} m(a) = 1, m(a) \geq 0, xzssss \forall a \subseteq \Theta; m(\emptyset) = 0 \end{array} \tag{7}$$

Furthermore, the proposed method addresses the problem of inconsistency. In fact, if the preference relations are consistent, then the optimization problem is feasible. Otherwise no solutions will be found. Thus, the expert may be guided to reformulate his preferences.

In the following section, we propose a method that deals with Ben Yaghlane et al. approach. Our model introduces new preference relations.

4 The Preference Modeling in the Belief Function Framework

We present now one way of introducing the qualitative belief approach to model and process preference information. It leads to a model which can be seen as an extension of the crisp model obtained by replacing pseudo-orders (I: indifference; Q: weak preference; P: preference) by belief informations.

Let us detail the typical features of these belief preference structures and their interpretations as significant quantitative information.

4.1 Preference Articulation

Let A be a set of alternatives, where a and b are two alternatives. Besides, crisp binary relations are based on two basic relations called strict preference P and indifference I [8]. They are defined as follows:

1. a is preferred to b ($(a, b) \in P$) iff $(a \succ b) \wedge \neg(b \succ a)$
2. a is indifferent to b ($(a, b) \in I$) iff $(a \succ b) \wedge (b \succ a)$

However, by using our model, we want to response to the question "The alternative a is at least as good as the alternative b?". We can have then the following answers:

- Either yes or no. The decision maker responses to the previous question by "yes" or "no".
- I don't know: The decision maker can also express his ignorance.
- Answers including the intensity of preference: for example, "a has strongly - weakly, moderately - preferred to b".

For these reasons, a richer model other than standard binary relation is a crucial step. We will assume that the comparison of a and b gives a choice between two other possible cases:

- a is weakly preferred to b ($(a, b) \in Q$) iff $(a \succeq b)$, means that the decision maker thinks that a is at least as good as b;
- the relation between a and b is unknown;

From this relation $\succeq$, we can derive two other important relations on A:

1. Strict preference relation, $\succ$, defined by:

$$a \succ b \Leftrightarrow a \succeq b \text{ and not } (b \succeq a)$$

2. Strict Indifference relation, $\sim$, defined by:

$$a \sim b \Leftrightarrow a \succeq b \text{ and } b \succeq a$$

Under the previous approach [1], in general, when comparing two alternatives a and b, the expert uses two binary relations the preference and indifference relations. Not matter of how large the difference is.

In real-life problems, however, a small positive difference of scores is not always a justification for a preference. A classical attitude is to assess discrimination thresholds to distinguish between significant and not significant differences of scores. Therefore, the indifference threshold ε was introduced [1]. If the performances of two alternatives differ by less than ε, then there is an indifference relation (see Equation (6)) and not a preference relation.

However, this model presents some drawbacks [4]. Suppose two alternatives a and b are such that:

$$bel(a) - bel(b) = \varepsilon - \frac{\mu}{2}. \tag{8}$$

where μ is a positive quantity very small compared to ε.

If a slightly superior score (μ) was attached to a, we would obtain:

$$bel(a) - bel(b) = \varepsilon + \frac{\mu}{2}. \tag{9}$$

transforming the previous indifference ($a\ I\ b$) into strict preference ($a\ P\ b$).

We may overcome these difficulties by separating the preference and the indifference relations by inserting an intermediate zone called weak preference relation [7]. A possible interpretation is an hesitation between strict preference and indifference.

Formally, one may consider a strict preference threshold γ to distinguish between strict preference and weak preference. This strict preference threshold is a value such as if the performances of a and b differ by at least γ, then we are in a situation when one alternative is strongly preferred to the other. This is illustrated as follows:

$$a \succ b \Leftrightarrow bel(a) - bel(b) \geq \gamma. \tag{10}$$

$$a \succeq b \Leftrightarrow 0 \leq bel(a) - bel(b) \leq \gamma. \tag{11}$$

However, when comparing two alternatives, we might want to use both the indifference and the strict preference thresholds, where $\gamma \geq \varepsilon$:

$$a \succ b \Leftrightarrow bel(a) - bel(b) \geq \gamma \tag{12}$$

$$a \succeq b \Leftrightarrow \varepsilon \leq bel(a) - bel(b) \leq \gamma \tag{13}$$

$$a \sim b \Leftrightarrow |bel(a) - bel(b)| \leq \varepsilon \tag{14}$$

Nevertheless, there exist different ways for choosing the preference and indifference threshold. For instance, Roy et al. [7] believe that the fixing of thresholds involves not only the estimation of error in a physical sense, but also a significant subjective input by the decision-maker himself. They assume that these two thresholds can be constant values or can take the linear form. Besides, in other works [7], γ and ε are derived from mathematical equations.

In this work, we assume that the thresholds γ and ε can be constant values. We interpret the indifference threshold as the minimum margin of uncertainty associated with a given alternative, and the preference threshold as the maximum margin of error associated with the alternative in question.

4.2 Computational Procedure

Now and after modeling the different preference relation, we propose to use the same model as Ben Yaghlane et al. method [1]. We transform these preferences relations into constraints as presented in section 3.2. We get:

$$
\begin{array}{c}
Max_m UM(m) \\
s.t. \\
bel(a) - bel(b) \geq \gamma \quad \forall (a,b) \text{ for which } \; a \succ b \\
bel(a) - bel(b) \leq \gamma \quad \forall (a,b) \text{ for which } \; a \succeq b \\
bel(a) - bel(b) \geq \varepsilon \quad \forall (a,b) \text{ for which } \; a \succeq b \\
bel(a) - bel(b) \leq \varepsilon \quad \forall (a,b) \text{ for which } \; a \sim b \\
bel(a) - bel(b) \geq -\varepsilon \quad \forall (a,b) \text{ for which } \; a \sim b \\
\sum_{a \in \mathcal{F}(m)} m(a) = 1; m(a) \geq 0; \forall a \subseteq \Theta; m(\emptyset) = 0
\end{array} \tag{15}
$$

where the first constraint of the model is derived from the preference relation. The second and third constraints model the weak preference relation. The fourth and fifth constraints correspond to the indifference relation.

ε and γ are constants specified by the expert before beginning the optimization process.

The choice of thresholds intimately affects whether a particular binary relationship holds. While the choice of appropriate thresholds is not easy, in most realistic decision making situations there are good reasons for choosing non-zero values for ε and γ.

Figure 1 summarizes the obtained transformation. These thresholds define five different intervals in the domain of preference of two alternatives.

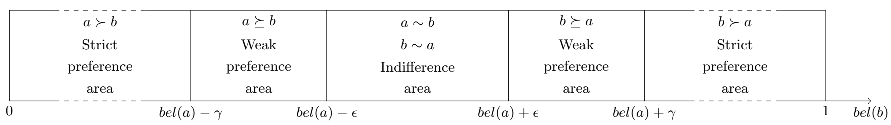

Fig. 1. Belief relations built from thresholds and crisp scores.

5 Illustrative Example

Let us consider a problem of eliciting the weight of the candidate alternatives. The problem involves five alternatives:

$$\Theta = \{a, b, c, d, e\}.$$

The focal elements are:

$$F1 = \{a\},\ F2 = \{a, b, c\},\ F3 = \{b, d\},\ F4 = \{e\},\ F5 = \{a, e\}.$$

Next, the expert opinions should be elicitated. For this purpose, an interview with the expert is realized in order to model his preferences. Consequently, he has validated the following relations:

$$F2 \succ F1 \ , \ F1 \succeq F3 \ , \ F4 \sim F1, \ F5 \succ F1 \ , \ F5 \succ F4,$$

Now and after eliciting his preferences, the next step is to transform the obtained relations into optimization problem according to our proposed method.

We assume that $\varepsilon = 0.01$, $\gamma = 0.02$ and the uncertainty measures is H since it has a unique maximum as defined in Equation (4).

The following step is then to transform the obtained relations into constraints. We obtain:

1. $F2 \succ F1 \Leftrightarrow bel(F2) - bel(F1) \geq 0.02$
2. $F1 \succeq F3 \Leftrightarrow bel(F1) - bel(F3) \leq 0.02$
3. $F1 \succeq F3 \Leftrightarrow bel(F1) - bel(F3) \geq 0.01$
4. $F4 \sim F1 \Leftrightarrow bel(F4) - bel(F1) \leq 0.01$
5. $F4 \sim F1 \Leftrightarrow bel(F4) - bel(F1) \geq -0.01$
6. $F5 \succ F1 \Leftrightarrow bel(F5) - bel(F1) \geq 0.02$
7. $F5 \succ F4 \Leftrightarrow bel(F5) - bel(F4) \geq 0.02$

Then, we obtain the following optimization problem example:

$$\begin{array}{c} Max_m H(m) = m(F1) * log_2(1/m(F1)) + m(F2) * log_2(3/m(F2)) \\ +m(F3) * log_2(2/m(F3)) + m(F4) * log_2(1/m(F4)) \\ +m(F5) * log_2(2/m(F5)) + m(\Theta) * log_2(5/m(\Theta)); \\ s.t. \\ bel(F2) - bel(F1) \geq 0.02 \\ bel(F1) - bel(F3) \leq 0.02 \\ bel(F1) - bel(F3) \geq 0.01 \\ bel(F4) - bel(F1) \leq 0.01 \\ bel(F4) - bel(F1) \geq -0.01 \\ bel(F5) - bel(F1) \geq 0.02 \\ bel(F5) - bel(F4) \geq 0.02 \\ \sum_{Fi \in \mathcal{F}(m)} m(Fi) = 1, m(Fi) \geq 0, \forall Fi \subseteq \Theta; m(\emptyset) = 0, \end{array} \quad (16)$$

Finally, the obtained results are representing in Table 1.

Table 1. The obtained bba using the presented model

Criteria	$\{a\}$	$\{a, b, c\}$	$\{e\}$	$\{b, d\}$	$\{a, e\}$	Θ
m	0.092	0.203	0.082	0.082	0.203	0.338
bel	0.092	0.295	0.082	0.082	0.377	1

Table 1 gives the results of all ordered couples on the basis of their preference relation. Besides, we show that a new subset Θ is introduced that express the part of ignorance.

Indeed, using our model the expert expresses his assessments freely. By applying our presented solution, it is easy to see that our method aggregates all the elicited data.

Here, in the present example, the expert expressed his assessments only in some pairs of alternatives. Thus, a quantitative information is constructed from these incomplete and even uncertain preference relations.

We are then able to represent all the expert knowledge and to transform this information into quantitative data. We have obtained encouraging results since we have the same ranking of alternatives as expressed by the expert.

6 Conclusion

This paper is concerned with preference models including four relations: strict preference (P), weak preference (Q), indifference (I) and incompleteness (J).

The purpose was to establish conditions allowing to represent these four relations by numerical functions and thresholds under the belief function framework. Under this perspective, the paper proposes a new method based on Ben Yaghlane et al. approach and takes into account distinct levels of preferences.

As a future work, we will apply our method in multi-criteria decision making field, which can be interesting in eliciting expert judgments. Our proposed model will be applied through real application: Catering selection problem.

References

1. Ben Yaghlane, A., Denoeux, T., Mellouli, K.: Constructing belief functions from expert opinions. Proceedings of the 2nd International Conference on Information and Communication Technologies: from Theory to Applications (ICTTA06), Damascus, Syria pp. 75–89 (2006)
2. Bryson, N., Mobolurin, A.: A process for generating quantitative belief functions. European Journal of Operational Research 115, 624–633 (1999)
3. Ennaceur, A., Elouedi, Z., Lefevre, E.: Introducing incomparability in modeling qualitative belief functions. Proceedings of the International Conference on Modeling Decisions for Artificial Intelligence, Springer-Verlag, Catalonia, Spain pp. 382–393 (2012)
4. Fodor, J., Roubens, M.: Fuzzy strict preference relations in decision making. Proceedings of the Second IEEE International Conference on Fuzzy Systems pp. 1145–1149 (1993)
5. Pal, N., Bezdek, J., Hemasinha, R.: Uncertainty measures for evidential reasoning I: A review. International Journal of Approximate Reasoning 7, 165–183 (1992)
6. Pal, N., Bezdek, J., Hemasinha, R.: Uncertainty measures for evidential reasoning II: A review. International Journal of Approximate Reasoning 8, 1–16 (1993)
7. Perny, P., Roy, B.: The use of fuzzy outranking relations in preference modelling. Fuzzy Sets and Systems 49(1), 33–53 (1992)
8. Roubens, M., Vincke, P.: Preference modelling. Springer-Verlag, Berlin (1985)
9. Roy, B.: Pseudo-orders: Definition, properties and numerical representation. Mathematical Social Sciences 14, 263–274 (1987)
10. Shafer, G.: A Mathematical Theory of Evidence. Princeton University Press (1976)
11. Smets, P.: The application of the Transferable Belief Model to diagnostic problems. International Journal of Intelligent Systems 13, 127–158 (1998)

12. Smets, P., Kennes, R.: The Transferable Belief Model. Artificial Intelligence 66, 191–234 (1994)
13. Wong, S., Lingras, P.: Representation of qualitative user preference by quantitative belief functions. IEEE Transactions on Knowledge and Data Engineering 6, 72–78 (1994)