Abstract— The multi-source information holds a great importance in processing complex and imprecise data. Unfortunately, it requires an adequate formalism capable to modelize and to fuse several information. The evidence theory distinguishes from all formalism by its capacity to modelize and treat imprecise and imperfect data. In this context, the high resolution images represent a huge amount of data and needs multi-source information to perform pattern recognition. In this paper, we present an adaption of the distance operator introduced by Denoeux for estimating belief functions. This proposed approach will be used to classify forest image remote sensing by identifying the tree crown classes.

Keywords: multi-source, evidence theory, classification

I. INTRODUCTION

The evolution of embedded technologies in satellites has revolutionized the remote sensing domain as well as its applications. One of the specialties that has undertaken a notable progress in recent years is pattern recognition. The increase of the sizes, the qualities and the resolutions of the remote sensing images has boosted research on the development of various models and theories for their treatments.

The manipulation of those kinds of images is crippled by an indeterminism in their processing. To curtail the potential risk of imprecision, we opt for the multi-sources approaches. However, this kind of approaches is characterized by a modeling complexity of the process of sources fusion.

The pattern recognition domain has benefited from the progress of the modeling formalisms such as the evidence theory [1, 7]. We distinguish this theory from its counterparts by a better modeling of uncertainty. By comparing it for example, with the Bayesian theory, it integrates more operators allowing to combine and to fuse a wide range of sources of information.

In this context, we tackle a problem of high-resolution remote sensing classification of a forest typed images. We take advantage of the evolution of the evidence theory to combine data of forest nature suffering from imprecision problems and uncertainty. The structure of the current paper is as follows: The first section is keen on presenting an approach of estimation belief functions based on Denoeux for estimating belief functions. This proposed approach will be used to classify forest image remote sensing by identifying the tree crown classes.

II. STATE OF ART OF BELIEF FUNCTION ESTIMATION

Many works on belief function estimation were led. Two kinds of methods to initialize belief functions were proposed. The approach proposed by Appriou [2] considers the belief structure must be compatible with several axioms leading to compatibility with the Bayesian approach [13]. The second approach was proposed by Denoeux [6], uses neighborhood information. Each nearest neighbor of a pattern to be classified is considered as an item of evidence.

III. BELIEF FUNCTIONS THEORY

The origin of belief functions theory started with work of Dempster which related to the theory of the statistical inference generalizing the Bayesian inference. Shafer proposed functions of belief as general framework of representation of uncertainties, including the theory of probability as a particular case. The belief functions theory was labelled at the beginning with the name of its authors: Dempster and Shafer [9]. Extensions to the Dempster–Shafer theory (DST) contributed to the enrichment of the belief functions theory [3, 4, 6, 10]. Ph. Smets suggested a model named transferable belief model (TBM) providing coherent non-probabilistic interpretation of the DST and clarifying the concept subjacent with it [11].

The belief functions theory is one of the theories largely used for information sources fusion considering the fact that it takes into account simultaneously the uncertainty of the
sources and the inaccuracy of information that it provided. It is reduced to the theory of probability and the theory of the possibilities in particular cases [8].

A. Information sources and power set

Bearing in mind the fact that each source of information is generally imperfect and has drawbacks, it is interesting to combine several sources in order to have thorough knowledge of the “world”. We consider that we have n sources of information S_i with i ∈ {1, …, n}.

These sources must make a decision on an observation x in a whole of k decisions C_1, …, C_k. Let Θ = {C_1, …, C_k} be the set of definition made up of k hypotheses and by the elements A_i, events of the frame of discernment \( 2^\Theta \) of the parts of Θ.

B. Belief mass functions

The belief mass function or the basic belief function \( m(A) \) (generally noted BBA) of an event A is the confidence strictly attributed to A without this one being able to be divided on the hypothesis which makes it up. The focal elements are the elements of \( 2^\Theta \) of not empty masses. If the source is perfect, information is precise and sure, there is thus a single hypothesis such \( H_i \) as \( m([H_i]) = 1 \).

The mass functions are then defined on each subspace of the set of disjunctions of \( 2^\Theta \) to values in [0,1]. The distribution of mass is written according to (1):

\[
m : 2^\Theta \rightarrow [0,1]

A \rightarrow m(A)
\]

Dempster proposes a conjunctive rule of combination between sources called conjunctive sum. This combination causes to assign the mass to propositions of which the number of elements is less than that of the original propositions. For two sources S_1 and S_2, having respectively \( m_1 \) and \( m_2 \) as BBA, we write the conjunctive sum \( [\cap] \) in the following form:

\[
m_{\cap} = m_1 \cap m_2
\]

which is written for an event A like:

\[
m_{\cap}(A) = \sum_{B \cap C = A} m_1(B) \times m_2(C)
\]

Evidential modeling makes it possible to represent at the same time the inaccuracy and uncertainty through two functions of credibility and plausibility, derived from the mass functions.

The Dempster-Shafer’s theory allows the fusion of several independent sources using the Dempster’s combination rule. It is defined like the following equation:

\[
m_{\oplus} = m_1 \oplus m_2
\]

For two sources S_1 and S_2, the aggregation of evidence can be written as follows:

\[
m_{\oplus}(A) = \frac{1}{1-K} \sum_{B \cap C = A} m_1(B) \times m_2(C) \ \forall A \subseteq \Theta, A \neq \emptyset
\]

Where K is defined by:

\[
K = \sum_{B \cap C = \emptyset} m_1(B) \times m_2(C)
\]

K reflects the conflictual mass between the combined evidence function. This mass varies in [0,1] relatively to their nature. If the sources are in agreement then K=0. On the contrary, if both functions are contradictory, then K=1 and the information cannot be fused.

C. Discounting

Discounting BBA is a vital pre-requisite step in the case we are treating contradictory sources of information. It aims at compromising one or several sources susceptible to contradict each other or even sometimes the reality. The first works on discounting belief function theory was developed by Shafer [13], axiomatized by Smets [14] and generalized by Mercier and Denoeux [16] and defined by:

\[
\begin{align*}
\{m(B) = (1 - \alpha) \cdot m(B) \ \forall B \subseteq \Theta \\
\{m(\Theta) = (1 - \alpha) \cdot m(\Theta) + \alpha
\end{align*}
\]

A discount rate equal to 1, means that the source is not reliable and the piece of information it provides cannot be taken into account. On the contrary, a null discount rate indicates that the source is fully reliable.

IV. ESTIMATION OF EVIDENCE-MASS FUNCTION

Several works have been done in evidence-mass function modeling. In this section we will focus on based distance modeling approach introduced by Denoeux [17].

A. Approach based on distance method

Let us consider a vector x with a known learning vector U corresponding to an object that we want to classify, if any vector x_i in the learning base is sufficiently close to x with regard to a distance d and found by the use of the K Nearest Neighbor (KNN) algorithm on the learning base L is deemed as a piece of evidence. This method will provide a basic belief function composed only of two elements which the class \( H_n \) of the neighbor founded in L and the ignorance \( \Theta \). Part of the belief will be given to the class \( H_n \) while the rest will be assigned to \( \Theta \).

\[
\begin{align*}
m([H_n]) = \alpha_i \phi_i(d_i) \\
m(\Theta) = 1 - \alpha_i \phi_i(d_i)
\end{align*}
\]

Where \( 0 < \alpha_i < 1 \) is a constant. \( \phi_i(\cdot) \) is a decreasing function that verify \( \phi_i(0) = 1 \) and \( \lim_{d \to \infty} \phi_i(d) = 0 \).

\( di \) is
the Euclidean distance between the vector \( x \) and its prototype. The function can be in an exponential form:

\[
\phi_i(d_j) = \exp(-\gamma_i(d_j)^2)
\]  

(9)

Fixing and optimizing \( \gamma_i \) are treated in [17].

B. Separable distance based method

In section A we have dealt with the modeling problem by using the vector \( x \) with all information it contains. Another strategy consists in modeling the information according to every characteristic \( x_j \) (with \( j \in \{1, J\} \)) of the vector \( x \) to classify.

\[
\begin{cases}
m_{ij}(H_a) = \alpha_{ij} \phi_i(d_{ij}) \\
m_{ij}(\Theta) = 1 - \alpha_{ij} \phi_i(d_{ij})
\end{cases}
\]  

(10)

Where \( d_{ij} \) represent the distance between the constituent \( x_j \) of the vector \( x \) and the component \( j \)th of the prototype \( i \) and where the function \( \phi_i \) can be expressed in the following way:

\[
\phi_i(d) = \exp(-\gamma_i(d)^2)
\]  

(11)

The use of Dempster combination operator allows merging those \( J \) belief functions. \( m_i \) is the resulting belief function:

\[
m_i = \bigoplus_{j \in \{1, J\}} m_{ij}
\]  

(12)

The unique belief function \( m \) is obtained by the same principle:

\[
m = \bigoplus_{i \in \{1, I\}} m_i
\]  

(13)

V. ESTIMATION OF EVIDENCE-MASS

In this section, we present our approach to adapt the separable method of evidence-mass function estimation for calculating belief function in a forest typed image. The modeling will be applied for the classification of trees through the information extracted from their crowns. Indeed, a belief function will be associated with every crown reflecting its degree of membership to the studied classes.

Let’s consider \( \Theta \) the frame of discernment constituted by four classes \( \Theta = \{C_1, C_2, C_3, C_4\} \). The nature of the studied image has leaded us to consider three sources \( S = \{S_1, S_2, S_3\} \) each one is a composite regarding the number of characteristics forming it. The proposed approach is constituted by two phases:

- The intra-source phase: in this phase, we consider the crown membership owing to its neighbors found in a learning base. Each neighbor is found by the use of the KNN algorithm and according to the studied characteristic of the source.
- The inter-source phase: The formed sources, initially calculated in the intra-source phase, are combined through the conjunctive rule. The final resulting evidence mass expresses the opinion of our system to the membership of the analyzed crown.

A. Intra-source fusion

The use of the separable distance approach is adequate thanks to the composite nature of the used sources. In intra-source phase, the fusion of sources via conjunctive operator of combination is used twice: the first use is the fusion of the evidence functions of our crown neighbors found in our learning base \( L \). Every neighbor found via the K Nearest Neighbors algorithm, will have its own evidence function conceived as the equation 10.

The second use of the conjunctive operator is realized when all composite characteristics belief functions of our source are finished. This fusion process has as purpose the combination of all these various evidence mass to express the view of the source \( S_i \) on the crown Membership.

B. Inter-source fusion

In this phase, we present the second stage of the fusion process: the inter-sources fusion allows us to construct the final belief function. Once the belief functions of crowns are calculated, we combine them with the Dempster operator. Finally, we obtain the belief function of the crown from which the tree can be classified. The use of the pignistic probability [15] transforms the resulting BBA to usual probabilities.

VI. ESTIMATION OF EVIDENCE-MASS FUNCTION FOR FOREST HIGH-RESOLUTION IMAGE

In this section, we present the practical aspect and the results of our work.

A. The used sources

In the belief function estimation process, we used three sources of information. Those sources that are characterized by their complementarities are:

- Level of grey information: this source study the crowns relatively to its level of grey mean.
- Texture information: a composite source which analyses the tree crowns by their level of grey organization.
- Forms information: a composite source which analyses the tree crowns by their structure.

B. Learning base

The modeling of the learning base requires attributing every saved crown with its characteristics. The heeded characteristics for texture source of information are mean, variance, contrast, entropy, energy and homogeneity. For the form source, we consider the area, diameter, perimeter and wellipsy. Finally for the grey level source, we consider the level grey mean as information.
C. Determination of the tree crown belief function

Let’s consider a crown Cr which we propose to identify its BBA by our estimating approach. We use the frame of discernment $\Theta$ defined II.A and a composite information source $S_i$ from those defined in the section III.A. We provide the calculated value of Cr for each characteristic $Char_j$ for every considered source of information.

The application of the KNN algorithm on the learning base crowns proportionally to the considered characteristic. The K crown neighbors given by KNN are considered as source of information. A fusion of those BBA with conjunctive operator gives us a single BBA which express the view of membership of Cr to the considered class proportionally to a characteristic of one source of information. This process is repeated for all characteristics of the source.

At this stage, we obtained J BBA each one corresponds to a characteristic. The fusion of those BBA gives us a single BBA corresponding to the considered source $S_i$. Everything done until now, represented in Figure 1, constitutes the intra-source phase.

At this stage, we obtained J BBA each one corresponds to a characteristic. The fusion of those BBA gives us a single BBA corresponding to the considered source $S_i$. Everything done until now, represented in Figure 1, constitutes the intra-source phase.

The inter-source phase, as shown in Figure 2, needs the I sources of information to merge into a single BBA which correspond to our crown Cr. This BBA helps us find to which class Cr belongs to.

D. Discounting the information sources

In the intra-source phase, we associate to each composite source BBA a coefficients resulting from the unreliability of certain sources characteristic. These coefficients are obtained studying the result of the image classification using only the studied characteristic. The percentage of good classification is our discounting coefficients.

![Figure 1. The intra-source phase diagram](image1.png)

![Figure 2. The inter-source phase diagram](image2.png)

| TABLE I. DISCOUNTING COEFFICIENT FOR SOURCE TEXTURE CHARACTERISTICS |
|---------------------------|-----------------|----------------|-----------------|----------------|
| Mean | Variance | Energy | Contrast | Entropy |
| Discounting coefficient | 0,4 | 0,4 | 0 | 0 |
| 0,5 |

| TABLE II. DISCOUNTING COEFFICIENT FOR SOURCE FORM CHARACTERISTICS |
|---------------------------|-----------------|----------------|-----------------|----------------|
| Area | Diameter | Perimeter | Wellepsy |
| Discounting coefficient | 0,4 | 0,4 | 0 | 0 |

In the inter-source phase we also attribute a discounting coefficient to each source $S_i$ with regards to its individual result in classification. The discounting coefficients of sources are shown in table 3.

| TABLE III. DISCOUNTING COEFFICIENT FOR THE CONSIDERED SOURCES |
|---------------------------|-----------------|-----------------|----------------|
| Texture | Level of grey mean | Form |
| Discounting coefficient | 0,4 | 0 | 0,2 |

VII. APPLICATION AND RESULTS

A. Classification of forest typed image

The proposed approach of belief function estimation is applied to a classification of forest typed image problem. Our study zone is a forested region in the administrative district of Jendouba in Tunisia, more specifically the town of Ain-
Drahim. We consider as discernment frame $\Theta$ the following class:

$\{C_1 =$ chene zen, $C_2 =$ chene liège, $C_3 =$ arboretum ; $C_4 =$ forest résineux\}

The image is segmented by the brownmien motion approach [8]. The choice of learning zones was based on the information contained in the forest inventory.

B. Results

To corroborate and validate our approach of classification we proceed to a set of tests. The tests are carried out on images which represent Trees plantations of the same species $C_i$. The figure 3 represents a chene zen area image that on which the classifier will be applied. The percentage of crowns belonging to the class $C_i$, represents the good classification rate.

Figure 3. Chene zen area

Figure 4 represents the classification of an area typically chene zen. The proposed classifier has no problem identifying the chene zen class even the chene liege class. We remark that there are some classification mistakes due essentially to segmentation errors.

Figure 4. Chene zen area classified

Figure 7 represents the result of classification of an arboretum area. Some errors in detecting the arboretum class crop up. Those errors are due to the conflict (similarity) existing between the arboretum class and the other’s in the frame of discernment which is proved by figure 8.

Figure 7. Arboretum area classified

The table IV shows the efficiency of our classification approach where the rate of good classification for chene zen and chene liege class is acceptable. For the arboretum and forêt résineux class the result shown in table below is due to inadequate use of the conflict management.

<table>
<thead>
<tr>
<th>species</th>
<th>Chene zen</th>
<th>Chene liège</th>
<th>Arboretum</th>
<th>Forêt résineux</th>
</tr>
</thead>
<tbody>
<tr>
<td>Tested crown</td>
<td>221</td>
<td>208</td>
<td>211</td>
<td>228</td>
</tr>
<tr>
<td>Chene zen</td>
<td>184 (83,25%)</td>
<td>47 (22,60%)</td>
<td>4 (1,89%)</td>
<td>38 (16,66%)</td>
</tr>
<tr>
<td>Chene liège</td>
<td>30 (13,57%)</td>
<td>123 (59,13%)</td>
<td>51 (24,17%)</td>
<td>51 (22,36%)</td>
</tr>
<tr>
<td>Arboretum</td>
<td>0 (0%)</td>
<td>28 (13,46%)</td>
<td>84 (39,81%)</td>
<td>51 (22,36%)</td>
</tr>
<tr>
<td>Forêt résineux</td>
<td>7 (3,18%)</td>
<td>10 (4,81%)</td>
<td>72 (34,13%)</td>
<td>88 (38,62%)</td>
</tr>
</tbody>
</table>

Our method was tested comparatively, as shown in figure 10, with a punctual classifier (a pixelized oriented approach for classification), denoted PC, usually used in similar problems. We had also compared our method to another variant of our approach, denoted SCC, using only the spectral information as unique source. The results show that our
method exceeds the punctual classifier in detecting all class. The results of our approach have also exceeded the SCC method in detecting chene zen, chene liege and arboretum but was surpassed in forest resinuous classification due to the spectral specificity of this class. This result not only proves the importance of the source fusion in classification process but also the complementarities of the chosen sources.

![Figure 10. Comparative graph of tested classifying methods](image)

**VIII. CONCLUSION**

In this work, we proposed a method to classify a forest typed high resolution image. Therefore, we adapt the based distance evidence function estimation approach to build belief function for each tree crown extracted from the image. We presented some results of our classifying model using the evidence function framework. The results are not as propitious as expected for some class due to the conflict mass resulting from the combination. In this paper, we used the Dempster's combination rule but his performance does not perfectly suit our treated problem. In further research papers, we will propose a new approach of conflict management to improve the percentages of good classed crown.

**REFERENCES**


