ABOUT THE USE OF DEMPSTER-SHAFER THEORY FOR COLOR IMAGE SEGMENTATION

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ABSTRACT

This paper focuses on the advantages of data and information fusion in color image segmentation. We propose an unsupervised segmentation method based on the Dempster-Shafer's Theory of Evidence. This theoretical framework provides a convenient tool which allows modeling uncertainty in situations where the available evidence is limited or weak. The tristimuli R, G and B, given by the sensor, are considered as three independent information sources. The basic idea consists in modeling the color information in order to have the statistical features of each region in the image. This model, obtained on training sets extracted from the three color planes, allows to reduce the classification errors concerning each pixel of the image. The number of regions in the color image is deduced from the three color planes by means of an automatic procedure allowing to determine the frame of discernment of the belief structures. The proposed segmentation algorithm has been applied to synthetic images to illustrate the proposed methodology and the advantages of the information fusion procedure in the color image segmentation problem. An application of this methodology on biomedical images is illustrated in order to detect a kind of skin cancer (melanoma).
Keywords: *Color Image, Segmentation, Dempster-Shafer's Theory, Information Fusion, Biomedical Images.*

1. INTRODUCTION

In color image segmentation, color of a pixel is given as three values corresponding to the well known tristimuli R (Red), G (Green) and B (Blue). Different kinds of colors spaces have been developped by several authors [1, 2, 3, 4]. They are derived from this representation of the color using linear and nonlinear transformations. In the framework of segmentation, each color model is more or less convenient, efficient or reliable [5]. The major problem consists in choosing the adapted color model for a specific application.

In our study, we choose to work only with the tristimuli (R, G and B) given by the sensor. Each color plane is considered as an information source which can be imprecise or uncertain. The basic idea of our purpose consists in combining these three information sources using the **Dempster-Shafer's Theory of Evidence** [6].

Traditionally, probability theory, which is inadequate in some cases as well known [7], is used for dealing with uncertain data. In the recent past, other models have been developed for handling imprecise knowledge (theory of fuzzy sets [8], possibility theory [9]or uncertain information (probability theory, theory of belief functions [6]). The use of belief functions as an alternative to subjective probabilities for representing uncertainty was later justified axiomatically by Smets [10] who introduced the Transferable Belief Model, providing a clear and coherent interpretation of the various concept underlying the theory. This well known tool in classification problemsprovides a convenient framework which allows modeling uncertainty in situations where the available evidence is limited or weak.

In the context of color image segmentation, we propose to emphasize a new unsupervised segmentation scheme based on the Dempster-Shafer theory. An original basic belief assignment is first introduced. The refinement concept introduced by Shafer [6] are used in order to determine the *a priori* unknown number of regions in a color image. In addition, the attenuation concept is given in order to eventually take into account the confidence level of each source of in-

formation that is to say the confidence level of each color plane.

This paper is organized as follows. In section 2, we introduce theoretical aspects allowing to describe the belief function theory of Evidence. Section 3 is devoted to present the proposed methodology of the segmentation procedure. Finally, belief functions are aggregated using the well known Dempster's operator of combination (section 4). This work has been applied to synthetic images to demonstrate the effectiveness of the segmentation procedure (section 5). An application of this methodology on biomedical images is then illustrated in order to detect a kind of skin cancer (malignant melanoma).

2. BELIEF FUNCTION THEORY

Reasoning under uncertainty is a well researched field of artificial intelligence with a number of different methods being suggested by researchers over the years based on Zadeh's possibility theory [8], Dempster-Shafer's Evidence theory, Bayesian probability theory, ... We choose to use Dempster-Shafer's Theory of Evidence which is regarded as an useful framework for representing and manipulating uncertain knowledge. It provides flexible input requirements and an efficient method for combining information obtained from multiple sources. In this section, a brief overview of the Evidence Theory, initially introduced by Dempster and reprise axiomatically by Shafer [6] and later by Smets [10], is provided.

2.1. Background

Let Θ represents the set of hypotheses H_n, called the **frame of discernment**. The knowledge about the problem induces a basic belief assignment which allows to define a **mass function** m from 2^Θ to $[0, 1]$ as :

$$m(\emptyset) = 0 \tag{1}$$

$$\sum_{H_n \subseteq \Theta} m(H_n) = 1. \tag{2}$$

Subsets H_n of Θ such that $m(H_n) > 0$ are called **focal elements** of m. The intersection of all the focal elements of a mass function is called the **core** of the mass function and defined as :

$$\mathcal{F} = \{A \subseteq \Theta \mid m(A) > 0\}. \tag{3}$$

From this basic belief assignment m, a **credibility function** $Bel()$ and **plausibility function** $Pl()$ can be computed. These functions (m, Bel and Pl) are derived from the concept of lower and upper bounds for a set of compatible probability distributions. In order to aggregate several information sources, Smets proposes two kinds of operators. For two pieces of evidence m_i and m_j, a conjunctive rule of combination

can be proposed :

$$\forall H_n \subseteq \Theta \qquad (m_i \cap m_j)(H_n) = \sum_{A \cap B = H_n} m_i(A).m_j(B) \tag{4}$$

or a disjunctive one. Under the closed-world assumption, some kind of normalization has to be performed. The conjunctive sum operation followed by Dempster normalization is the orthogonal sum operation (Dempster's combination operator). The belief function quantifies the impact of the Q pieces of evidence.

2.2. Refinements and coarsenings

In this subsection, we deal with the problem of information sources whose frames of discernment are different but compatible. It can be the case when features of the information sources are discriminating for different hierarchical level [6]. The main idea here, is that a frame of discernment Θ is obtained from another frame of discernment Ω by splitting some of all the elements of Θ [6]. Mathematically, it leads to specify for each element H_n of Θ for a source S_i, the subset $\omega\{H_n\}$ of Ω consisting of those possibilities into which H_n has been split. The sets $\omega\{H_n\}$ should build a disjoint partition of the frame of discernment Ω, and following the three axioms :

$$\forall H_n \in \Theta \qquad \omega\{H_n\} \neq \emptyset \tag{5}$$

$$\omega\{H_n\} \cap \omega\{H'_n\} = \emptyset \quad if \quad H_n \neq H'_n \tag{6}$$

$$\bigcup_{H_n \in \Theta} \omega\{H_n\} = \Omega. \tag{7}$$

In addition, we have :

$$\forall A \subseteq \Theta \qquad \omega\{A\} = \bigcup_{H_n \in A} \omega\{H_n\}. \tag{8}$$

So, $\omega\{A\}$ consists in all the possibilities in Ω which are obtained by splitting the elements of A. It involves an application which relies 2^Θ to 2^Ω such as :

$$\omega : 2^\Theta \rightarrow 2^\Omega. \tag{9}$$

This application thus defined is the **refining**. So, Ω is called the refinement of Θ. On the other side, Θ is a coarsening of Ω. A proposition defined by a subset A of Θ will also be represented by a subset $\omega\{A\}$ of Ω. In terms of propositions, we can say that 2^Θ may be considered as a subset of 2^Ω. In practice, the refinement of a frame of discernment has to be made in order to produce a new point of view about the knowledge we have. The frames of discernment provides by refinements or coarsenings are different but compatible. One has to call of a family of frames of discernment.

2.3. Belief function attenuation

An additional aspect of the theory concerns the attenuation of the basic belief assignment m by a coefficient α. For a source j, the **attenuated belief function** $m_{(\alpha,j)}$ can be written as :

$$m_{(\alpha,j)}(H_n) = \begin{cases} \alpha_j.m_j(H_n) & \text{if } H_n \neq \Theta \\ 1 - \alpha_j + \alpha_j.m_j(\Theta) & \text{otherwise.} \end{cases} \tag{10}$$

The Dempster-Shafer Theory of Evidence is a rich model for uncertainty handling as it allows the expression of partial belief. The main difficulty consists in the adjustment of the attenuation factor α_j.

2.4. Decision Analysis

The problem of decision making may refer to the credibility function, the plausibility function or to a specific probability distribution called the pignistic probability [10]. The decision rule is based on the decision function δ which assigns a feature to classify X' to the hypothesis H_n following :

$$\delta(X') = H_n \text{ iif } H_n = \arg\max_{H_i \in \Theta}(Pl(H_i)). \tag{11}$$

In this paper, we propose to use the theoretical framework defined by the belief function theory in order to classify the pixels into homogeneous regions based on the color information.

3. THE SEGMENTATION SCHEME

A segmentation of an image I is a partition of I into disjoint nonempty subsets $\mathcal{R}_u$ for $u = 1, 2, ..., U$ such as :

$$I = \bigcup_{u=1}^{U} \mathcal{R}_u. \tag{12}$$

The proposed method is based on the color information contained in the image. It is decomposed in four steps :

- Finding the number of regions U,

- Modeling the belief on the training sets,

- Combining the Q information sources with the Dempster's rule,

- Taking a decision to classify each pixel to a region $\mathcal{R}_u$.

For our segmentation scheme, we choose to work with $Q = 3$ where the different information sources are the three color planes R, G and B. The first step consists in determining the number of regions in an image, that is to say determining U. The second one corresponds to the basic belief assignment based on the

frame of discernment extracted from the color image. Finally, the belief structures defined for each source of information are aggregated in order to decrease significantly the uncertainty for the later classification process.

3.1. Determination of the number of regions in an image

Each color plane may distinguish a number of regions which can be different from a color plane to another that is to say that each color plane may have a frame of discernment which can be **different** but **compatible** with the two others. The problem consists in finding a single frame of discernment compatible with each color plane. Let be respectively Θ_R, Θ_G and Θ_B the frames of discernment corresponding to each color plane and defined by :

$$\Theta_R = \{\mathcal{R}_r\} \text{ for } r = 1, 2, ..., R, \tag{13}$$

$$\Theta_G = \{\mathcal{R}_g\} \text{ for } g = 1, 2, ..., G, \tag{14}$$

$$\Theta_B = \{\mathcal{R}_b\} \text{ for } b = 1, 2, ..., B. \tag{15}$$

We can define the number of regions that a color plane is able to distinguish by analysing the grey level histogram of this plane by means of a *Maximum Entropy Principle* (MEP) as in [11]. Then, by splitting the compatible regions in each color plane as long as it is necessary (that is to say by refining the frame of discernment), we obtain a unique frame of discernment for all the color planes. Finally, we obtain the number of regions U and the associated frame of discernment Θ defined by :

$$\Theta = \{\mathcal{R}_u\} \text{ for } u = 1, 2, ..., U \tag{16}$$

Figure Fig.1 illustrates the three compatible frames of discernment extracted from the color planes with respectively three for the red plane, and two for the green and blue planes. Finally, the resulting num-
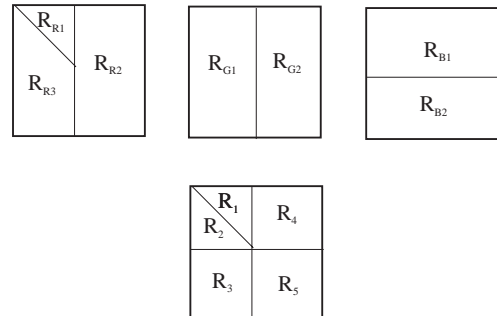

Figure 1: Determination of the number of regions U

ber of regions is deduced from the intersection of the different regions $\mathcal{R}_r, \mathcal{R}_g, \mathcal{R}_b$. Only compatible regions are preserved to lead to the final frame of discernment Θ with five regions.

3.2. Modeling the belief on the training sets

In order to assign the belief functions, each color plane is assimilated to an information source S_q for $q \in 1, ..., Q$. Let us consider a basic belief assignment m_q defined as :

$$m_q : 2^\Theta \longmapsto [0, 1] \qquad (17)$$

with $m_q(\emptyset) = 0$ and $\sum_{\mathcal{R}_u \subseteq \Theta} m_q(\mathcal{R}_u) = 1$. We make the hypothesis that the data extracted from one information source S_q among Q sources can be represented as a gaussian distribution. Under this assumption, a membership probability to a region $\mathcal{R}_u$ can be written :

$$P(x_q/\mathcal{R}_u) = \frac{1}{\sigma_u \sqrt{2\pi}} \exp^{-\frac{(x_q - \mu_u)^2}{2\sigma_u^2}} \qquad (18)$$

where x_q is a realization of a Q-dimensional random variable X. In our case, x_q is the value of a pixel $P_{(i,j)}$ for one of the three color planes. The values $\mu_u = E(X)$ and $\sigma_u^2 = E(X - E(X))^2$ are respectively the mean and the variance on the region $\mathcal{R}_u$. These values are replaced by their statistical approximations computed on all the pixels contained in the region $\mathcal{R}_u$. The advantage of Dempster-Shafer theory lies in representing uncertainty by means of a belief on the whole frame of discernment 2^Θ. To built each membership function of the subsets in 2^Θ, we use the min operator as it follows :

$$P(x_q'/\mathcal{R}) = \min(P(x_q'/\mathcal{R}_u), \ldots, P(x_q'/\mathcal{R}_v))) \qquad (19)$$

with : $\mathcal{R} = \bigcup_{i \in [u,v]} \mathcal{R}_i$.

3.3. Basic Belief Assignment

The previous modeling allows to generate the belief functions m_q. For each source S_q, the belief m_q given for each hypothesis $\mathcal{R}_u$ depends on the membership probability with respect to :

$$\forall \mathcal{R}_u \in 2^\Theta \qquad m_q(\mathcal{R}_u) = R_q.P(x_q'/\mathcal{R}_u) \qquad (20)$$

where x_q' corresponds to the tristimuli values of the pixel $P_{(i,j)}'$ to classify. The coefficient R_q is a normalization coefficient. It allows to verify the condition given by equation (2). It is defined as :

$$R_q = \frac{1}{\sum_{\mathcal{R}_u \in 2^\Theta} P(x_q'/\mathcal{R}_u)}. \qquad (21)$$

3.4. Attenuation

In order to quantify the uncertainty of a belief function, uncertainty measures derived from the Shannon entropy can be introduced. Klir [12] proposes two kind of uncertainty measures for a belief structure

$m_q()$. We choose to use the confusion measure which is defined as :

$$C(m_q) = - \sum_{\mathcal{R}_u \in \mathcal{F}} m_q(\mathcal{R}_u).\log_2 Bel(\mathcal{R}_u). \qquad (22)$$

This measure defines the confusion between the masses attributed to the hypotheses and gives an information about the mass distribution within a source. It is maximum when the mass function is equally distributed on the singleton hypotheses and has a lower bound equal to 0. We propose to use the confusion measure for discounting the belief structure in order to attenuate the information source which produces ambiguity for the mass distribution. We propose to determine the confusion measure $C(m_q)$ for each source S_q and normalize it as follows :

$$\alpha_q = 1 - \frac{C(m_q)}{C_{max}} \qquad (23)$$

with $q \in \{1, \ldots, Q\}$. In the equation (23), C_{max} represents the maximum of the confusion related to the considered frame of discernment. It depends on $Card(\Theta)$ [12]. With this normalization, the source S_q which is the most confused will be considered as to be the less informative and then the attenuation factor will be the lower. On the contrary, for the less confused source, α_q will tend to 1. This coefficient α_q allows then to determine the new basic belief assignment by means of the equation (10).

4. FUSION AND DECISION

The Dempster-Shafer's theory allows the fusion of the three sources using the Dempster's combination operator. For all $\mathcal{R}_u \in 2^\Theta$, the resulting mass function is defined as :

$$m(\mathcal{R}_u) = m^{S_1}(\mathcal{R}_u) \oplus m^{S_2}(\mathcal{R}_u) \oplus m^{S_3}(\mathcal{R}_u) \qquad (24)$$

where S_1, S_2 and S_3 are the three color planes. The resulting normalization coefficient $\mathcal{K}$ evaluates the conflict between the three sources. Its value is essential in order to improve the later decision rule. Classically, three criteria can be used as decision functions : the maximum of credibility Bel, the maximum of plausibility Pl or the maximum of pignistic probability P_p. We choose to base our decision rule with a function δ which assigns a pixel to classify $P_{(i,j)}'$ to the hypothesis $\mathcal{R}_u$ following :

$$\delta(P_{(i,j)}') = \mathcal{R}_u \text{ iif } \mathcal{R}_u = \arg\max_{\mathcal{R}_i \in \Theta}(Pl(\mathcal{R}_i)). \qquad (25)$$

5. SIMULATIONS

This section is devoted to present some results concerning the segmentation scheme in order to evaluate the methodology. The proposed approach is applied to synthetic and biomedical images.

5.1. Synthetic images

The synthetic color image used to check our segmentation scheme is presented in Fig. 2. It corresponds to a color card composed of four regions (left part). The associated luminance image is given in the right part. The three color planes respectively R, G and B
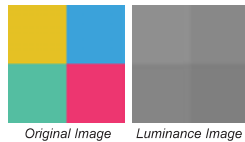

Original Image   Luminance Image

Figure 2: Synthetic images

are given in figure Fig. 3. These three images allow to understand the difficulty to distinguish the four regions contained in the color image. A gaussian noise

Red color plane   Green color plane   Blue color plane
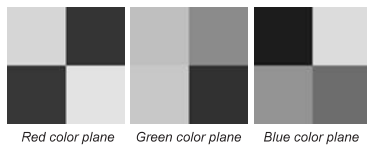
Figure 3: Red, Green and Blue color planes

with different signal-to-noise ratio is added on the original image. Some examples are presented in figure Fig. 4. We propose an evaluation of our method

Figure 4: Noisy synthetic images

and a comparison with a classical algorithm based on the fuzzy C-means procedureThe figure Fig. 5 gives the good classification rate obtained for each checked algorithm. We compute these results with two versions of our algorithm. The first version corresponds to the use of the attenuation factor in the segmentation scheme and the second one do not use this factor. To summarize, it is obvious to note that the use of an attenuation factor decreases the good classification rate. Indeed, it is due to the very poor number of information sources (3). Let us note, however, that this attenuation procedure improves significantly results in other situation (see [13]).

The figure Fig. 6 gives the good classification rate obtained for the algorithm without attenuation factor and for the same algorithm with a spatial filtering. This filter significantly improves the good classification rate.
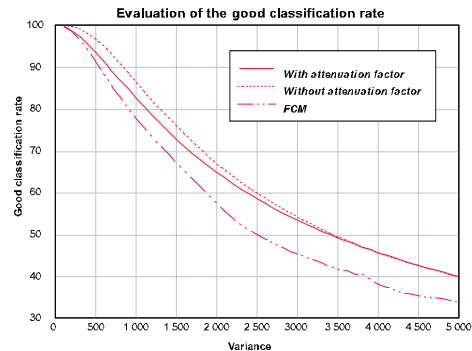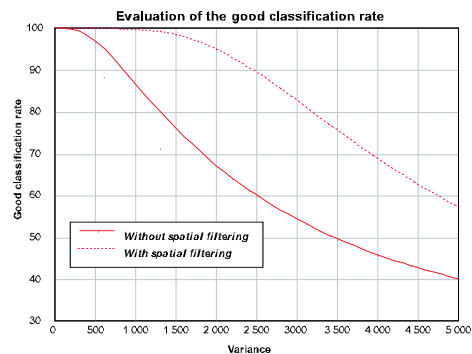

Figure 5: Good classification rate

Figure 6: Spatial filtering evaluation

5.2. Biomedical images

In Dermatology, melanoma is an increasing form of cancer. It has increased twice times for 15 years in Canada and it is now 3% of cancers in the USA. The rates of clinical diagnostic accuracy are about 65% at the very best. In particular, it is very difficult to distinguish some atypical lesions - which are benign - from melanoma because they have the same properties according to the well known ABCDE rules used by dermatologists [14]. There is a visual inspection problem for the atypical lesions class. Unnecessary excisions are often practise for these lesions. The variability of colours and shapes (see Figure 7) can lead to several interpretation by different dermatologists. However, melanoma is well suited for color image pro-
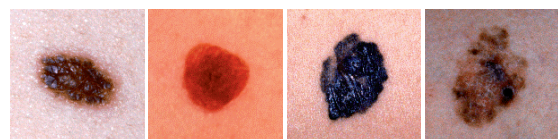

Figure 7: Original images of lesions

cessing because it is on the skin. Some researches [15] have shown the advantages to use image processing in dermatology. The essential difficulty is to design robust and relevant parameters to ensure the separation

between melanoma and benign lesions, in particular the atypical lesions (benign), called naevus, which can be clinically mistaken for melanoma. The first step of the processing consist in the segmentation of the lesion from the surrounding skin.

Some images, in the context of dermatology, are presented in the figure Fig. 8. First row corresponds to the original color images. The respective two other rows represent the segmentation scheme results with the decision concerning the maximum of credibility (second row) and the maximum of plausibility (third row). We can note that the lesion (red color) is correctly extracted from the safe skin (white color). The blue color corresponds to pixels which cannot be classify either to the safe skin or to the lesion. The
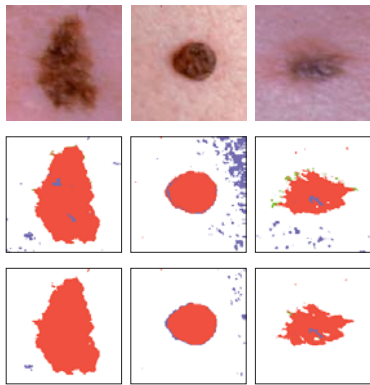

Figure 8: Results images

algorithm was checked on a database made of 300 color images of dermatological lesions. Based on this database, the good segmentation result is over 95%. The encountered segmentation problems are essentially due to images where the lesions are partially cluttered.

6. CONCLUSION

This paper focuses on the advantages of data and information fusion in color image processing, especially in the framework of segmentation. We have presented an original color image segmentation procedure using Dempster-Shafer's theory. The proposed methodology consists first in initializing the belief functions with probability densities obtained by learning. An unsupervised procedure has been proposed in order to determine the frame of discernment composed with the number of regions in the color image. In a second step, the opportunity to attenuate the belief structures has been studied. Its limits have been underlined. By means of uncertainty measures, we determine the attenuation of the belief assignment based on the confusion measure. The three information sources extracted from the three color planes are then aggregated to classify each pixel of the image.

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