Conflict management with dependent information sources in the belief function framework

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***Abstract*—Several rules were proposed in the context of evidence combination to deal with the conflict generated between the combined information sources. However, in the belief function framework, as far as we know only one rule exists for managing dependent bodies of evidence which is the cautious rule. Unfortunately, this rule does not give the conflict its initial alarm role and does not overcome the absorbing effect of the conflictual mass. The Combination With Adapted Conflict (CWAC) was proposed as a solution to this problem but only when dealing with independent information sources. Based on the cautious rule and inspired from the behaviour of CWAC towards the conflict, our contribution in this paper is to propose such a solution in the context of combining dependent sources.**

I. INTRODUCTION

In almost every domain, we need to use data. It represents a basic element in achieving processes and taking decisions. That is why it has to be well managed and explored in a way that makes the information more useful and adapted to the problem. Within the framework of data management, multiple studies have been carried out in different domains specially to manage uncertainty and imprecision in data [2], [7].

To quantify these kinds of data, the belief function theory [3], [6], also named Dempster-Shafer theory, was proposed. This theory can treat both imprecision and uncertainty and also enables to combine multiple sources of information in order to have valuable information. In some cases, sources of information cannot be useful apart, in contrary, they have to be combined in order to get a more valuable one. Unfortunately, when combining sources, information can be non compatible, by having non common knowledge or by an existing contradiction between them, what is called a conflict. In this case, the conflict expressed by a mass on the empty set, also named as the conflictual mass, must play the role of a problem indicator between the sources and behave as an alarm signal, so, we can detect where the problem stands.

Multiple combination rules have been proposed, in the belief function framework, but not all seem to well behave towards the conflict between the sources. Several ones were proposed when dealing with independent sources of information but a main one, the cautious rule [17], was proposed for dependent sources. Although it exists it does not enable to manage the absorbing effect of the conflictual mass. A solution, the Combination With Adapted Conflict (CWAC) [4], [5] was proposed to overcome this problem when dealing with independent sources.

This rule is an efficient tool, when the objective is to give the conflict its main role of contradiction indicator between the sources. The performance of this rule was proved when compared to the conjunctive rule [12], [14], a basic rule of combination, that affects a value on the mass of the empty set and does not deal with its minimization in case the value is high.

In this paper, the proposed approach is a similar solution, inspired from the behaviour of the CWAC rule, when combining dependent sources [10], [19]. Managing the conflict must not be neglected because of the importance of its value which is essential to know what information or opinion to trust and which not.

Our objective is then to ensure that the conflict plays its main role of an alarm signal. We, hence, propose a combination rule that gives back its role to the conflict and inhibits the high value of the conflict that is given after the combination of multiple dependent sources.

This paper is organized as follows. In Section 2, we introduce some basic concepts needed in our work when using the belief function theory, more details can be found in [6]. In Section 3, we present some classical combination rules studied in the belief function framework. Then, our proposed contribution will be explained in Section 4. Tests on synthetic belief functions highlighting the performance and the efficiency of the proposed rule are presented in Section 5. Finally, we conclude with a brief summary and advances some future works.

II. BASIC CONCEPTS OF THE BELIEF FUNCTION THEORY

The belief function theory [3] is a formal framework for representing, managing and reasoning with uncertain knowledge.

A. representing information

Let $\Omega = \{h_1, h_2, ..., h_K\}$ be the frame of discernment, a non empty set including the elementary hypotheses of a given problem.

A basic belief assignment (*bba*) is defined as follows:

$$m : 2^{\Omega} \longrightarrow [0,1]$$

$$\sum_{A \subseteq \Omega} m(A) = 1 \quad (1)$$

$m(A)$ is defined as a basic belief mass denoted *bbm*, representing the part of belief exactly assigned to the subset A of Ω. When we have $m(A) > 0$, these subsets are called the focal elements of m. As for representing $m(\emptyset)$, on one hand Shafer has proposed a normalized version of representing it as: $m(\emptyset) = 0$ [6]. On the other hand, for Smets [14], [15] it represents the amount of conflict between the sources, so the value of $m(\emptyset)$ must be kept because it reflects a problem in the fusion process.

B. Special belief functions

In this paper, we only present some notions of the belief function theory and here are some special belief functions that will be used while building our proposed rule.

1) Commonality function: We call a commonality function denoted q the total mass attached to the largest possible subset of Ω [9]. A commonality function is defined as follows:

$$q\text{: } 2^{\Omega} \rightarrow [0,1]$$

$$q(A) = \sum_{A \subseteq B} m(B) \quad (2)$$

2) Simple support function (ssf): A Simple support function is the mass function with only two focal elements A and Ω and ω in [0,1] [11]. It is defined as follows:

$$m(A) = 1 - \omega, \qquad \forall A \subset \Omega \quad (3)$$

$$m(\Omega) = \omega \quad (4)$$

$$m(B) = 0, \qquad \forall B \neq A \subset \Omega \quad (5)$$

A simple bba can also be noted A^{ω} (this notation will be used further in this paper), with A focal element and $\omega \in$ [0,1].

3) Non-dogmatic belief function: We call a function non-dogmatic when $m(\Omega) > 0$.

III. COMBINING MULTIPLES INFORMATION SOURCES

As we mentioned in the introduction, the purpose of our approach is to find a solution to give the conflict the role of a problem indicator between dependent sources and that by resolving the problem of the absorbing effect generated by the conflictual mass. To purchase this goal, we need to fuse information to obtain a synthetised and more valuable information. Multiple combination rules were proposed to fuse information. We find two main categories: combination rules dealing with independent information sources and combination rules dealing with dependent ones.

A. Combining independent pieces of evidence

In this paper, we choose to present only the used combination rules while building our approach. We, hence, studied their behaviour towards the conflict and management of its increasing value throughout the combinations.

1) Conjunctive combination rule: The conjunctive combination rule [6], [8] is one of the rules used when sources are considered reliable and distinct. It is a rule where the value of the mass of the empty set is kept and considered as the degree of conflict between the combined sources. The conjunctive combination rule is defined as follows:

$$(m_1 \bigcirc\!\!\!\!\cap\, m_2)(A) = \sum_{B,C \subseteq \Omega : B \cap C = A} m_1(B).m_2(C) \quad (6)$$

The value of the $m(\emptyset)$ represents the degree of the conflict that exists between the combined sources.
When using this rule only the common belief between these sources are taken into consideration, the non shared ones are ignored, which represent the conflict when the sources are in contradiction.
We used this rule as a tool to highlight the absorbing effect of the conflictual mass. The absorbing effect of the conflictual mass is defined as the fact of the increase of the value of the empty set by the increase of the number of the sources to combine.

2) Dempster's combination rule: The conjunctive rule is considered as the unnormalized version of the Dempster rule [6] [16]. The Dempster rule is hence used when combining evidence of different sources.
The value of the conflictual mass, according to this rule, is redistributed over the focal elements. This is made by using a normalization factor denoted $K = (1 - m_1 \bigcirc\!\!\!\!\cap\, m_2(\emptyset))^{-1}$.
The Dempster rule of combination is defined as follows:
$\forall A \subseteq \Omega, A \neq \emptyset \ (m_1 \oplus m_2)(\emptyset) = 0$

$$(m_1 \oplus m_2)(A) = K.(m_1 \bigcirc\!\!\!\!\cap\, m_2)(A) \quad (7)$$

If $(m_1 \bigcirc\!\!\!\!\cap\, m_2)(\emptyset)$= 1, then we can say that the mass functions are completely opposite. In such a case, Dempster's rule cannot be defined.
If $(m_1 \bigcirc\!\!\!\!\cap\, m_2)(\emptyset) \neq 0$, then this mass must be redistributed over the other focal elements.

3) Combination With Adapted Conflict (CWAC): After studying the behaviour of the conjunctive combination rule, it showed that it has the problem of the absorbing effect of the conflictual mass which makes the conflict losing its initial role of alarm signal. With the increase of the value of the empty set, we cannot say if it reflects a real problem between the sources. The idea of the Combination With Adapted Conflict [4], [5] is to give back the role of the conflict as a problem indicator by overcoming this absorbing effect

generated by the conjunctive combination.
The CWAC rule applied to an important number of sources to combine, adapts the value of the conflict to make it a real problem indicator. The conflict is kept during the fusion.
The CWAC rule is based on the use of a distance measure to compute the similiarity between sources.
We, so, define the CWAC rule as follows: $\forall A \subseteq \Omega$ and $m_{\bigcirc\!\!\!\!\cap}(\emptyset) \neq 1$

$$m_{\ominus}(A) = (\ominus_i m_i)(A) = D m_{\bigcirc\!\!\!\!\cap}(A) + (1-D) m_{\oplus}(A) \quad (8)$$

with D= $max_{i,j}[d(m_i, m_j)]$ with $i \in [1,N]$ and $j \in [1,N]$.

The CWAC rule uses the Jousselme et al.s distance [1] as it represents one of the most used distance in the belief function framework. It is defined as follows:

$$d(m_1, m_2) = \sqrt{(m_1 - m_2)^t \mathcal{D}(m_1 - m_2)}$$

with $\mathcal{D}$ the Jaccard index:

$$\mathcal{D}(A, B) = 1 \;\; if \;\; A = B = \emptyset \quad (9)$$

$$\mathcal{D}(A, B) = \frac{A \cap B}{A \cup B} \;\; \forall \;\; A, B \in 2^{\Omega} \quad (10)$$

We can then define the Combination With Adapted Conflict to be an adapted weighting between the conjunctive combination and the Dempster one.
So, when $d(m_1, m_2) = 0$, the rule acts towards the conflict the same way as the Dempster rule (the sources are in agreement and does not generate a conflict).
When $d(m_1, m_2) = 1$, the sources are in disagreement, a conflict is generated and the rule behaves as the conjunctive one towards the conflict.

B. Combining dependent pieces of evidence: Cautious combination rule

When considering sources to be non distinct, we apply the Cautious combination rule proposed by Denoeux [17]. This rule relies on the conjunctive combination for reliable dependent sources [13] and that for non dogmatic belief functions. As it relies on the conjunctive combination, it has the same behaviour towards the conflict. In a further example, we will show that with an increase of the number of bbas to combine, the value of the conflict increases as well.

The cautious combination rule can be simply defined as the conjunctive combination of simple support functions (ssf) with the minimum of weight, after the canonical decomposition of the ssf [11].

The canonical decomposition enables one to consider any belief function as the result of the combination of distinct and non distinct bodies of evidence. For our work, we will only need the conjunctive canonical decomposition.
We have a unique representation of the canonical decomposition of m, if m in non dogmatic. We use separable belief functions [11], where a separable bba is the result of the $\oplus$ combination of simple bbas and it is defined as follows:

$$m = \oplus A^{\omega(A)} \quad (11)$$

Let us define the process of the cautious rule:

- First, it transforms the masses into commonalities (equation (2)).
- Second, it computes the weight functions of simple generalized mass functions.
  Let $\omega(A)$ be the weight obtained through the use of the commonalities as follows:

$$\omega(A) = \prod_{B \supseteq A} q(B)^{(-1)^{|B|-|A|+1}} \quad (12)$$

$$= \begin{cases} \frac{\prod_{B \supseteq A, |B| \notin 2N} q(B)}{\prod_{B \supseteq A, |B| \in 2N} q(B)} & \text{if } |A| \in 2N \\ \frac{\prod_{B \supseteq A, |B| \in 2N} q(B)}{\prod_{B \supseteq A, |B| \notin 2N} q(B)} & \text{otherwise} \end{cases} \quad (13)$$

  with $2N$ the set of even natural numbers.
  Based on the obtained weights, we look for the minimum weight. We can obtain then new generalized mass functions.

- Finally, to obtain the final mass function, as result of the cautious rule, it uses the fusion operator to combine these new simple generalized mass functions.

The combination phase is the main important step in the cautious rule, which gives the final bba, with which we can evaluate if the empty set is an alarm signal or not, and so then deduce the importance of the conflict between the sources.

1) Unnormalized cautious rule: As we mentioned the cautious rule has two versions. The first one, we introduce here is the conjunctive version [18].
According to the announced principles of the cautious rule, after computing the weight functions, we compute $m_1 \bigcirc\!\!\!\!\wedge\, m_2$ as the $\bigcirc\!\!\!\!\cap$ combination of GSBBAs $A^{\omega_1(A) \wedge \omega_2(A)}$, $\forall \; A \subset \Omega$ such that $\omega_1(A) \wedge \omega_2(A) \neq 1$, defined as follows:

$$m_1 \bigcirc\!\!\!\!\wedge\, m_2 = \bigcirc\!\!\!\!\cap_{A \subset \Omega} A^{\omega_1(A) \wedge \omega_2(A)} \quad (14)$$

with $\wedge$ denotes the minimum operator, $\bigcirc\!\!\!\!\wedge$ the cautious rule, and GSBBAs (generalized simple bba) is the notion used to extend the canonical decomposition of a separable bba to any non dogmatic bba.
While combining mass functions with this version of the cautious rule, we obtain a value on the empty set and by applying it on an important number of masses to combine the value of the conflictual mass increases. We, then, conclude that the cautious rule has the problem of the absorbing effect generated by the conjunctive combination (see next example).

2) The normalized cautious rule: After studying the conjunctive version of the cautious rule, let us introduce the normalized version which is inspired from the Dempster rule to overcome the value of the conflictual mass. It is defined as follows:

$$m_1 \owedge^* m_2 = \bigoplus_{\emptyset \neq A \subset \Omega} A^{\omega_1(A) \wedge \omega_2(A)} \tag{15}$$

The cautious rule has an unnormalized version which is based on a conjunctive combination, and so, it has the same behaviour towards the conflict, and a normalized version of cautious rule which does not deal with the conflict. Both versions will be used in our proposed approach.

Example: Figure 1 presents the results of the cautious combination rule applied on multiple sources. Let us have: $\Omega = \{\omega_1, \omega_2\}$ and N bbas to combine.
The belief functions are defined such as follow:

$$m(\{\omega_1\}) = \epsilon_1 \qquad m(\{\omega_2\}) = \epsilon_2 \qquad m(\Omega) = 0.1$$

with $\epsilon_1 \geq 0$, $\epsilon_2 \geq 0$ and $\epsilon_1 + \epsilon_2 = 0.9$.

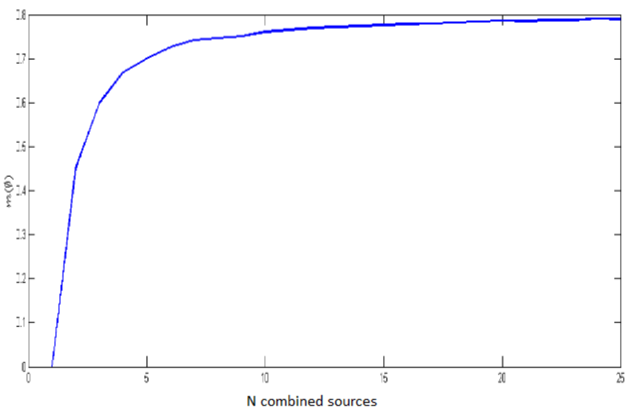

Figure 1. Conflict evolution of the cautious combination of N bbas

This figure shows that we have a degree of conflict between the sources when we combine N bbas. However, the value of this conflict increases by combining a bigger number of bbas which induces us to say that applied to an important number of functions to combine the cautious rule shows a problem of an absorbing effect. This absorbing effect prevents us to say whether it exists a real problem between the sources or not. The conflict does not play its role of alarm signal.

IV. Cautious Combination With Adapted Conflict

The idea of our proposed rule is to treat non distinct bodies of evidence and give the conflict its initial role as an alarm signal by overcoming the absorbing effect generated by the cautious rule as showed in the previous example. In the following, sources are considered to be dependent. We propose a combination rule called the cautious combination with adapted conflict. This rule must give better results then the cautious one, so the value of the conflict can have a meaning and reflect a real problem between the combined sources.

The Cautious CWAC rule, as its name mentions, is based on the cautious rule and the combination with adapted conflict one:

- First, we use the Combination With Adaptative Conflict rule (CWAC) to take the advantage of its behaviour towards the conflict when overcoming the high value of the conflictual mass, knowing that it gives good results applying on the conjunctive combination.

- Second, we use the cautious combination rule, with both its normalized and unnormalized version because it is the most commonly used rule that treats dependent sources and it has the same problem as the conjunctive rule when it comes to the conflict management, where it does not handle the value of the mass of the empty set.
  That is to say that the mass of the conflict increases with the increase of the number of combined sources. In this case, the conflict is an absorbing element.

We will though compare the behaviour of these both rules with the behaviour of our proposed cautious CWAC rule. The main goal here is that for a given problem, we have to distinguish between a true problem and the effect generated due to the absorbing element which is the empty set.
In the case of the cautious rule, the conflict is not an indicator but an absorbing element as well as in the conjunctive rule, where a series of conjunctive combinations generates a high value of conflict. It tends to bring a value equal to 1 to the empty set. So, it would become hard to distinguish between a real problem or an absorbing effect.

Taking into consideration these ascertainments, the first idea is to apply the CWAC rule, which uses distance measures to compute the similarity between sources, more particularly Jousselme distance as well as a weighting function, on the conjunctive and Dempster's rule to inhibit the effect of the absorbing element and make it be an alarm signal that enables us to detect the real problem.
Inspired from this behaviour of the CWAC rule, we propose a cautious CWAC rule where we apply the CWAC rule on the cautious to remedy the increasing of the mass of the empty set, and this, not for distinct sources, but for non distinct one as well as non dogmatic belief functions.

To build our proposed rule, we compare the results obtained by using the conjunctive combination and those obtained by the unnormalized version of the cautious rule. The comparison made on numerical examples shows that both rules have the problem of the absorbing effect. However, the unnormalized version of the cautious combination gives lower value on the conflictual mass than the conjunctive one. So, the first idea we adapt is to replace the conjunctive operator used in the CWAC rule by the cautious conjunctive one.

As a second step, after giving the results of the normalized cautious combination, studied previously, and that acts in the same way as the Dempster combination rule for the mass of

the conflict, the idea is to substitute the use of Dempster's rule in CWAC by the use of the cautious normalized rule.
Applying the changes on the CWAC rule, we obtain our proposed Cautious Combination With Adapted Conflict. It represents an adaptive weighting between the unnormalized version of the cautious rule and its normalized one.
The cautious CWAC rule is then defined as follows:
$\forall A \subseteq \Omega$ and $m_{\owedge}(\emptyset) \neq 1$:

$$m_1 \odot m_2(A) = (d(m_1, m_2) m_{\owedge}(A)) + (1 - d(m_1, m_2)) m_{\owedge^*}(A) \quad (16)$$

In a more general way:
$\forall A \subseteq \Omega$ and $m_{\owedge}(\emptyset) \neq 1$

$$m_{\odot}(A) = (\odot_i m_i)(A) = D m_{\owedge}(A) + (1 - D) m_{\owedge^*}(A) \quad (17)$$

with $D = max_{i,j}[d(m_i, m_j)]$ is the distance measure between m_i and m_j and $\forall A \subseteq \Omega$ and $m_{\owedge}(\emptyset) \neq 1$:

$m_{\owedge}(A) = (\owedge_i m_i)(A)$ and $m_{\owedge^*} = (\owedge_i^* m_i)(A)$ $\forall i \in [1, N]$

Properties:

- Commutativity: The cautious CWAC rule is based on both conjunctive cautious rule and normalized cautious rule, which are both commutative, moreover it represents a weighted sum of these rules based on distance measure which is also commutative. So, the cautious CWAC rule is commutative.
- Associativity: The cautious CWAC rule is not associative.
- Idempotency: For two same mass functions, the distance between these two functions is zero. In this case, if we take equation (17) only the term of the normalized cautious remains. We already know that the cautious rule is idempotent, hence the cautious CWAC is idempotent as well but only according to this condition: the mass function must be initially normalized ($m(\emptyset)$= 0). If this is not the case, the rule is not idempotent.

V. RESULTS

To highlight the performance and the efficiency of the proposed rule, we compare the behaviour of the cautious CWAC towards the conflict with the one of the cautious rule, on synthetic data. In this section, we present the results of this comparison. Two main tests are run for this comparison.

A. Example 1

In the first test, two sources are considered. For a first case, we study their combination when they are in agreement. These distributions and the combination results by the operators $\owedge$ and $\odot$ are given in Table I.

Table I
RESULTS OF THE FUSION OF TWO SOURCES IN AGREEMENT

	m_1	m_2	$m_1 \owedge m_2$	$m_1 \odot m_2$
$\{\omega_1\}$	0.6	0.45	0.44	0.51
$\{\omega_2\}$	0.1	0.25	0.18	0.22
Ω	0.3	0.3	0.22	0.25
$\emptyset$	0	0	0.16	0.02

The conflict generated after applying the cautious combination is higher then the conflict induced by applying the cautious CWAC rule (0.16 against 0.02). Continuing with combining two mass functions, as a second case we consider them in disagreement.

Table II represents the results of applying both rules.
According to the given results, the proposed cautious CWAC

Table II
RESULTS OF THE FUSION OF TWO SOURCES IN DISAGREEMENT

	m_1	m_2	$m_1 \owedge m_2$	$m_1 \odot m_2$
$\{\omega_1\}$	0.8	0.2	0.2	0.32
$\{\omega_2\}$	0.1	0.7	0.18	0.28
Ω	0.1	0.1	0.02	0.04
$\emptyset$	0	0	0.6	0.36

rule gives better results then the cautious one. Indeed, when you combine two sources in agreement, the conflict obtained with the proposed rule is lower than the cautious rule. When two sources in desagreement are combined, the value of the conflict increases significantly (0.02 to 0.36).

B. Example 2

In this example, we consider N bbas to combine, randomly generated, with N going from 2 to 25.
As a fisrt case, we consider the bbas in agreement and defined with Ω=$\{\omega_1, \omega_2\}$ with ϵ a random value going from -0.1 to 0.1:

$$m(\{\omega_1\}) = 0.25 + \epsilon \quad m(\{\omega_2\}) = 0.65 - \epsilon \quad m(\Omega) = 0.1$$

The conflict evolution for both cautious and cautious CWAC operators ($\owedge$, $\odot$) is presented in Figure 2.
When analysing the two curves, we can say that the value of the mass of the empty set increases with the increase of the number of combination, when sources are in agreement but the absorbing element problem did not completely disappear.

As a second case, we consider $N-1$ not contradictory sources with one in disagreement which is defined as follows Using the same frame of discernment, we get the following:

$$m(\{\omega_1\}) = 0.65 + \epsilon \quad m(\{\omega_2\}) = 0.25 - \epsilon \quad m(\Omega) = 0.1$$

Figure 3 presents the evolution of the conflict when sources are in disagreement.
The given results from these tests show that the cautious

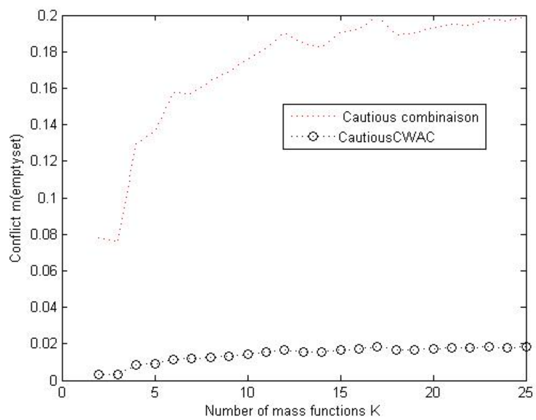

Figure 2. Conflict evolution of the combination of N not contradictory bbas

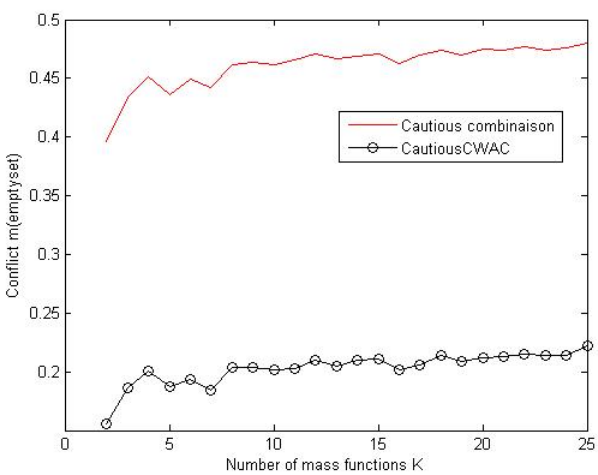

Figure 3. Conflict evolution of the combination of $N-1$ not contradictory bbas and one in conflict

CWAC gives a lower value of the conflict compared with the cautious rule.

Our proposal enables one to detect if a source is in conflict with others. Because between the two cases (sources in agreement and sources disagreement) there is a ratio of 10 at the conflicting mass. For example, for 20 concurrent sources $m_{\odot}(\emptyset)$ = 0.02 and for different sources $m_{\odot}(\emptyset)$ = 0.2. If we use the cautious rule this ratio equals 2. That is to say that a larger report provides a greater ability to detect different sources.

The behaviour of the cautious CWAC rule is better then the cautious one. Through these tests and figures, the accuracy and efficiency of the proposed rule of combination is proved.

VI. Conclusion and Future works

In this paper, we have presented our proposed cautious combination with adapted conflict with one main purpose to manage the conflict induced from the combination of several information sources in a way that gives it its initial role as an alarm signal. The cautious combination with adapted conflict is a weighted sum between the conjunctive cautious version and its normalized one, using the Jousselme et al's distance, which treats dependent sources of information and values the main role of the conflict by inhibiting the absorbing effect of the resulting conflict.

As future works, it is possible to study more the limits of this proposed rule, by increasing the number of combined bbas or by changing the used distance measure.

Moreover, it would also be interesting to focus on the application of our cautious CWAC to different domains related to data fusion like intrusion detection problem.

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