Qualitative AHP Method Based on Multiple Criteria Levels Under Group of Experts

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Abstract. In this paper, we consider a multi-criteria group decision making problem. We propose a novel version of the Analytic Hierarchy Process under the belief function theory. The presented approach uses groups of experts to express their assessments regarding the evaluation criteria and the evaluation alternatives. It considers also more complex multi-criteria decision problems that have multiple criteria levels. The presented method is illustrated with some examples.

1 Introduction

The multi-criteria decision-making (MCDM) can be defined as a field which refers to making decisions in the presence of multiple and conflicting criteria [11]. In this research, we focus on one of the most popular MCDM approach, namely the Analytic Hierarchy Process (AHP) [6]. Within the AHP context, many extensions were introduced [10]. Their main objective is to handle uncertainty under the expert assessments [4]. For instance, in some cases, decision maker is unable to express his judgements using crisp numbers and to provide a complete pair-wise comparison procedure. Qualitative AHP is then one of the very useful tools to tackle this drawback [3].

Even though selecting single expert-based alternatives according to conflicting criteria has received significant attention [11], handling such problems with group of experts and based on multiple criteria levels is still an open subject. Consequently, many extensions, under the belief function framework, have been introduced [1, 10]. Regarding these methods, the expert may be unable to provide quantitative numbers to describe his opinions. For that reason, we propose to extend the Qualitative AHP into a group decision making context based on multiple criteria levels. In this work, we present additional usefulness of Ennaceur et al.'s method [3] for handling more complex MCDM problems. In fact, the Qualitative AHP method combines the standard AHP and the belief function theory to adequately model uncertain human judgments and to represent the expert assessment easily. However, in many decision problems, the expert is able to decompose the problem into: goal, criteria, sub-criteria and alternatives. Therefore, an improved version of the Qualitative AHP is proposed to take into account the fact that many multi-criteria problems might be modeled under multiple criteria levels. Moreover, this new approach is extended into a group decision making environment.

The rest of the paper is organized as follows: In Section 2, we introduce the belief function theory. Next, Section 3 describes the Qualitative AHP method. In Section 4 and 5, we detail an improved version of our Qualitative AHP under multiple criteria levels and based on a group decision-maker context. Section 6 concludes this paper.

2 Belief function theory

For brevity, we will not consider in detail what this model is. The interested reader should refer to [7]. We present the basic concepts as interpreted by the Transferable Belief Model (TBM). Let Θ be a finite set of elementary hypotheses, called a frame of discernment. Let 2^{Θ} be all the subsets of Θ [7]. The basic belief assignment (bba) is a function m, that represents the portion of belief committed exactly to the event A. The belief function theory offers many interesting tools. The discounting technique allows to take in consideration the reliability of the information source that generates the bba m [9]. Also, to combine beliefs induced by distinct pieces of evidence, we can use the conjunctive rule of combination [8]. Moreover, It is necessary when making a decision, to select the most likely hypothesis. One of the most used solutions within the belief function theory is the pignistic probability. More details can be found in [7].

3 AHP method with belief preference relations

In this section, we consider a revised version of the AHP model, namely Qualitative AHP. To describe the approach, we present its different steps:

Step 1: Model the problem as a hierarchy based on three levels. At the highest level, we find the main objective. Then, in the middle, the sets of criteria $\Omega = \{c_1, \ldots, c_n\}$ for evaluating the sets of alternatives $\Theta = \{a_1, ..., a_m\}$, which will be in the lowest level. Then, we define the subsets of criteria and alternatives. As presented in [3], we put together criteria (or alternatives) that have the same degree of preference.

Step 2: Establish priorities among the elements of the hierarchy. Each element is paired and compared. In this context, Ennaceur et al. model [2] is used to transform preferences relations into constraints of an optimization problem. Its resolution, according to an uncertainty measure (UM) [5] generates the most uncertain belief functions. For instance, we consider the criterion based on pair-wise comparison matrix, we get:

$$
\begin{gathered}
\text{Max } H(m^{\Omega}) = m^{\Omega}(C_1) * log_2(|C_1|/m^{\Omega}(C_1)) + m^{\Omega}(C_2) log_2(|C_2|/m^{\Omega}(C_2)) \\
+ ... + m^{\Omega}(C_n) * log_2(|C_n|/m^{\Omega}(C_n)) + m^{\Omega}(\Omega) * log_2(|\Omega|/m^{\Omega}(\Omega)); \\
s.t. \\
bel^{\Omega}(C_i) - bel^{\Omega}(C_j) \geq \gamma \quad \forall (C_i, C_j),\, C_i \succ C_j \\
bel^{\Omega}(C_i) - bel^{\Omega}(C_j) \leq \gamma \quad \forall (C_i, C_j),\, C_i \succeq C_j \\
bel^{\Omega}(C_i) - bel^{\Omega}(C_j) \geq \varepsilon \quad \forall (C_i, C_j),\, C_i \succeq C_j \\
bel^{\Omega}(C_i) - bel^{\Omega}(C_j) \geq -\varepsilon \quad \forall (C_i, C_j),\, C_i \sim C_j \\
bel^{\Omega}(C_i) - bel^{\Omega}(C_j) \leq \varepsilon \quad \forall (C_i, C_j),\, C_i \sim C_j \\
\sum_{C_i \in \mathcal{F}(m^{\Omega})} m^{\Omega}(C_i) = 1, m^{\Omega}(A) \geq 0, \forall A \subseteq \Omega; m^{\Omega}(\emptyset) = 0
\end{gathered}
$$

where H is the uncertainty measure. The preference relation is represented by the first constraint. Next, the weak preference relation is illustrated by the second and third constraints. The indifference relation corresponds to the fourth and fifth constraints. The expert has to specify the indifference threshold ε and the preference threshold γ as two constants.

Step 3: Assume that criteria weights and alternatives scores are described by a bba defined on the possible responses. Thus, m^{Ω} denotes the criterion bba and $m^{\Theta}_{c_k}$ denotes the alternative bba, according to c_k.

Step 4: Use the pignistic probabilities. At the level of criteria, the bba m^{Ω} is transformed into pignistic probabilities as follows :

$$
BetP^{\Omega}(c_i) = \sum_{C_j \subseteq \Omega} \frac{|c_i \cap C_j|}{|C_j|} \frac{m^{\Omega}(C_j)}{(1 - m^{\Omega}(\emptyset))}, \quad \forall c_i \in \Omega \tag{1}
$$

Step 5: Consider each pignistic probability ($BetP^{\Omega}(c_i)$) as a measure of reliability. For each specific criterion c_i, β_i is its corresponding measure of reliability. For each $i, k = 1, \ldots, n$:

$$
\beta_i = \frac{BetP^{\Omega}(c_i)}{max_k BetP^{\Omega}(c_k)} \tag{2}
$$

Step 6: Synthesize the overall judgment. We have to update the alternatives priorities with their corresponding criteria weight. The obtained bba's are discounted such as:

$$
{}^{\alpha}m^{\Theta}_{c_k}(A_j) = \beta_k . m^{\Theta}_{c_k}(A_j), \ \forall A_j \subset \Theta \tag{3}
$$

$$
{}^{\alpha}m^{\Theta}_{c_k}(\Theta) = (1 - \beta_k) + \beta_k . m^{\Theta}_{c_k}(\Theta) \tag{4}
$$

Step 7: Combine the overall bba's to get a single representation by using the conjunctive rule ($m^{\Theta} = \bigcirc\!\!\!\!\cap\, {}^{\alpha}m^{\Theta}_{c_k}$) and choose the best alternatives by computing its pignistic probabilities.

Example Let us consider a problem of buying a car. This case study involves four criteria: $\Omega = \{\text{Style } (c_1), \text{ Price } (c_2), \text{ Fuel } (c_3), \text{ Reliability } (c_4)\}$ and three selected alternatives: $\Theta = \{\text{Peugeot } (p), \text{Renault } (r), \text{Ford } (f)\}$. The expert has identified three subsets of criteria: $\{c_1\}$, $\{c_4\}$ and $\{c_2, c_3\}$. Along with the qualitative pair-wise comparison, the preference relations defined in Table 1 was obtained. After deriving the criteria weight, the corresponding bba is transformed into pignistic probabilities as presented in Table 1. At the level of alternatives, the same steps is repeated.

Table 1. Preference relations matrix

Criteria	$\{c_1\}$	$\{c_4\}$	$\{c_2, c_3\}$
$\{c_1\}$	-	$\succ$	$\succ$
$\{c_4\}$	-	-	$\succ$
$\{c_2, c_3\}$	-	-	-

Subsets of criteria	$\{c_1\}$	$\{c_4\}$	$\{c_2, c_3\}$	Ω
m^{Ω}	0.228	0.218	0.208	0.346
Criteria	$\{c_1\}$	$\{c_4\}$	$\{c_3\}$	$\{c_2\}$
$BetP^{\Omega}$	0.315	0.305	0.190	0.190

We move now to the next stage to calculate the reliability measure. We obtain $\beta_{\{c_1\}} = 1$, $\beta_{\{c_4\}} = 0.96$, $\beta_{\{c_3\}} = 0.6$ and $beta_{\{c_2\}} = 0.6$.

Next, we need to update the alternatives priorities by the criteria weight. The discounted bba's are defined in Table 2.

Table 2. The discounted bba's

	${}^{\alpha_1}m^{\Theta}_{c_1}(.)$		${}^{\alpha_4}m^{\Theta}_{c_4}(.)$		${}^{\alpha_3}m^{\Theta}_{c_3}(.)$		${}^{\alpha_2}m^{\Theta}_{c_2}(.)$
$\{p\}$	0.505	$\{p\}$	0.306	$\{r\}$	0.303	$\{f\}$	0.303
$\{p, r, f\}$	0.495	$\{r, f\}$	0.514	$\{p, r, f\}$	0.697	$\{p, r, f\}$	0.697
		$\{p, r, f\}$	0.180				

The next step is to combine the overall evidences and the alternative ranking is, finally, obtained according to the pignistic transformation. We obtain $BetP^{\Theta}(p) = 0.410$, $BetP^{\Theta}(r) = 0.295$ and $BetP^{\Theta}(f) = 0.295$.

4 Qualitative AHP method under multiple criteria levels

At this stage, our main aim is to propose an extension of Qualitative AHP model under multiple levels of criteria. The originality of this work is to introduce a new hierarchy level, namely sub-criteria, in order to handle more complex multi-criteria problem. Let us consider a case when there are two criteria levels. Our MCDM problem is defined as follows: $\Theta = \{a_1, \ldots, a_m\}$ represents the set of alternatives, $\Omega = \{c_1, \ldots, c_n\}$ is a set of criteria. For each criterion, we have Ω_l, the set of k^l sub-criteria (denoted by sc^l_j with $j = 1, \ldots, k^l$) corresponding to the criterion c_l. For example, the second criterion c_2 has three sub-criteria denoted by sc^2_1, sc^2_2 and sc^2_3. We start by computing their relative scores. At each hierarchy level,the expert has to assess the relative importance of each criterion regarding the main objective. His evaluations have to be modeled using preference relations in order to convert them into an optimization problem. A basic belief assignment is computed. The resulting basic belief mass, denoted by

$m^{\Omega}(C_j)$ (where C_j is a single or a group of criteria), has to satisfy the following relation: $\sum_{C_j \subseteq \Omega} m^{\Omega}(C_j) = 1$.

The same process is repeated at the sub-criterion level to get the local priorities. Suppose that the j-th criterion is selected by the expert to evaluate its corresponding sub-criteria. The relative bba should satisfy: $\sum_{SC_i^j \subseteq \Omega_j} m^{\Omega_j}(SC_i^j) = 1$. where $i = 1, \dots, k^j$ with k^j is the number of sub-criterion according to the j-th criterion and $j = 1, \dots, n$.

We move to the alternative level to get $m^{\Theta}(A_k)$ (which represents the belief about the subset of alternatives A_k regarding each sub-criterion).

Now, we calculate the result and synthesize the solution by aggregating all the obtained bba's. Therefore, we must start by computing the global priority of each sub-criterion. Since we know how much the priority of each sub-criterion contributes to the priority of its parent, we can now calculate their global priorities. That will show us the priority of each sub-criterion regarding the goal.

As with Qualitative AHP method, we start by the criterion level. We transform m^{Ω} to its relative pignistic probabilities $BetP^{\Omega}$, since our beliefs are defined on groups of criteria. The same technique is repeated on the sub-criterion level to get the pignistic probabilities. We convert m^{Ω_j} into $BetP^{\Omega_j}$.

For each sub-criterion, we can now compute its global priority by: $GP(sc_i^j) = BetP^{\Omega}(c_j) * BetP^{\Omega_j}(sc_i^j)$. Then, to compute the alternatives ranking, we must apply the Qualitative AHP model from step 5 in Section 3, to compute the reliability measures, to update the alternatives priorities and to combine the overall bba's to select the highest alternatives.

Example Let us continue with the same problem. We consider that the criterion c_1 has two sub-criteria sc_1^1 and sc_2^1. The necessary calculations have already been presented in the previous example. We obtain the following local priorities presented in Table 3. After computing the corresponding $BetP^{\Omega_1}$, we can deduce the global priority as defined in Table 3.

Table 3. The local priorities assigned to the criteria and sub-criteria

Criteria	$BetP^{\Omega}$	Sub-criteria	$BetP^{\Omega_1}$	**GP**
c_1	0.315	sc_1^1	0.310	0.098
		sc_2^1	0.690	0.217
c_4	0.305			0.305
c_2	0.190			0.190
c_3	0.190			0.190

In this case, one more step need to be made. In fact, our aim here is to quantify the priority of each criterion regarding the main objective. The global priorities throughout the hierarchy will add up to 1 (see Table 3).

5 Belief AHP method under multiple criteria levels based on group of experts

In many complex problems, collective decisions are made based on a group of expert. Accordingly, we propose to extend the Qualitative AHP under a group decision-making environment. The objective is then to aggregate evidence from a member of group, to select the most appropriate alternative.

Step 1: Expert weights The main aim of this step is to give weights to experts to quantify their importance. In this case, pair-wise comparison is suggested to generate their importances. We consider the experts as the set of alternatives that are compared regarding the reliability criterion. As presented previously, the belief pair-wise comparison generates a bba, that represents the part of belief committed exactly to each expert. This bba is transformed into a reliability measure, denoted by β_i corresponding to expert i.

Step 2: Expert elicitation Each expert starts by identifying all the focal elements. Then, he compares all the identified elements through binary relations. The next step is to transform the obtained assessments into an optimization problem and to generate the least informative bba. A bba (m_i) is generated corresponding to each expert i.

Step 3: Aggregation process We proceed to the aggregation of the obtained bba's. First, we start by the discounting technique. Indeed, the idea is to measure most heavily the bba evaluated according to the most importance expert and conversely for the less important ones. So, each obtained bba is discounted by its corresponding measure of reliability, as follows:

$$P_i(A) = m_i(A) * \beta_i, \forall A \subseteq \Theta \tag{5}$$

$$P_i(\Theta) = (1 - \beta_i) + \beta_i.m_i(\Theta) \tag{6}$$

where $i = 1, \ldots, \eta$ with η is the number of experts.

Then, we move to compute the final decision. An intuitive definition of the strategy to fuse these bba's will be through the conjunctive rule of combination and the ranking of alternatives is obtained using the pignistic probabilities.

Example Let us consider the previous problem with four decision makers. The first step is to assign a degree of importance to each expert. We have $\beta_1 = 1$, $\beta_2 = 0.9$, $\beta_3 = 0.2$ and $\beta_4 = 0.1$. After applying the Qualitative AHP method, we obtain Table 4.

Table 4. The final result using the Qualitative AHP approach regarding each decision maker

Alternatives	$\{p\}$	$\{r\}$	$\{f\}$	$\{p, r\}$	$\{p, f\}$	$\{r, f\}$	$\emptyset$	Θ
m_1	0.193	0.073	0.073			0.124	0.495	0.042
m_2	0.154	0.063	0.080	0.106		0.100	0.395	0.102
m_3	0.090	0.108	0.034	0.171			0.455	0.142
m_4	0.073	0.073	0.144			0.102	0.592	0.016

Next, each bba corresponding to each expert is discounted according to its measure of reliability. Then, we combine the different bba's in order to generate the collective decision. We obtain: $m_{\text{global}}(\{p\}) = 0.070$, $m_{\text{global}}(\{r\}) = 0.042$, $m_{\text{global}}(\{f\}) = 0.032$, $m_{\text{global}}(\{f, r\}) = 0.032$, $m_{\text{global}}(\{p, r\}) = 0.002$, $m_{\text{global}}(\emptyset) = 0.815$ and $m_{\text{global}}(\Theta) = 0.009$. The obtained ranking is illustrated as follows: $BetP_{\text{global}}(p) = 0.395$, $BetP_{\text{global}}(r) = 0.334$ and $BetP_{\text{global}}(f) = 0.271$.

Now, we analyze the different results. At this step, we propose to combine the obtained priority except one, in order to study the influence of the decision made by this expert on the decision of the group. This process is repeated for all the expert priorities. The aim of the process is to identify experts who provide preferences that are significantly different from the group, and to provide these experts with the opportunity to update these preferences to be closer to the majority.

Table 5. The final result using the Qualitative AHP approach

Alternatives	$P_1 \cap P_2 \cap P_3$	$BetP_{P_1 \cap P_2 \cap P_3}$	$P_1 \cap P_2 \cap P_4$	$BetP_{P_1 \cap P_2 \cap P_4}$
$\emptyset$	0.799		0.790	
$\{p\}$	0.076	0.400	0.079	0.392
$\{r\}$	0.046	0.334	0.045	0.324
$\{f\}$	0.034	0.266	0.038	0.284
$\{f, r\}$	0.033		0.037	
$\{p, r\}$	0.002		0.002	
Θ	0.010		0.009	
Alternatives	$P_1 \cap P_3 \cap P_4$	$BetP_{P_1 \cap P_3 \cap P_4}$	$P_2 \cap P_3 \cap P_4$	$BetP_{P_2 \cap P_3 \cap P_4}$
$\emptyset$	0.590		0.468	
$\{p\}$	0.155	0.406	0.117	0.377
$\{r\}$	0.067	0.307	0.059	0.334
$\{f\}$	0.059	0.287	0.067	0.289
$\{f, r\}$	0.094		0.062	
$\{p, r\}$	0.001		0.069	
Θ	0.034		0.158	

The main objective is to show that the best alternative is supported by the majority of expert and there is not a contradictory alternative. We can notice that experts 1 and 2 are considered as reliable sources of information, since they have almost the highest reliability measure. However, experts 3 and 4 are treated as not fully reliable. As we can see, we have the P_1 and P_2 support the same alternative and they consider that the best one is p and the worst is f. However, P_3 and P_4 present a contradictory information.

Consequently, we can notice that by combining the preferences of experts 1, 2 and 3 or experts 1, 2 and 4, have generated almost the same ranking of alternatives (see Table 5). We find that the most preferred alternative is p even if experts 3 and 4 prefer the alternatives r and f respectively. This is because experts 3 and 4 are not considered reliable. Besides, as shown in Table 5, when combining evidences $P_1 \circledcirc P_3 \circledcirc P_4$ or $P_2 \circledcirc P_3 \circledcirc P_4$, we know that experts 3 and 4 are not fully reliable. Therefore, we can consider only the preference induced from expert 1 and 2 respectively. So, we can conclude that the most preferred alternative is p.

6 Conclusion

In this paper, we have formulated qualitative AHP method in an environment characterized by imperfection. Our approach deals with qualitative reasoning to model the uncertainty related to experts assessment. The advantage of this newly proposed model is its ability to handle multi-criteria level problem. It is also able to manage more complex problem by solving a multi-criteria group decision making problem. A future research idea is to study the effect of changing the weight of the groups of experts by a sensitivity analysis. In fact, the idea of assigning importance to a group of expert has been investigated, with arguments given as to the need for and against its utilization.

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