A Recourse Approach for the Capacitated Vehicle Routing Problem with Evidential Demands

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Abstract. The capacitated vehicle routing problem with stochastic demands can be modelled using either the chance-constrained approach or the recourse approach. In previous works, we extended the former approach to address the case where uncertainty on customer demands is represented by belief functions, that is where customers have so-called evidential demands. In this paper, we propose an extension of the recourse approach for this latter case. We also provide a technique that makes computations tractable for realistic situations. The feasibility of our approach is then shown by solving instances of this difficult problem using a metaheuristic algorithm.

Keywords: Vehicle routing problem, Stochastic programming with recourse, Belief function

1 Introduction

In the Capacitated Vehicle Routing Problem (CVRP), one aims at finding a set of routes of minimum cost, such that a fleet of vehicles initially located at a depot, collect goods from a set of customers with deterministic collect demands, while respecting the capacity restrictions of the vehicles. The CVRP with Stochastic Demands (CVRPSD) [14] is a modified version of this problem, where customers have stochastic demands such that, in general, the vehicle capacity limit has a non zero probability of being violated on any route. It is a stochastic integer linear program, which can be modelled by two main approaches: Chance Constrained Programming (CCP) and Stochastic Programming with Recourse (SPR) [1]. Modelling the CVRPSD via CCP consists in having constraints specifying that vehicle capacity limit on any route must not be violated with a high probability. While an SPR model for the CVRPSD allows so-called recourse actions to be performed along a route, such as returning to the depot to unload, in order to bring to feasibility a violated capacity limit. The cost of these actions is considered directly in the problem objective [14]. Specifically, the total expected travel cost is subject to minimisation, this cost covering the classical travel cost, *i.e.*, the cost of travel if no recourse action is performed, as well as the expected cost of the recourse actions. SPR models of the CVRPSD have a wider range of applications than CCP models, but they are generally more involved.

Recently [7], another variant of the CVRP was considered: the CVRP with Evidential Demands (CVRPED), where *evidential* means that uncertainty on customer demands is represented by belief functions [11]. Belief function theory is an alternative framework to probability theory for modelling uncertainty, and it can naturally account for uncertainty on customer demands in various situations, such as when pieces of information on customer demands are partially reliable. In [7], the CVRPED was modelled using an extension of the CCP approach used for CVRPSD, and subsequently solved using a metaheuristic, which is a classical means to tackle the CVRP, because it is NP-hard. In this paper, the CVRPED is modelled using an extension of the other main approach to modelling stochastic programs, that is by extending the SPR approach used for the CVRPSD, and then it is also solved using a metaheuristic algorithm.

Note that, to the best of our knowledge, this is the first time that an integer linear program involving uncertainty represented by belief functions is tackled using such a modelling approach. Indeed, besides [7], other works [9,12,8] handled optimisation problems involving uncertainty represented by belief functions in the case of *continuous* linear programs, which are usually much less difficult to solve than their discrete counterparts. In particular, Masri and Ben Abdelaziz [8] extended both CCP and SPR to model linear programs involving belief functions (so-called *belief* linear programs).

This paper is organised as follows. Necessary background on SPR modelling of CVRPSD and on belief function theory is recalled in Section 2. An extension of the recourse approach for the CVRPED is presented in Section 3. Experiments on CVRPED instances solved using a simulated annealing metaheuristic adapted from [6], are reported in Section 4. Section 5 concludes the paper.

2 Background

2.1 CVRPSD Modeled by SPR

In the CVRP, a fleet of m identical vehicles with a given capacity limit Q, initially located at a depot, must collect[1] goods from n customers, with $0 < d_i \leq Q$ the indivisible deterministic collect demand of client i, $i = 1, \ldots, n$. The objective is to find a set of m routes with minimum cost to serve all the customers such that i) total customers demands on any route must not exceed Q; ii) each route starts and ends at the depot; and iii) each customer is serviced only once; we refer to [2] for a formal description of these constraints. Let R_k be the route associated to vehicle k and $c_{i,j}$ be the cost of traveling from customer i to customer j. The

[1] The problem can also presented in terms of delivery, rather than collection, of goods.

objective is thus to

$$\min \sum_{k=1}^{m} C(R_k),$$

where

$$C(R_k) = \sum_{i=0}^{n} \sum_{j=0}^{n} c_{i,j} w_{i,j,k}, \tag{1}$$

with $w_{i,j,k}$ a binary variable that equals 1 if vehicle k travels from i to j and serves them, and 0 if it does not.

In the CVRPSD, each client demand d_i, $i = 1, \ldots, n$, becomes a random variable, such that $P(d_i \leq Q) = 1$. As a consequence, a vehicle might not be able to load all of the actual customer demands on any given route having more than one customer. The SPR approach deals with this issue by permitting recourse actions, such as allowing vehicles to return to the depot to unload when they are full. These actions lead to extra costs for routes, which we call penalty costs, and it is generally possible to compute the expected penalty cost of a route induced by the stochastic demands. A general expression for SPR models of CVRPSD is then the following. The objective is to find a set of routes that

$$\min \sum_{k=1}^{m} C_{\mathrm{E}}(R_k),$$

where $C_{\mathrm{E}}(R_k)$ is the expected cost of R_k defined by

$$C_{\mathrm{E}}(R_k) = C(R_k) + C_{\mathrm{P}}(R_k),$$

with $C(R_k)$ the cost defined by (1) representing the cost of traveling along R_k if no recourse action is performed, and $C_{\mathrm{P}}(R_k)$ the expected penalty cost on R_k – $C_{\mathrm{P}}(R_k)$ may be defined in many different ways depending on the recourse policy used (see, *e.g.*, [5,4]).

2.2 Belief Function Theory

Let us recall the concepts of belief function theory needed in this study. Let x be a variable taking its values in a domain X. In this theory, uncertain knowledge about x may be represented by a *Mass Function* (MF) defined as a mapping $m^X : 2^X \to [0,1]$ such that $m^X(\emptyset) = 0$ and $\sum_{A \subseteq X} m^X(A) = 1$. The quantity $m^X(A)$, for some $A \subseteq X$, represents the probability of knowing only that $x \in A$. Subsets $A \subseteq X$ such that $m^X(A) > 0$ are called focal sets. A MF whose focal sets are singletons, *i.e.*, $m^X(A) > 0$ iff $|A| = 1$, corresponds to a probability mass function and is called a *Bayesian* MF. Furthermore, a variable x whose true value is known in the form of a MF will be called an evidential variable.

Finally, given a MF m^X and a function $h : X \to \mathbb{R}^+$, it is possible to compute its *upper* expected value $E^*(h, m^X)$ defined as [3]

$$E^*(h, m^X) = \sum_{A \subseteq X} m^X(A) \max_{x \in A} h(x).$$

3 Recourse approach for the CVRPED

In this section, a recourse approach is proposed for the case where uncertainty on customer demands in the CVRP is represented by belief functions.

3.1 Formalisation

Assume customer demands d_i, $i = 1, \ldots, n$, are no longer deterministic or random, but evidential, *i.e.*, the actual demand of customer i is known with some uncertainty represented by a MF. In such case, one obtains a new problem called CVRPED. As shown in [7], this problem can be addressed via a constrained programming approach. However, similarly to what has been done for the case of belief linear programs [8], this problem may be also addressed using an extension of the other main approach to modelling stochastic programs, that is by extending the recourse approach of CVRPSD to CVRPED.

Specifically, we propose to extend the recourse approach, for the following policy and assumptions studied for the stochastic case in [5,4]. Each actual customer demand cannot exceed the vehicle capacity. In addition, when a vehicle arrives at a customer on its planned route, it is loaded with the actual customer demand up to its remaining capacity. If this remaining capacity is sufficient to pick-up the entire customer demand, then the vehicle continues its planned route. However, if it is not sufficient, *i.e.*, there is a failure, then the vehicle returns to the depot, is emptied, goes back to the client to pick-up the remaining customer demand and continues its originally planned route.

Consider a given route R containing N customers and, without lack of generality, that the i-th customer on R is customer i. According to the above setting, a failure cannot occur at the first customer on R. However, it can occur at any other customer on R, and there may even be failure at multiple customers on R (at worst, if the actual demand of each customer is equal to the capacity of the vehicle, failure occurs at each customer except the first one).

Formally, let us introduce a binary variable r_i that equals 1 if failure occurs at the i-th customer on R and 0 otherwise (by problem definition $r_1 = 0$). Then, the possible failure situations that may occur along R may be represented by the vectors $(r_2, r_3, \ldots, r_N) \in \{0,1\}^{N-1}$. To simplify the exposition, we define the set $\Omega = \{\omega_1, \ldots \omega_{2^{N-1}}\}$ representing the possible failure situations along R, with failure situation $(r_2, r_3, \ldots, r_N)$ being in one-to-one correspondence with ω_j where $j = 1 + \sum_{i=2}^{N} r_i \times 2^{i-2}$. For instance, when R contains only $N = 3$ customers, we have $\Omega = \{\omega_1, \omega_2, \omega_3, \omega_4\}$, where ω_j, $j = 1, \ldots, 4$, mean that the vehicle needs to perform a round trip to the depot, respectively, "never", "when it reaches the second customer", "when it reaches the third customer", and "when it reaches both the second and third customers".

Furthermore, let $g : \Omega \to \mathbb{R}^+$ be a function representing the cost of each failure situation $\omega \in \Omega$. Since the penalty cost upon failure on customer i is $2c_{0,i}$

(a failure implies a return trip to the depot), the cost associated to failure ω_j is

$$g(\omega_j) = \sum_{i=2}^{N} r_i 2c_{0,i},$$

using the one-to-one correspondence $\omega_j \leftrightarrow (r_2, r_3, \ldots, r_N)$.

Let m^{Ω} be a MF representing uncertainty towards the actual failure situation occurring on R – as will be shown in the next section, evidential demands may induce such a MF.

Then, adopting a similar pessimistic attitude as in the recourse approach to belief linear programming [8], the upper expected penalty cost $C^*_{\mathrm{P}}(R)$ of route R may be obtained as $C^*_{\mathrm{P}}(R) = E^*(g, m^{\Omega})$. Accordingly, the upper expected cost $C^*_{\mathrm{E}}(R)$ of route R may be defined as

$$C^*_{\mathrm{E}}(R) = C(R) + C^*_{\mathrm{P}}(R),$$

with $C(R)$ the cost (1) of travelling along route R when no failure occurs.

The CVRPED under the above recourse policy, may then be modelled as the problem of finding a set of m routes optimising the following objective function

$$\min \sum_{k=1}^{m} C^*_{\mathrm{E}}(R_k). \tag{2}$$

Since $C^*_{\mathrm{E}}(R)$ is the upper, *i.e.*, worst, expected cost of a route, we note that optimising (2) has some similarities with the protection against the worst case popular in robust optimisation [13].

The evaluation of the objective function (2) requires the computation for each route, of the MF m^{Ω} representing uncertainty on the actual failure situation occurring on the route. This is detailed in the next section.

3.2 Uncertainty on Recourses

We assume customer demands to be positive integers. Hence, evidential demands are defined on the finite set $\Theta = \{1, \ldots, Q\}$.

Consider again a route R containing N customers. In addition, let us first assume that MF m_i^{Θ} representing the evidential demand of the i-th client, $i = 1, \ldots, N$, on R is such that $\exists \theta_i \in \Theta$, $m_i^{\Theta}(\{\theta_i\}) = 1$, *i.e.*, client demands are known without any uncertainty. Then, it is clear that the above recourse policy amounts to the following definition for the binary failure variables r_i:

$$r_i = \begin{cases} 1, & \text{if } q_{i-1} + \theta_i > Q, \\ 0, & \text{otherwise,} \end{cases} \qquad \forall i \in \{2, \ldots, N\} \tag{3}$$

where q_j, $j = 1, \ldots, N$, denotes the load in the vehicle after serving the j-th customer such that $q_j = \theta_1$ for $j = 1$ and, for $j = 2, \ldots, N$,

$$q_j = \begin{cases} q_{j-1} + \theta_j - Q, & \text{if } q_{j-1} + \theta_j > Q, \\ q_{j-1} + \theta_j, & \text{otherwise.} \end{cases}$$

In other words, when it is known that the demand of the i-th customer is θ_i, $i = 1, \ldots, N$, then it can be deduced that the failure situation $\omega_j \leftrightarrow (r_2, r_3, \ldots, r_N)$, with r_i defined by (3), occurs. This can be encoded by a function $f : \Theta^N \to \Omega$, s.t. $f(\theta_1, \ldots, \theta_N) = \omega_j$, with ω_j the failure situation induced by demands θ_i. For example, suppose we have $N = 3$ customers on route R, with respective demands $\theta_1 = 3, \theta_2 = 3$ and $\theta_3 = 5$, and the vehicle capacity limit is $Q = 5$. In such case, failure situation $\omega_4 \leftrightarrow (r_2 = 1, r_3 = 1)$ occurs, hence $f(\theta_1, \theta_2, \theta_3) = \omega_4$.

Assume now that MF m_i^Θ, $i = 1, \ldots, N$, on R is such that $m_i^\Theta(A_i) = 1$, with $A_i \subseteq \Theta$, *i.e.*, client demands are known imprecisely. In such case, it can only be inferred that the failure situation on R belongs to the subset $B \subseteq \Omega$ defined as (using a common abuse of notation for the image of a set)

$$B = f(A_1, \ldots, A_N) = \bigcup_{(\theta_1, \ldots, \theta_N) \in A_1 \times \cdots \times A_N} f(\theta_1, \ldots, \theta_N). \tag{4}$$

More generally, assume that MF m_i^Θ, $i = 1, \ldots, N$, have arbitrary numbers of focal sets and that the joint probability of knowing only that demands of customers $i = 1, \ldots, N$, belong, respectively, to $A_i \subseteq \Theta$, $i = 1, \ldots, N$, is equal to $\prod_{i=1}^N m_i^\Theta(A_i)$ (this latter equality is not necessary in our approach, but it simplifies the exposition and corresponds to the case considered in our experiments in Section 4). Then, uncertainty on the actual failure situation on R is represented by a MF m^Ω defined as

$$m^\Omega(B) = \sum_{f(A_1, \ldots, A_N) = B} \prod_{i=1}^N m_i^\Theta(A_i). \tag{5}$$

Computing m^Ω defined by (5) involves evaluating $f(A_1, \ldots, A_N)$ for all possible combinations of focal sets of MF m_i^Θ, $i = 1, \ldots, N$. Evaluating $f(A_1, \ldots, A_N)$ for some A_i, $i = 1, \ldots, N$, implies $|A_1| \times \cdots \times |A_N|$ (and thus at worst Q^N) times the evaluation of function f at some point $(\theta_1, \ldots, \theta_N) \in \Theta^N$. Hence, computing Eq. (5) is generally intractable. Nonetheless, in the particular and realistic case where the focal sets of MF m_i^Θ, $i = 1, \ldots, N$, are all intervals of positive integers (which will be the case in our experiments in Section 4), it becomes possible to compute $f(A_1, \ldots, A_N)$, and thus Eq. (5), with a much more manageable complexity. This is detailed in the next section.

We remark that if evidential demands of all customers are Bayesian, then we are actually dealing with a CVRPSD. In addition, m^Ω is in this case Bayesian on any given route R. Hence, the upper expected penalty cost $C_{\mathrm{P}}^*(R)$ reduces to the classical (probabilistic) expected value of cost function g with respect to the probability mass function m^Ω, and thus our recourse modelling of the CVRPED clearly degenerates into the recourse modelling of the aforementioned CVRPSD.

Finally, we showed in [7] that the constrained programming modelling of CVRPED can be converted, in a particular case, into an equivalent CVRPSD modelled via constrained programming, by transforming each evidential demand represented by MF m_i into a stochastic demand represented by probability mass function p_i such that $p_i(\overline{A}) = m_i(A)$, $\forall A \subseteq \Theta$, with $\overline{A}$ the greatest value in A.

It can be shown that under the recourse approach, this latter transformation cannot be used in general to convert a CVRPED into an equivalent CVRPSD.

3.3 Interval demands

Let us consider a route R with N customers, such that the demand of customer i, $i = 1, \ldots, N$, is known in the form of an interval of positive integers, which we denote by $[\![\underline{A_i}; \overline{A_i}]\!]$, where $\underline{A_i} \geq 1$ and $\overline{A_i} \leq Q$. In this case, as explained above, the failure situation on R belongs to $f\left([\![\underline{A_1}; \overline{A_1}]\!], \ldots, [\![\underline{A_N}; \overline{A_N}]\!]\right) \subseteq \Omega$. Hereafter, we provide a method to efficiently compute $f\left([\![\underline{A_1}; \overline{A_1}]\!], \ldots, [\![\underline{A_N}; \overline{A_N}]\!]\right)$.

In a nutshell, this method consists in generating a rooted binary tree, which represents synthetically yet exhaustively what can possibly happen on R in terms of failure situations.

More precisely, this tree is based on the following remark. Suppose a vehicle traveling along R and all that is known about its load when it arrives at the i-th customer on R is that its load belongs to an interval $[\![\underline{q}; \overline{q}]\!]$. Let us denote by q_i its load after visiting the i-th customer. Then, there are three exclusive cases:

1. either $\overline{q} + \overline{A_i} \leq Q$, hence there will surely be no failure at that customer and all that is known is that $q_i \in [\![\underline{q}; \overline{q}]\!] + [\![\underline{A_i}; \overline{A_i}]\!]$;
2. or $\underline{q} + \underline{A_i} > Q$, hence there will surely be a failure at that customer and all that is known is that $q_i \in [\![\underline{q}; \overline{q}]\!] + [\![\underline{A_i}; \overline{A_i}]\!] - Q$;
3. or $\underline{q} + \underline{A_i} \leq Q < \overline{q} + \overline{A_i}$, hence it is not sure whether there will be or not a failure at that customer. However, we can be sure that if there is no failure at that customer, *i.e.*, the sum of the actual vehicle load and of the actual customer demand is lower or equal to Q, then it means that $q_i \in [\![\underline{q} + \underline{A_i}; Q]\!]$; and if there is a failure at that customer, then it means that $q_i \in [\![1; \overline{q} + \overline{A_i} - Q]\!]$.

By applying the above reasoning repeatedly, starting from the first customer and ending at the last customer, whilst accounting for and keeping track of all possibilities and their associated failures (or absence thereof) along the way, one obtains a binary tree. The tree levels are associated to the customers according to their order on R. The nodes at a level i represent the different possibilities in terms of imprecise knowledge about the vehicle load after the i-th customer, and they also store whether these imprecise pieces of knowledge about the load were obtained following a failure or an absence of failure at the i-th customer. The pseudo code of the complete tree induction procedure is provided in Algorithm 1, which is illustrated by Example 1.

Example 1. Let us illustrate Algorithm 1 on a route R where $Q = 10$ and containing 3 customers, with $[\![4; 8]\!]$, $[\![5; 7]\!]$ and $[\![7; 9]\!]$ the imprecise demands of the first, second and third customers, respectively. Since the demand of the first customer is $[\![4; 8]\!]$, and there is no failure by definition at the first customer, and the customer following the first customer is the second customer, the tree is obtained with $RT([\![4; 8]\!], 0, 2)$ and is shown in Fig. 1.

Algorithm 1 Induction of Recourse Tree (RT)

Input: interval load $\llbracket \underline{q}; \overline{q} \rrbracket$, Boolean failure variable r, next customer number i
Output: final tree $Tree$

create a root node containing interval load $\llbracket \underline{q}; \overline{q} \rrbracket$ and Boolean failure r
if $i = N + 1$ **then**
  return $Tree = \{\text{root node}\}$
else if $\overline{q} + \overline{A_i} \leq Q$ **then**
  $Tree_L = RT(\llbracket \underline{q}; \overline{q} \rrbracket + \llbracket \underline{A_i}; \overline{A_i} \rrbracket, 0, i + 1)$
  attach $Tree_L$ as left branch of $Tree$
else if $\underline{q} + \underline{A_i} > Q$ **then**
  $Tree_R = RT(\llbracket \underline{q}; \overline{q} \rrbracket + \llbracket \underline{A_i}; \overline{A_i} \rrbracket - Q, 1, i + 1)$
  attach $Tree_R$ as right branch of $Tree$
else
  $Tree_L = RT(\llbracket \underline{q} + \underline{A_i}; Q \rrbracket, 0, i + 1)$
  attach $Tree_L$ as left branch of $Tree$
  $Tree_R = RT(\llbracket 1; \overline{q} + \overline{A_i} - Q \rrbracket, 1, i + 1)$
  attach $Tree_R$ as right branch of $Tree$
end if

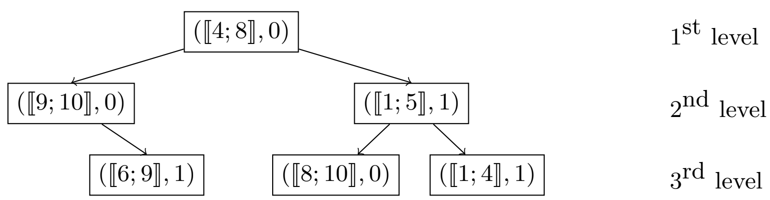

Fig. 1. Recourse tree constructed for Example 1

For a given branch of the tree, by concatenating in a vector the Boolean failure variable r_i at level i, $i = 2, \ldots, N$, we obtain the failure situation $\omega_j \leftrightarrow (r_2, r_3, \ldots, r_N)$. Hence, all the branches of the tree yield the subset $B \subseteq \Omega$. For instance, the rightmost branch of the tree in Fig. 1 yields the failure situation $(r_2 = 1, r_3 = 1) \leftrightarrow \omega_4$, the leftmost branch yields $(r_2 = 0, r_3 = 1) \leftrightarrow \omega_3$ and the remaining branch yields $(r_2 = 1, r_3 = 0) \leftrightarrow \omega_2$. The tree in this example yields thus the set $B = \{\omega_2, \omega_3, \omega_4\}$.

Proposition 1. *The set B built using the tree generated by Algorithm 1 verifies* $B = f\left(\llbracket \underline{A_1}; \overline{A_1} \rrbracket, \ldots, \llbracket \underline{A_N}; \overline{A_N} \rrbracket\right)$.

Worst-case complexity to obtain set B is $\mathcal{O}(2^{N-1})$ on a route R with N clients, which is the maximum number of leaf nodes in the tree.

4 Experiments

We used the CVRPED instances described in [7] and deriving from those of Augerat set A for the CVRP [10]. These CVRPED instances are obtained as follows. A customer deterministic demand d^{det} in the Augerat instances is transformed into an evidential demand with associated MF m^{Θ} defined by

$$m^{\Theta}(\{d^{det}\}) = \alpha, \quad m^{\Theta}(\llbracket \lfloor d^{det} - \gamma \cdot d^{det} \rfloor; \lceil d^{det} + \gamma \cdot d^{det} \rceil \rrbracket) = 1 - \alpha, \tag{6}$$

where $\alpha \in (0, 1)$ and $\gamma \in [0, 1]$. This transformation corresponds to assuming that the deterministic demand of each customer has been provided by a source, which is reliable with probability α and approximately (at $\pm\gamma * 100\%$) reliable with probability $1 - \alpha$. In addition, we assumed that these latter sources have independent probabilities of reliability.

Proposition 2. *For any α, the upper expected cost of an optimal set of routes for a CVRPED instance generated from a CVRP instance through transformation* (6) *and modelled via the recourse approach, is non decreasing in γ.*

Proposition 2 basically shows that the more a decision maker is uncertain (cautious) with respect to actual customer demands, *i.e.*, the greater γ is, the greater will be the (upper expected) cost of the optimal solution to his associated optimisation problem. Proposition 2 also yields a lower bound on the cost of the optimal solution to any CVPRED instance built using (6): it is obtained by solving to optimality under the recourse approach the corresponding Augerat set A instance, since such instance corresponds to setting $\gamma = 0$ in (6).

In order to solve CVRPED instances under the recourse approach, we adapted a simulated annealing metaheuristic algorithm originally introduced for CVRP in [6]. However, we do not describe this adaptation here due to space limitation.

In our experiments, parameters α and γ of the CVRPED instances were set arbitrarily to 0.8 and 0.1, respectively. Each instance was solved 30 times and the best, average and standard deviation of costs are reported in Table 1. In addition, the contribution of the expected penalty costs to the overall costs of the best solutions is provided as percentages: as can be seen, it varies between 0% to 8%. Finally, the last column of Table 1 provides the best costs obtained with our metaheuristic when solving CVRPED instances with $\gamma = 0$ - these costs may be seen as an approximation of the lower bounds on the costs of the optimal solutions of the CVPRED instances generated through transformation (6).

5 Conclusions

Belief function theory was used to represent uncertainty on customer demands in the capacitated vehicle routing problem. We handled this problem by extending the recourse modelling approach of stochastic programming. In addition, we provided a technique that makes computations tractable in realistic cases. Instances of such cases were then solved using a simulated annealing algorithm. Future works include studying more elaborate recourse policies and improving the solving algorithm.

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Table 1. Results of the simulated annealing algorithm for the CVRPED instances

Instance	Best cost	Penalty cost	Avg cost	Stand. dev.	Avg runtime	Best cost $\gamma = 0$
A-n32-k5	843,06	0.03%	874,18	9,19	1837s.	839,18
A-n33-k5	705,69	0.37%	724,11	8,39	2241s.	697,12
A-n33-k6	773,55	0.75%	793,07	10,42	2271s.	758,36
A-n34-k5	820,37	1.40%	837,04	9,19	2975s.	812,16
A-n36-k5	884,51	0.34%	914,85	13,84	2715s.	869,10
A-n37-k5	722,57	0%	753,51	12,86	2634s.	720,85
A-n37-k6	1044,27	3.06%	1071,27	12,74	3111s.	995,07
A-n38-k5	781,69	8.36%	816,67	18,44	4525s.	748,64
A-n39-k5	890,88	1.57%	935,58	19	5068s.	885,04
A-n39-k6	896,60	0.34%	916,91	16.11	3196s.	884,09
A-n44-k6	1051,21	2.46%	1104,58	24,88	3922s.	1019,07
A-n45-k6	1091,72	6.01%	1129,21	18,98	5444s.	1006,90
A-n45-k7	1296,37	0.94%	1348,57	23,02	3237s.	1246,14
A-n46-k7	1060,47	0.05%	1087,16	16	2865s.	1045,93
A-n48-k7	1241,33	0.11%	1274,24	20,97	3119s.	1227,79

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