A generic framework for resolving the conflict in the combination of belief structures

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Abstract - ***Within the framework of Dempster-Shafer theory of evidence, the data fusion is based on the building of single belief mass by combination of several mass functions resulting from distinct information sources. This combination called Dempster's combination rule (or orthogonal sum) has several interesting mathematical properties like commutativity or associativity. Unfortunately, it badly manages the existing conflict between the various information sources at the step of normalization. In this paper, we introduce traditional combination operators used within the framework of evidence theory. We propose other combination operators allowing an arbitrary redistribution of the conflicting mass on the propositions. These various combinations operators were tested on sets of synthetic and real masses.***

Keywords: Data Fusion; Dempster-Shafer Theory of Evidence; Combination Rules.

1 Introduction

In pattern recognition, the information extracted from the sensors (numeric or symbolic) is often represented by a degree of belief resulting from imprecise and uncertain data. The multi-sensor data fusion [1] is an interesting solution in order to reach information overall more reliable. The complementarity and the redundancy of information provide by the sensors are among the imperative reasons of this effect. In the framework of evidence theory (or Dempster-Shafer theory of evidence) [2, 3], the data fusion rests on the building of single belief mass by combination of several mass functions from distinct information sources. The implementation of this combination requires a normalization step in order to ensure the properties of mass functions. In [4], Zadeh presents a situation where the step of normalization used by Dempster's combination rule to results against intuitive. In order to cure this problem, other combination operators have been proposed [5, 6, 7]. In this paper, we will see a brief overview on the already existing combination rules and we will propose new operators. It will be organized in as if follow. Initially, we present the basic concepts of evidence theory (Section 2). In section 3, we will see the various combination operators which were developed within framework of this theory. We will introduce the new combination operators in section 4. Finally, simulations on synthetic masses will be presented in section 5.

2 Dempster Shafer theory

The Dempster Shafer theory of evidence is based on the concept of lower and upper bounds for a set of compatible probability distributions introduced by Dempster [2]. On this basis, Shafer [3] showed the advantage of using belief functions for modeling the uncertain knowledge. The use of belief function as an alternative to subjective probabilities was later justified axiomatically by Smets [8] who introduced the *Transferable Belief Model* providing a clear and consistent interpretation of the various concept underlying the theory.

2.1 Knowledge modelling

Let Θ represents a finite set of N hypothesis. The set Θ is called *frame of discernment* and is defined by :

$$\Theta = \{H_1, \cdots, H_n, \cdots, H_N\}. \quad (1)$$

The property of exhaustive assumption, called *closed-world* assumption is in opposition to *open world* assumption presented by Smets [7]. We go back over in detail this notion in section 3.1.2. 2^Θ represents the set of the $2^N - 1$ propositions $\mathcal{H}$ of Θ :

$$2^\Theta = \{\mathcal{H}/\mathcal{H} \subseteq \Theta\} = \{H_1, \cdots, H_N, H_1 \cup H_2, \cdots, \Theta\}. \quad (2)$$

A piece of evidence that influences our belief concerning the true value of a proposition $\mathcal{H}$ can be represented by a basic belief assignment $m(.)$. For each source S_j for $j = \{1, \ldots, J\}$, a mass function $m_j(.)$ is defined by :

$$m_j : 2^{\Theta} \rightarrow [0, 1] \quad (3)$$

and satisfying following properties :

$$m_j(\emptyset) = 0 \quad (4)$$

$$\sum_{\mathcal{H} \subseteq \Theta} m_j(\mathcal{H}) = 1. \quad (5)$$

The subsets $\mathcal{H}$ of Θ such that $m_j(\mathcal{H}) > 0$ are called the *focal elements* of $m_j(.)$. The union of all focal elements of $m_j(.)$ is called the *core* of the mass function. The core of an information source S_j is noted $\mathcal{F}_j$. From the basic belief assignment $m_j(.)$, a credibility function and plausibility function can be computed using the following equations :

$$Cr_j(\mathcal{H}) = \sum_{\mathcal{H}' \subseteq \mathcal{H}} m_j(\mathcal{H}') \quad (6)$$

$$Pl_j(\mathcal{H}) = \sum_{(\mathcal{H} \cap \mathcal{H}') \neq \emptyset} m_j(\mathcal{H}') = 1 - Cr_j(\overline{\mathcal{H}}) \quad (7)$$

where $\overline{\mathcal{H}}$ denotes the complement of $\mathcal{H}$. The value $Cr_j(\mathcal{H})$ quantifies the strength of the belief that event $\mathcal{H}$ occurs. The plausibility function $Pl_j(\mathcal{H})$ provides a measure of no doubt about the hypothesis $\mathcal{H}$. The main difficulty consists in modeling knowledge to initialize the basic belief assignment $m_j(.)$. Usually, this modeling depends on the application. In [1], Appriou uses two methods to manage the learning uncertainty with Dempster Shafer theory. These methods are consistent with the Bayesian approach, when the belief is only assigned to a singleton hypothesis. Many modeling methods are proposed [9], of which the methods using neighborhood information introduced by Denoeux [10].

2.2 Combination rules

In case of imperfect data (uncertain, imprecise, incomplete), fusion of multi-sensors is an interesting solution in order to reach more reliable informations. The evidence theory applies very well within the framework of the data fusion. Indeed, starting from the belief functions $m_j(.)$ resulting from each source S_j, a combination rule of these functions makes it possible to obtain a single belief function $m(.)$. This single belief function makes it possible to use a decision rule by taking into account the whole of information sources. In the following section, we will present the various aggregation rules of belief which one can find in the literature.

3 Combination operators

The different rules of combination that have been presented in the literature can be distinguished in two categories. These two categories represent two different philosophies of the fusion technique. The first type of combination operators, presented in the section 3.1, imposes the hypothesis of reliability of all sources which have to be aggregated. These conjunctive operators have been introduced by Dempster [2] and Smets [7]. The second category imposes that at least one of the information sources is reliable. The second kind includes the disjunctive operators that have been presented by Yager [5], Dubois and Prade [6].

3.1 Combination operators of reliable sources

3.1.1 Dempster combination rule

It is the first rule of information combination that has been used in the framework of the evidence theory. A necessary condition for using this combination is that the information sources are independent. The Dempster operator of combination, or orthogonal sum, proves to be commutative and associative. The resulting mass function from the Dempster's rule will be noted $m^0(.)$ and defined by :

$$\forall\, \mathcal{H} \in 2^{\Theta} \qquad m^0(\mathcal{H}) = m_1(\mathcal{H}) \oplus \ldots \oplus m_J(\mathcal{H}) \quad (8)$$

where $\oplus$ represents the operator. In a case of two sources noted S_i and S_j, the combination can be written as :

$$m^0(\mathcal{H}) = \frac{1}{1 - m(\emptyset)} \sum_{(\mathcal{H}' \cap \mathcal{H}'') = \mathcal{H}} m_i(\mathcal{H}').m_j(\mathcal{H}'') \quad (9)$$

where $m(\emptyset)$ is defined by :

$$m(\emptyset) = \sum_{(\mathcal{H}' \cap \mathcal{H}'') = \emptyset} m_i(\mathcal{H}').m_j(\mathcal{H}''). \quad (10)$$

In the equation (9), the coefficient $m(\emptyset)$ reflects the conflict between the two sources S_i and S_j. When this factor is equal to 1, the sources are totally in conflict and the information sources cannot be aggregated. On the contrary, when $m(\emptyset)$ is equal to 0, the sources agree. This combination rule has some interesting properties like associativity and commutativity but it is not idempotent. Neutral element is the mass function of total ignorance ($m(\Theta) = 1$) and the absorbing element is the mass function that puts the whole mass on a hypothesis singleton ($m(H_n) = 1$).

This first rule carries out two major problems. The first problem comes from the idempotence, that is to say that the combination of two dependent sources allows to reinforce the propositions that these sources sustain abusively. The second appears in case of conflict between the sources. In this case, the Dempster combination rule proceeds a step of normalization with the help of the coefficient $m(\emptyset)$. This problem, known as sensitivity of the Dempster rule, has been presented by Zadeh [4]. It clearly appears in the following example.

Let $\Theta = \{H_1, H_2, H_3\}$ be a frame of discernment, and two information sources S_1 and S_2 producing respectively two mass function m_1 and m_2 defined as it follows :

$$\begin{array}{cc} m_1(H_1)=\epsilon & m_2(H_1)=1-k-\epsilon \\ m_1(H_2)=k & m_2(H_2)=k \\ m_1(H_3)=1-k-\epsilon & m_2(H_3)=\epsilon \end{array} \quad (11)$$

with $0 \leq k \leq 1$. In the case where ϵ is equal to 0, the combination with the Dempster rule allows to write the following result :

$$m^0(H_1)=0 \quad m^0(H_2)=1 \quad m^0(H_3)=0. \quad (12)$$

In the general case, the application of the Dempster operator gives the following result :

$$m^0(H_1)=m^0(H_3)=\frac{\epsilon(1-k-\epsilon)}{k^2+2\epsilon(1-k-\epsilon)}, \quad (13)$$

and :

$$m^0(H_2)=\frac{k^2}{k^2+2\epsilon(1-k-\epsilon)}. \quad (14)$$

Also, taking $k = 0.1$ and $\epsilon = 0.01$, we obtain the following mass function :

$$m^0(H_1)=m^0(H_3)=0.32 \qquad m^0(H_2)=0.36 \quad (15)$$

whereas for $k = 0.1$ and $\epsilon = 0.001$, we have :

$$m^0(H_1)=m^0(H_3)=0.08 \qquad m^0(H_2)=0.84. \quad (16)$$

Therefore, we can observe that the Dempster rule is very sensitive to the variations of the value of ϵ. This sensitivity is due to strong variations of the normalization coefficient $\frac{1}{1-m(\emptyset)}$. In Figure 1, we have represented the variations of the normalization coefficient according to the values of the conflict $m(\emptyset)$. We can see that in the neighborhood of $m(\emptyset)=1$, a weak variation of $m(\emptyset)$ involves a strong variation of the normalization coefficient. In order to solve this problem, Smets proposes another interpretation of the conflict mass $m(\emptyset)$ in the framework of reliable sources.

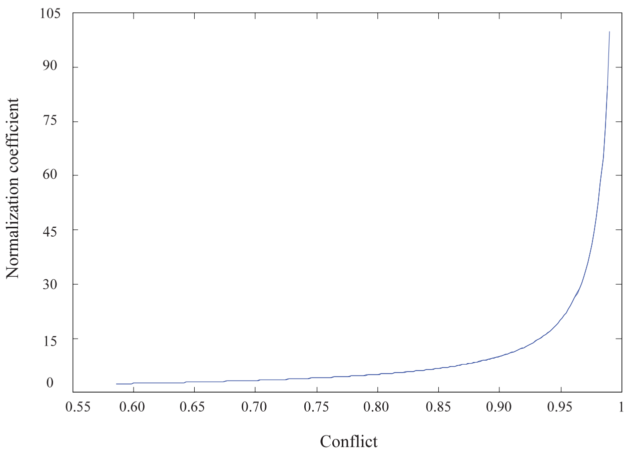

Figure 1: Normalization coefficient vs. Conflict.

3.1.2 Smets combination operator

The solution proposed by Smets [7, 11] considers that the information sources are completely reliable. The conflict between the sources can only comes from that one does not take into account one or several hypotheses in the frame of discernment. This solution consists therefore in not normalizing the function mass and so affecting the conflict mass $m(\emptyset)$ to the empty set $\emptyset$. That is the notion of *open - world* proposed by Smets [8]. The mass resulting from the combination of Smets will be noted $m^1(.)$. This operator can be written :

$$\forall\ \mathcal{H} \in 2^{\Theta} \qquad m^1(\mathcal{H}) = m_{\cap}(\mathcal{H}) \quad (17)$$

with :

$$m_{\cap}(\mathcal{H}) = \sum_{(\mathcal{H}'\cap\mathcal{H}'')=\mathcal{H}} m_i(\mathcal{H}').m_j(\mathcal{H}''). \quad (18)$$

The empty set $\emptyset$ may be interpreted as a reject class. This combination has the same properties (associativity and commutativity) than the rule of Dempster. In addition, in [12], Smets proposes a method which unifies the conjunctive and disjunctive rules of combination.

3.2 Combination operators of unreliable sources

Other solutions for the interpretation of the conflict have been introduced. These methods are based on the assumption that at least one of the sources to aggregate is reliable (e.g. see [6]). In this section, we propose to develop a generic framework in order to unify several operators of combination. This framework allows us to propose other operators.

3.2.1 Proposed Framework of Operators

Each information source S_j gives a belief mass to each proposition $\mathcal{H}_j$ in the core $\mathcal{F}_j$. When $\mathcal{H}_j$ extracted from the sources S_j are compatible, that is to say when the intersection between the subsets $\mathcal{H}_j$ is non hopeless, he product of the masses granted to these subsets is affected to the intersection of these subsets. If the hypotheses $\mathcal{H}_j$ are incompatible, that is to say that their intersection is equal to the empty set, we have a partial conflicting mass called m^* given by the following relation :

$$m^* = \prod_{\substack{j=1,\ldots,J \\ \bigcap_{j=1}^{J}\mathcal{H}_j=\emptyset}} m_j(\mathcal{H}_j) \qquad \begin{array}{r}\forall\ j=1,\ldots,J \\ \forall\ \mathcal{H}_j \in \mathcal{F}_j.\end{array} \quad (19)$$

The total conflict $m(\emptyset)$ is derived from these partial conflicting mass according to :

$$m(\emptyset) = \sum{}^{*} m^* \quad (20)$$

where $\sum^*$ represents a countable sum of elements. The aim of the combination operators, proposed in this section, is to redistribute a partial conflicting mass m^* on a set of propositions. The set of all propositions $\mathcal{H}$ where the partial conflict masses have been redistributed will be noted $\mathcal{P}^*$.

A part of the mass m^* will be assigned to each proposition $\mathcal{H}$ according to a weighting factor noted w^*. In addition, the set of all propositions where the conflicting mass have been redistributed will be noted $\mathcal{P}$, with :

$$\mathcal{P} = \{\mathcal{P}^*\}. \quad (21)$$

So the total mass got after fusion for a proposition $\mathcal{H}$ will be the sum of two masses. It will be written as it follows :

$$m(\mathcal{H}) = m_\cap(\mathcal{H}) + m^c(\mathcal{H}) \quad (22)$$

In the equation (22), the first term, $m_\cap(.)$, is derived from the conjunctive rule of combination defined by equation (18). The second one, noted $m^c(.)$, is the part of the conflict mass granted to the proposition $\mathcal{H}$. This value can be written as :

$$\forall\, \mathcal{H} \subseteq \mathcal{P} \qquad m^c(\mathcal{H}) = \sum m^{c*}(\mathcal{H}) \quad (23)$$

where $m^{c*}(\mathcal{H})$ is the part of the partial conflicting masses m^* assigned to the proposition $\mathcal{H}$:

$$\forall\, \mathcal{H} \subseteq \mathcal{P}^* \qquad m^{c*}(\mathcal{H}) = w^*(\mathcal{H}).m^*. \quad (24)$$

This generic framework allows to rewrite the operator proposed by Yager in [5].

3.2.2 Yager combination operator

The method proposed by Yager [5] follows this principle. While considering that at least one of the sources concerned with the fusion is reliable, but without knowing which is reliable, the conflict is then distributed on the set Θ. With respect to the generic framework presented in the section 3.2.1, we get a set $\mathcal{P}$ made of the whole set of hypotheses, that means :

$$\mathcal{P} = \{\Theta\}. \quad (25)$$

The weight associated to this set is equal to 1 ($w^*(\Theta) = 1$). The conflicting mass is placed therefore on Θ. This method has the effect of separating the totality of the conflicting mass, and so of more to make intervene it in the discernment of the hypotheses. This rule of combination is commutative. Unfortunately, it is not associative. It is necessary to define an order of fusion of the sources therefore. Let us note $m^2(.)$ the resulting mass function obtained with this combination operator.

3.2.3 Dubois and Prade combination rule

In the same way as for the Yager's combination, the operator of combination of Dubois and Prade [6] rests on the hypothesis that at least one a source intervening in the process of combination tells the truth. The principle of combination, for two sources of information, can be explained as it follows. Let S_i a source supporting $\mathcal{H}'$ with a mass $m_i(\mathcal{H}')$ and a S_j source supporting $\mathcal{H}''$ with a mass $m_j(\mathcal{H}'')$. If the propositions $\mathcal{H}'$ and $\mathcal{H}''$ are in contradiction, that is to say if $(\mathcal{H}' \cap \mathcal{H}'') = \emptyset$ then one does not know what source is right and one has to consider that the solution is one of the two propositions. The mass $m_i(\mathcal{H}').m_j(\mathcal{H}'')$ will be assigned then to the proposition $(\mathcal{H}' \cup \mathcal{H}'')$. The resulting mass, got with the rule of combination of Dubois and Prade, will be noted $m^3(.)$. In the general setting of this type of combination, we get a proposition $\mathcal{H}$, where the partial conflicting masses have been distributed, such as :

$$\mathcal{H} = \mathcal{H}' \cup \mathcal{H}'' \quad (26)$$

and :

$$\mathcal{P}^* = \{\mathcal{H}\}. \quad (27)$$

The totality of the partial conflicting mass is assigned to this proposition that means $w^*(\mathcal{H}) = 1$. This combination rule is more precise in the redistribution of the conflict and therefore more informative than the rule proposed by Yager. Besides, in the step of decision the conflicting mass having to be redistribute will intervene in the discernment of the hypotheses. One can notice that this combination uses a conjunctive approach when the sources agree and an disjunction approach in case of conflict. As for the rule of combination of Yager, the operator of fusion proposed by Dubois and Prade is commutative but is not associative. A strategy permitting to combine the sources in a certain order should be defined therefore.

4 New combination operators proposal

With this generic framework, we can define a family of combination operators. Among this family, we introduce two operators more precisely. The aim of the following aggregation operators is to distribute the conflicting mass among the smaller subsets which involve conflict as far as possible. In order to do that, the conflict management will not be done globally but locally. This local conflict will be distributed among the subsets according to a weighting factor associated with each subset. These weighting factors will be computed from the masses of each subset involved in the local conflict. For the different proposed combination operators, we will assume that the contradictions arise out of the shortcoming of the information sources. In addition, we will assume that at least one source tells the truth. We will suppose that the basic belief assignment $\{m_1, \ldots, m_j, \ldots, m_J\}$ of the J information sources are known before the fusion process. So, we will aggregate all the information sources simultaneously. So, these new combination operators avoid to order the sources before the fusion process as it is necessary in Dubois or Yager fusion scheme.

4.1 Proposition 1

4.1.1 Principle

The J information sources give basic belief assignment to each subset $\mathcal{H}_j \in \mathcal{F}_j$ with $j = \{1, \ldots, J\}$. When the hypotheses are compatible, that is to say when the intersection of the subsets $\mathcal{H}_j$ is not an empty subset, then the mass product assigned to these subsets is given to the intersection subset. If the hypotheses

$\mathcal{H}_j$ are not compatible, that is to say when their intersection is equal to the empty set, we have a partial conflicting mass m^* we have to distribute among the subsets $\mathcal{H}_j$. The redistribution is made only on the sets which involves a contradiction $\mathcal{H}_j$. The sets where the partial conflicting mass will be redistributed are written as follows :

$$\mathcal{P}^* = \{\mathcal{H}/\mathcal{H} \subseteq \mathcal{H}_j\} \quad \forall j = 1, \ldots, J. \tag{28}$$

After defined the set $\mathcal{P}^*$ containing the subsets on which the conflicting mass has to be redistributed, we define a function $\lambda(.)$ as it follows:

$$\forall \mathcal{H} \in \mathcal{P}^* \qquad \lambda(\mathcal{H}) = \sum_{\substack{\mathcal{H} \subseteq \mathcal{H}j \\ j=1,\ldots,J}} m_j(\mathcal{H}_j). \tag{29}$$

We have then, for each set $\mathcal{H}$, the associated weighting factor $w^*(\mathcal{H})$ defined by :

$$w^*(\mathcal{H}) = \frac{\lambda(\mathcal{H})}{\sum_{\mathcal{H} \subseteq \mathcal{P}^*} \lambda(\mathcal{H})}. \tag{30}$$

We obtain, as for the first combination operator, pairs $(\mathcal{H}, w^*(\mathcal{H}))$. From these pairs, we redistribute the local conflict using equation (24). The local conflict redistribution is then done proportionally to the masses assigned to each subset involving the conflict. The sum of all these redistributions gives the fusion result obtained with this combination operator. The mass assignment associated to this operator will be written $m^4(.)$.

4.1.2 Properties

The described aggregation operator is commutative but is not associative. This last fact does not impose a fusion order in the aggregation process as for Dubois or Yager. Indeed, the knowledge of all the mass functions avoids this constraint.

4.2 Proposition 2

The aim of the second combination operator is to assign the local conflicting mass which may exists on all the possible disjunctions of hypotheses from the sets involving the conflict.

4.2.1 Principle

Let be $m_j(\mathcal{H}_j)$ the J belief assignments given by the J information sources to each subset $\mathcal{H}_j \in \mathcal{F}_j$ with $j = \{1, \ldots, J\}$. When these subsets are compatible that is to say when $\bigcap_j \mathcal{H}_j \neq \emptyset$, we will assign this mass to the conjunction of the $\mathcal{H}_j$. If the subsets $\mathcal{H}_j$ are not compatible, we define the sets which take place in the redistribution of the conflicting mass. The sets where the partial conflicting mass will be redistributed are then defined by :

$$\mathcal{P}^* = \{\mathcal{H}/\mathcal{H} \subseteq \{\mathcal{H}_1, \ldots, \mathcal{H}_j, \ldots, \mathcal{H}_J\}\}. \tag{31}$$

At each set $\mathcal{H}$ is associated a mass equal to the sum of the masses assigned to the sets $\mathcal{H}_j$ such as $\mathcal{H}$ is included in the set $\mathcal{H}_j$. This mass is expressed as $\lambda(.)$ using equation (29). From the masses function $\lambda(.)$, we define weighting factors given to each set $\mathcal{H}$ as it follows :

$$w^*(\mathcal{H}) = \frac{\lambda(\mathcal{H})}{\sum_{\mathcal{H} \subseteq \mathcal{P}^*} \lambda(\mathcal{H})}. \tag{32}$$

Then we obtain a set of pairs $(\mathcal{H}, w^*(\mathcal{H}))$. The redistribution of the conflicting mass noted m^* is given then by the equation (24). As previous operators, the fusion mechanism has to be applied to all the partial conflicting zones. The summation of the masses $m^{c*}(.)$, derived from conflicting distributions, give then the fusion result which will be written $m^5(.)$.

4.2.2 Properties

This aggregation operator is based on the same principle as presented in section 4.1.1. It has the same properties. This operator is commutative but it is not associative. Nevertheless, the fact that the operator does not have the property of associativity does not impose a fusion order as it is necessary for the Dubois or Yager aggregation rules.

5 Results

The aim of the tests is to observe the behavior of the operators, described previously, in different situations. Initially, we will see an example of conflicting mass redistribution realized using the two operators proposed in this paper. We will describe (Section 5.2) the results obtained with the various operators in the situation presented by Zadeh in [4] and which was described in section 3.1.1. Lastly, a test where the context of data resulting from an information source evolves will be presented in section 5.3.

5.1 Example

To illustrate the proposed combination operators, we consider two information sources $\{S_1, S_2\}$ and a frame of discernment with three hypotheses such as $\Theta = \{H_1, H_2, H_3\}$. The basic belief assignment is given in the TAB. 1. Using figure FIG. 2, we explain the principles of the conflict mass distribution used by the two fusion operators describes above.

Table 1: Basic belief assignment for the two information sources.

Source S_1	**Source S_2**
$m_1(H_1) = 0.35$	$m_2(H_1) = 0.2$
$m_1(H_2) = 0.1$	$m_2(H_2) = 0.2$
$m_1(H_3) = 0.05$	$m_2(H_3) = 0.1$
$m_1(H_1 \cup H_2) = 0.2$	$m_2(H_1 \cup H_2) = 0.2$
$m_1(H_1 \cup H_3) = 0.15$	$m_2(H_1 \cup H_3) = 0.1$
$m_1(H_2 \cup H_3) = 0.1$	$m_2(H_2 \cup H_3) = 0.1$
$m_1(\Theta) = 0.05$	$m_2(\Theta) = 0.1$

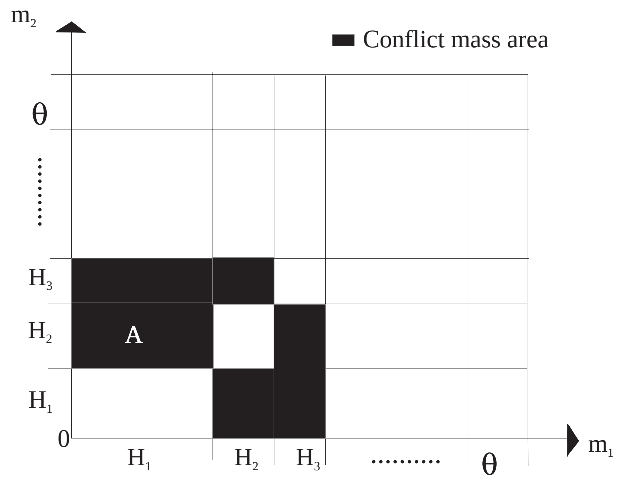

Figure 2: Representation of fusion for two sources.

5.1.1 Example Proposition 1

In the figure 2, the area noted A represented a partial conflict area between the information sources. So, the source S_1 asserts the hypothesis H_1 with a mass $m_1(H_1)$ equal to 0.35 and the source S_2 asserts H_2 with a mass $m_2(H_2)$ equal 0.2. The local conflicting mass is then :

$$m^* = m_1(H_1).m_2(H_2) = 0.07. \quad (33)$$

We are going to distribute this local conflicting mass proportionally to the mass affected to each source on the hypotheses H_1 and H_2. This mass will be redistributed on the sets :

$$\mathcal{P}^* = \{H_1, H_2\}. \quad (34)$$

Then, we obtain the following weighting factors :

$$w^*(H_1) = \frac{\lambda(H_1)}{\sum_{\mathcal{H} \subseteq \mathcal{P}^*} \lambda(\mathcal{H})} = \frac{m_1(H_1)}{m_1(H_1) + m_2(H_2)} = 0.64 \quad (35)$$

and :

$$w^*(H_2) = \frac{\lambda(H_2)}{\sum_{\mathcal{H} \subseteq \mathcal{P}^*} \lambda(\mathcal{H})} = \frac{m_2(H_2)}{m_1(H_1) + m_2(H_2)} = 0.36. \quad (36)$$

According to these weighting factors, the distribution of the conflicting mass is then :

$$m^{c*}(H_1) = w^*(H_1).m^* = 0.0448 \quad (37)$$

and :

$$m^{c*}(H_2) = w^*(H_2).m^* = 0.0252. \quad (38)$$

By applying this rule to the whole combination of hypotheses for two sources, we obtain the resulting mass assignment presented in Table 2.

5.1.2 Example Proposition 2

The value m^* of the first partial conflict between the two sources is equal to 0.07 (33). The distribution rule of the mass will be as it follows. At first, we define the masses assigned to each subset according to the equation (29). We obtain then :

$$\begin{aligned} \lambda(H_1) &= m_1(H_1) = 0.35 \\ \lambda(H_2) &= m_2(H_2) = 0.2 \\ \lambda(H_1 \cup H_2) &= m_1(H_1) + m_2(H_2) = 0.55\ . \end{aligned} \quad (39)$$

The weighting factors will be defined according to the equation (32). So, we obtain the following weighting factors :

$$\begin{aligned} w^*(H_1) &= \tfrac{0.35}{1.1} = 0.32 \\ w^*(H_2) &= \tfrac{0.2}{1.1} = 0.18 \\ w^*(H_1 \cup H_2) &= \tfrac{0.55}{1.1} = 0.5\ . \end{aligned} \quad (40)$$

Then, we obtain the distribution of the local conflicting mass m^* between the different propositions :

$$\begin{aligned} m^{c*}(H_1) &= m^*.w^*(H_1) = 0.0224 \\ m^{c*}(H_2) &= m^*.w^*(H_2) = 0.0126 \\ m^{c*}(H_1 \cup H_2) &= m^*.w^*(H_1 \cup H_2) = 0.035\ . \end{aligned} \quad (41)$$

If we apply this rule to all the partial conflicting areas and by computing the sum of the whole functions $m^{c*}(.)$, we obtain the fusion result. The complete fusion result of the two sources is given Table 2.

Table 2: Result of the fusion process.

	Resulting Masses	
Hypotheses	**Proposition 1**	**Proposition 2**
H_1	0.4782	0.4031
H_2	0.245	0.2041
H_3	0.1142	0.0920
$H_1 \cup H_2$	0.0788	0.140
$H_1 \cup H_3$	0.0444	0.0780
$H_2 \cup H_3$	0.0344	0.0520
Θ	0.005	0.0308

5.1.3 Remarks

We can see that operator of combination 1 we propose promote the hypotheses singletons at the cost of the composite hypotheses. On the contrary, operator of combination 2 promote the composite hypotheses. In addition, on the contrary of combination operators 1, the combination 2 redistributes conflicting mass on the whole set of composite hypotheses. With Table 3, we can compare the fusion results obtained with these two operators with those obtained with traditional combination operators.

5.2 Sensibility

We are now going to study the sensibility of the various operators to the conflict variations. This first study will be carried out using the mass distributions suggested by the equation (11). To study the sensibility, we will vary the value of ϵ what will involve conflict variations. The various masses resulting from the combination operators are presented in Figures 3 and 4.

Table 3: Result of the fusion process.

	Resulting Masses	
Hypotheses	**Dempster**	**Dubois**
H_1	0.4722	0.34
H_2	0.2361	0.17
H_3	0.1042	0.075
$H_1 \cup H_2$	0.0972	0.16
$H_1 \cup H_3$	0.0486	0.08
$H_2 \cup H_3$	0.0347	0.045
Θ	0.0069	0.13

In Figure 3, we can check the sensibility of Dempster's combination rule. Indeed in the case of a severe conflict, a small conflict variation involves strong variations of masses $m^0(H_1)$ and $m^0(H_2)$. On this same figure, we notice that the results obtained by the other combinations operators are less influenced by the conflict variations. Moreover, we note that the combination operator proposed by Dubois and the operator noted Proposition 2 have the same behavior roughly. Whereas Proposition 1 has a different behavior by not redistributing mass on disjunctions of hypotheses (Figure 4). This is due to its conjunctive behavior.

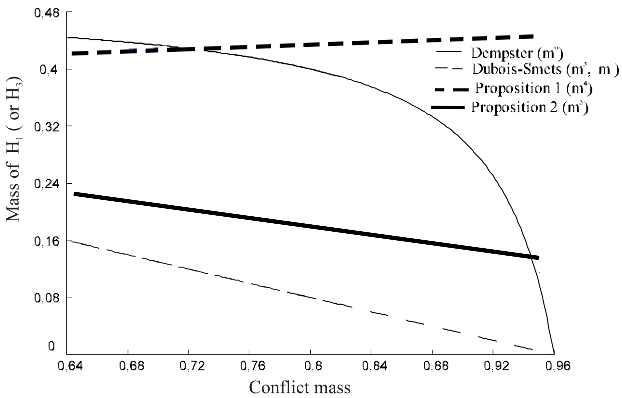

Figure 3: Mass of H_1 (or H_3) vs. conflict mass.

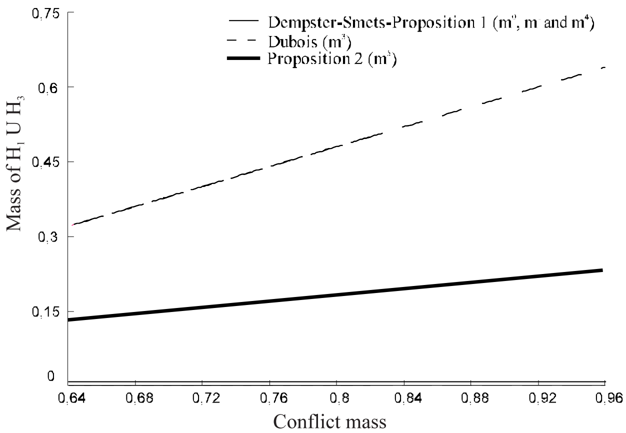

Figure 4: Mass of $H_1 \cup H_3$ vs. conflict mass.

5.3 Context evolution

For this second test, we take again one of the tests realized by Appriou in [1]. This test implements a problem with two sources and two hypotheses. These two sources have a good capacity of discrimination, but a doubt on the training concerning the hypothesis H_2 of the source S_2. The available learning are given by normal distribution :

$$\begin{array}{c} P(S_1/H_1) = P(S_2/H_1) = N(0,1) \\ P(S_1/H_2) = P(S_2/H_2) = N(6,1) \end{array} \quad (42)$$

while measurements actually simulated follow :

$$\begin{array}{c} P(S_1/H_1) = P(S_2/H_1) = N(0,1) \\ P(S_1/H_2) = N(6,1) \\ P(S_2/H_2) = N(S,1). \end{array} \quad (43)$$

So, in this test, the source S_1 has effectively a good knowledge, and the reliability of source S_2 varies in function of the signal S due to H_2. S represents a possible evolution of the context of this hypothesis. The masses first determined from simulated measurements are obtained using model 1 presented by Appriou [13]. The recognition of the assumption H_1 is not a problem because simulated measurements are far consistent to the training. It is not the case for the hypothesis H_2. Indeed, when the signal S is close to 0, the two sources are in conflict. The source S_1 asserts being in the presence of an hypothesis H_2 whereas the source S_2 supports the hypothesis H_1. On the contrary, when the signal S is close to value 6, the sources support the hypothesis H_2. In order to analyze the fusion result obtained using the operators described previously, we use two uncertain measurements introduced by Yager [14]. The first of these measurements is the confusion which translates ambiguity on the mass distributions within a belief structure. The second, the no-specificity, translates the fact that for belief structure given the mass is more or less distributed on sets of big size. Figure 5 represents the evolution of confusion according to the parameter S. In conflicting areas, we can note that the masses resulting from the fusion using the Dempster or Dubois operator have a weak confusion. With regard to the Dempster's combination rule in the event of raised conflict, normalization imposed by this operator becomes very significant. The conflict mass is then redistributed on only one hypothesis, whereas the sources are conflicting, which generates a weak confusion. For the Dubois's combination, the conflicting mass is redistributed only on the conjunction of the hypotheses generating the conflict what is characterized by a null confusion and a significant no-specificity (see Figure 6). The operators introduced in this paper have a very significant confusion (not understanding between the two assumptions) in the case of strong conflict. Moreover, Proposition 1 is more specific than Proposition 2 but it redistributes the conflicting mass only on the hypotheses singletons. Lastly, when the information sources are agree ($S \approx 6$) then the results obtained with the various operators are identical, because only the conjunctive aspect of these operators are taken into account.

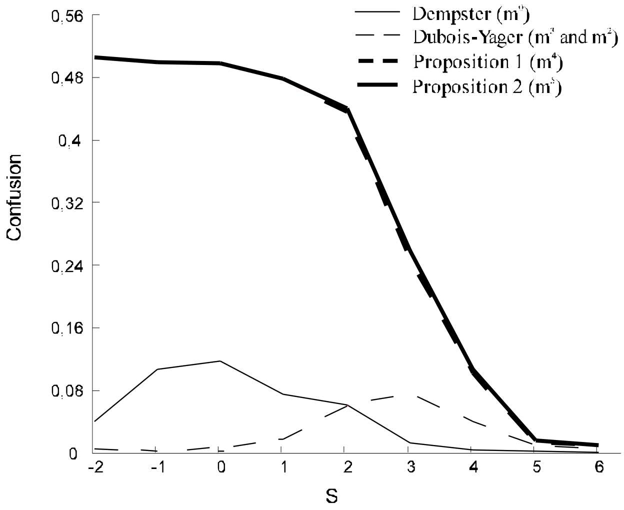

Figure 5: Confusion vs. S.

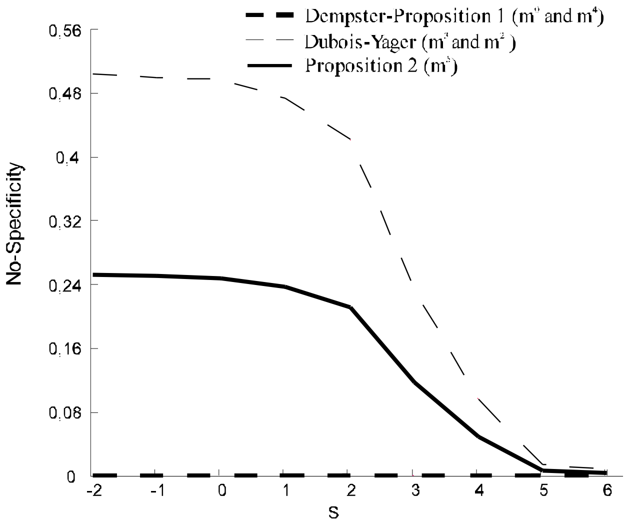

Figure 6: No-specificity vs. S.

6 Conclusion

In this paper, we introduced the problem of the sensibility in case of conflict with Demspter's combination rule. We have proposed a generic framework for the traditional operators fusion allowing to solve this problem. This framework allows us to propose two new combination operators of belief structures. These combinations, like the Dubois and Yager operators, are far from sensitive in conflicting situations. Moreover, on the contrary, the operators allows to distribute more precisely (in fact locally), the conflicting masses. Lastly, the no-associativity of the Dubois and Yager operators requires a fusion order in the aggregation process, whereas the employment of the operators proposed here does not impose this constraint. Indeed, the knowledge of all the mass functions avoids this constraint. Our future works consists to define an optimal decision rule, within the meaning of classification, for each proposed operator.

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