Reasoning under Uncertainty in the AHP Method using the Belief Function Theory

Amel Ennaceur[1], Zied Elouedi[1], and Eric Lefevre[2]

[1] LARODEC, University of Tunis, Institut Supérieur de Gestion, Tunisia
amel_naceur@yahoo.fr, zied.elouedi@gmx.fr
[2] Univ. Lille Nord of France, UArtois EA 3926 LGI2A, France
eric.lefevre@univ-artois.fr

Abstract. The Analytic Hierarchy Process (AHP) method was introduced to help the decision maker to express judgments on alternatives over a number of criteria. In this paper, our proposal extends the AHP method to an uncertain environment, where the uncertainty is represented through the Transferable Belief Model (TBM), one interpretation of the belief function theory. In fact, we suggest a novel framework that tackles the challenge of introducing uncertainty in both the criterion and the alternative levels, where the objective is to represent imperfection that may appear in the pair-wise comparisons and to model the relationship between these alternatives and criteria through conditional beliefs.

1 Introduction

Within the framework of Multi-Criteria Decision Making (MCDM) problems, many methods have been proposed and each one has its own characteristics [17]. We have then two major families. On the one hand, the outranking approach introduced by Roy where some methods like Electre and Promethee are developed [2], [5]. On the other hand, the value and utility theory approaches mainly started by Keeney and Raiffa [6], and then implemented in a number of methods [15]. Amongst the most well known ones is the Analytic Hierarchy Process (AHP) [8], [9], introduced by Saaty (1980) and based on preference judgments. In fact, in the AHP method, the problem is structured hierarchically at different levels. Within the same context, the purpose of constructing this hierarchy is to evaluate the influence of the criteria on the alternatives to attain objectives. In other words, the decision maker is required to provide his preferences by comparing all criteria, sub-criteria and alternatives with respect to upper level decision elements. This is accomplished through pair-wise comparisons.

The capability to deal with uncertainty and imprecision is the common problem of decision making. In fact, this imperfection can arise due to different situations: incomplete data for making decisions, imprecise judgments, etc.

However, standard AHP method was criticized because it does not well perform their task in such environment. Sometimes, the decision maker cannot ensure pair-wise comparisons between all the criteria and alternatives because the information about them may be incomplete due to the time pressure and the lack of data. In order to overcome this limitation, several extensions were developed such as referenced AHP [10], fuzzy AHP [7], etc.

In our work, we will focus on belief function framework. It is considered as a useful theory for representing and managing uncertain knowledge [11]. This theory provides a convenient framework for dealing with incomplete and uncertain information, notably those given by experts. So, a first work has been tackled by Beynon et al. have proposed a method called the DS/AHP method [1] comparing not only single alternatives but also groups of them. Besides, belief AHP approach was introduced by [4] which evaluates sets of alternatives according to sets of criteria. Also, several works has been proposed by Utkin [16], etc. Despite all the advantages of these two approaches, allowing different comparisons to be made for groups of alternatives and/or criteria, they do not take into account the conditional relationships between alternatives and criteria. In fact, alternatives do not always have a unique priority relationship between them. For instance, in a problem of buying a car, the expert might consider that "Peugeot" is evaluated to be more important than "Renault" regarding comfort criterion, but "Renault" is more important than "Peugeot" with respect to style criterion. As we can see, the alternative priorities are dependent on each specific criterion.

To solve the problem presented above, this paper presents a new AHP approach under uncertainty, a MCDM method adapted to incomplete and uncertain preferences but also it models conditional relationships between alternatives and criteria, where the uncertainty is represented by belief functions as defined in the Transferable Belief Model (TBM). The choice of the TBM seems appropriate as it allows experts that represent sources of information to express their believes about the cause-effect relationship degree not only in terms of elementary events but also in terms of subsets. Besides, belief function theory offers interesting tools to model the partial and total ignorance and to combine several pieces of evidence as the conjunctive and the disjunctive rules of combination and for conditioning. Our aim through this work is then to represent uncertainty and to more imitate the expert reasoning since he tries to express his preferences over the sets of alternatives regarding each criterion and not regardless of the criteria. Consequently, we try to represent the influences of the criteria on the evaluation of alternatives.

The remainder of this paper is organized as follows. In section 2, we focus on AHP method. Next, we present some useful definitions needed for belief function context. Section 4 represents our new AHP method based on conditional belief functions, and gives an example to show its application. Finally, section 5 concludes the paper.

2 Analytic Hierarchy Process

The AHP approach is a decision-making technique developed by Saaty [8], [9] to solve complex problems of choice and prioritization. The basic idea of the approach is to convert subjective assessments of relative importance to a set of overall scores or weights. The AHP decision problem is structured hierarchically at different levels. The purpose of constructing this hierarchy is to evaluate the influence of the criteria on the alternatives to attain objectives. So, an AHP hierarchy has at least three levels: The highest level consists of a unique element that is the overall objective. Then, each level of the hierarchy contains criteria or sub-criteria that influence the decision. Alternative elements are put at the lowest level.

Once the hierarchy is built, the decision maker starts the prioritization procedure to determine the relative importance of the elements on each level of the hierarchy (criteria and alternatives). Elements of a problem on each level are paired (with respect to their upper level decision elements) and then compared. This method elicits preferences through pair-wise comparisons which are constructed from decision maker's answers. Indeed, the decision maker can use both objective information about the elements as well as subjective opinions about the elements relative meaning and importance. The responses to the pair-wise comparison question use a nine-point scale [8], which translates the preferences of a decision maker into crisp numbers.

Next, the comparison matrix is formed by repeating the process for each level of the hierarchy. After filling all the pair-wise comparison matrices, the local priority weights are determined by using the eigenvalue method. The objective is then to find the weight of each criterion, or the score of each alternative by calculating the eigenvalue vector. With these values, the AHP method permits to compute a consistency ration to check if the matrix is consistent or not. When the matrix is considered inconsistent, the entries that are given by the decision maker have to be revised until a satisfactory consistency ratio is obtained. The last step of the AHP aggregates all local priorities from the decision table by a simple weighted sum. The global priorities thus obtained are used for final ranking of the alternatives and selection of the best one.

3 Belief Function Theory

In this section, we briefly review the main concepts underlying the Transferable Belief Model (TBM), one interpretation of the belief function theory [14].

3.1 Basic Concepts

The TBM is a model to represent quantified belief functions [14]. Let Θ be the frame of discernment representing a finite set of elementary hypotheses related to a problem domain. We denote by 2^{Θ} the set of all the subsets of Θ [11].

The impact of a piece of evidence on the different subsets of the frame of discernment Θ is represented by the so-called basic belief assignment (bba), called initially by Shafer basic probability assignment [11].

A bba is a function denoted by m that assigns a value in $[0,1]$ to every subset A of Θ such that:

$$\sum_{A \subseteq \Theta} m(A) = 1 \ . \tag{1}$$

The value $m(A)$, named a basic belief mass (bbm), represents the portion of belief committed exactly to the event A.

The belief function theory offers many interesting tools. For instance, to combine beliefs induced by distinct pieces of evidence, we can use the conjunctive rule of combination [13]. Also, to select the most likely hypothesis, one of the most used solutions is the pignistic probability [14].

3.2 Operations on the Product Space

Let $U = \{X, Y, Z, \ldots\}$ be a set of variables, where each variable has its frame of discernment. Let X and Y be two disjoint subsets of U. Their frames are the product space of the frames of the variables they include.

Vacuous Extension. Given a bba defined on X, its vacuous extension on $X \times Y$ denoted $m^{X \uparrow X \times Y}$ is given by [12]:

$$m^{X \uparrow X \times Y}(B) = \begin{cases} m^X(A) & \text{if } B = A \times Y, A \subseteq X, \\ 0 \text{ otherwise} \ . \end{cases} \tag{2}$$

Marginalization. A bba defined on a product space $X \times Y$ may be marginalized on X by transferring each mass $m^{X \times Y}(B)$ for $B \subseteq X \times Y$ to its projection on X [12]:

$$m^{X \times Y \downarrow X}(A) = \sum_{\{B \subseteq X \times Y | Proj(B \downarrow X) = A)\}} m^{X \times Y}(B), \forall A \subseteq X \ . \tag{3}$$

where $Proj(B \downarrow X)$ denotes the projection of B onto X.

Ballooning Extension. Let $m^X[y_i]$ represents your beliefs on X conditionnally on y_i a subset of Y, i.e., in a context where y_i holds. The ballooning extension is defined as:

$$m^X[y_i]^{\Uparrow X \times Y}(A \times y_i \cup X \times \bar{y_i}) = m^X[y_i](A), \forall A \subseteq X \ . \tag{4}$$

4 AHP Method in an Uncertain Environment

In this section, we introduce the concept of the AHP method within the belief function framework. We start by explaining how this method works, and then an example will be traced to further understand and illustrate our approach.

4.1 AHP Method in the Belief Function Context

Since impression and uncertainty are common characteristics in many decision-making problems, our new AHP method should be able to deal with this uncertainty. Within this context, a first work has been introduced by Beynon et al. [1]. They developed a method, called DS/AHP which compares not only one alternative but also groups of alternatives. With the comparison matrix, the eigenvector is computed and transformed into a bba. Then a combination rule is used to aggregate these bba's, and the belief and plausibility measures are used to choose the best alternatives. In addition, Ennaceur et al. [4] have proposed a method named belief AHP, a combination between the AHP method and the belief function theory as understood in the transferable belief model, that evaluates sets of alternatives according to sets of criteria. To choose the best alternative, this approach proposes to use the pignistic probabilities. From another perspective, Dezert et al. [3] have proposed to follow Beynon's approach, but instead of using the belief function theory, they investigate the possibility to use the Dezert-Smarandache theory. The DSmT/AHP method uses the PCR5 rule to combine the priorities vectors whereas the DS/AHP applies the Dempster's rule.

In spite of all the advantages of the proposed methods, they do not take into account the conditional relationships between alternatives and criteria. For instance, in the alternative level, the expert tries to estimate his opinions-beliefs about alternatives according to each criterion. That is why; representing uncertainty by bba's seems to be inconsistent with the expert reasoning because he tries to express his preferences regarding each criterion and not regardless of the criteria. As a result, in this work, we want to more imitate the human reasoning. Therefore, we suggest a new approach based on AHP method. Unlike belief AHP where the evaluation of each alternative with respect to each criterion is given by a bba, we try to model the evaluation of each subset of alternatives with respect to each criterion by conditional beliefs. Additionally, after eliciting the expert's preferences at the criterion level, we suppose that criteria weights are also expressed by means of a bba in order to represent the imperfect evaluation.

4.2 Uncertain AHP Approach

Identification of the Candidate Alternatives and Criteria. One of the key questions being issued over the implementation of any MCDM problem is the identification of the candidate alternatives and criteria. As in [4], the main aim of our proposed approach is the allowance for incompleteness in the judgments made by expert. Besides, in many complex problems decision makers are able to compare only subsets of criteria and cannot evaluate separate ones. To solve this problem, that means to reduce the number of criteria which decreases the number of comparisons, our method suggests to allow the expert to express his opinions on groups of criteria instead of single one. So, he chooses these subsets by assuming that criteria having the same degree of preference are grouped together. For instance, if an expert identifies a group of criteria, then we could suppose that all of them have the same importance.

Let $\Omega = \{c_1, \ldots, c_m\}$ be a set of criteria, we denote the set of all subsets of Ω by 2^Ω, and let C_k be the short notation of a subset of Ω. By generalization, these groups of criteria can be defined as:

$$C_k \succ C_j, \forall\ k, j | C_k, C_j \in 2^\Omega, C_k \cap C_j = \emptyset\ . \tag{5}$$

On the other hand and similarly to the criterion level, our method proposes not to consider all the alternatives but just to choose groups of them. So, we assume that $\Theta = \{a_1, \ldots, a_n\}$ is a set of alternatives, and we denote the set of all subsets of Θ by 2^Θ. In other terms and as explained in [4], the decision maker compares not only a single one but also sets of alternatives between each other.

By comparing subsets between each other, we provide a major benefit to the decision maker. In fact, our proposed approach has reduced the number of comparisons, because instead of using single elements, we have used subsets.

Pair-wise Comparisons and Preference Elicitation. Once the sets of criteria and alternatives are defined, the expert tries to specify his preferences in order to obtain the criterion weights and the alternative scores in terms of each criterion. In this study, we have adopted the Saaty's scale to evaluate the importance of pairs of grouped elements in terms of their contribution. Thus, the priority vectors are then generated using the eigenvector method and we have chosen the standard consistency index in order to ensure that uncertain AHP's pair-wise comparison matrix is consistent.

Updating the Alternatives Priorities. Having made all the pair-wise comparisons, we will be interested in this step by showing how to combine the obtained alternatives priorities with the importance of their corresponding criteria. In the first step of the approach, the uncertainty is introduced on the decision maker preferences. Besides, we propose to represent the imperfection over the sets of criteria. Within our framework, we have $C_i \subseteq 2^\Omega$ and we have the priority values of each C_i representing the opinions-beliefs of the expert about his preferences. We also notice that this priority vector sums to one which can be regarded as a bba. As a result, this bba can be denoted by m^Ω.

Furthermore, we propose to represent the uncertainty at the alternative level. Unlike the criterion level, the expert tries to express his preferences over the sets of alternatives regarding each criterion and not regardless of the criteria. Accordingly, and to more imitate the expert reasoning, we indicate that to define the influences of the criteria on the evaluation of alternatives, we might use a conditional belief. Given a pair-wise comparison matrix which compares the sets of alternatives according to a specific criterion, a conditional bba can be represented by:

$$m^\Theta[c_j](A_k) = w_k, \quad \forall A_k \subseteq 2^\Theta \text{ and } c_j \in \Omega\ . \tag{6}$$

where A_k represents a subset of 2^Θ, w_k is the eigen value of the k^{th} sets of alternatives regarding the criterion c_j. $m^\Theta[c_j](A_k)$ means that we know the belief about A_k regarding c_j.

As indicated above, our objective through this step is to combine the obtained conditional belief with the importance of their respective criteria to measure their contribution. In this context, our major problem here is that we have priorities concerning criteria and groups of criteria that are defined on the frame of discernment Ω, whereas the sets of decision alternatives are generally defined on another frame Θ. In order to solve this problem, we propose to standardize our frame of discernment. First, at the criterion level, our objective is then to redefine the bba that represents criteria weights. Indeed, we propose to extend this bba from Ω to $\Theta \times \Omega$:

$$m^{\Omega \uparrow \Theta \times \Omega}(B) = m^{\Omega}(C_i) \quad B = \Theta \times C_i, C_i \subseteq \Omega \ . \tag{7}$$

Second, at the alternative level, the idea was to use the deconditionalization process in order to transform the conditional belief into a new belief function. In this case, the ballooning extension technique is applied:

$$m^{\Theta}[c_j]^{\Uparrow \Theta \times \Omega}(A_k \times c_j \cup \Theta \times \bar{c_j}) = m^{\Theta}[c_j](A_k), \forall A_k \subseteq \Theta \ . \tag{8}$$

Once the frame of discernment $\Theta \times \Omega$ is formalized, our approach proposes to combine the alternative priorities. In fact, we assume that each pair-wise comparison matrix is considered as a distinct source of evidence, which provides opinions towards the preferences of particular decision alternatives. Then, based on the belief function framework, we can apply the conjunctive rule of combination. The obtained bba represents the belief in groups of alternatives based on the combined evidence from the decisions matrices.

Finally, we might combine the obtained bba with the importance of their respective criteria to measure their contribution. That is, we will apply the conjunctive rule of combination and we get:

$$m^{\Theta \times \Omega} = \left[\textcircled{\cap}_{j=1,\ldots,m} m^{\Theta}[c_j]^{\Uparrow \Theta \times \Omega} \right] \textcircled{\cap} m^{\Omega \uparrow \Theta \times \Omega} \ . \tag{9}$$

So, we obtain $m^{\Theta \times \Omega}$ reflecting the importance of alternatives to the given criteria.

Decision Making. To this end and after combining the resulting ballooning extension, a decision under uncertainty must be defined. In the sequel, the pignistic probabilities is used. However, our obtained beliefs are defined on the product space $\Theta \times \Omega$. To solve this problem, we propose to marginalize this bba on Θ (frame of alternatives) by transferring each mass $m^{\Theta \times \Omega}$ to its projection on Θ:

$$m^{\Theta \times \Omega \downarrow \Theta}(A_j) = \sum_{\{B \subseteq \Theta \times \Omega | Proj(B \downarrow \Theta) = A_j)\}} m^{\Theta \times \Omega}(B), \forall A_j \subseteq \Theta \ . \tag{10}$$

Finally, we can compute the pignistic probabilities to choose the best alternatives:

$$BetP(a_j) = \sum_{A_i \subseteq \Theta} \frac{|a_j \cap A_i|}{|A_i|} \frac{m^{\Theta \times \Omega \downarrow \Theta}(A_i)}{(1 - m^{\Theta \times \Omega \downarrow \Theta}(\emptyset))}, \forall a_j \in \Theta \ . \tag{11}$$

4.3 Example

To describe this approach, we consider the problem of purchasing a car. Suppose that this problem involves four criteria: $\Omega = \{$Comfort (c1), Style (c2), Fuel (c3), Quietness (c4)$\}$, and three selected alternatives: $\Theta = \{\text{Peugeot}(p), \text{Renault}(r), \text{Ford}(f)\}$.

The first stage is the identification of the groups of criteria and alternatives. Then, the expert can express his preferences over these subsets. At the criterion level, the following pair-wise matrix can be obtained (see Table 1). As indicated above, the criterion weights are expressed by a basic belief assessment (bba). In fact, after eliciting the expert's preferences, we get: $m^{\Omega}(\{c1\}) = 0.58$, $m^{\Omega}(\{c4\}) = 0.32$ and $m^{\Omega}(\{c2, c3\}) = 0.1$.

Table 1. The weights assigned to the criteria according to the expert's opinion

Criteria	$\{c1\}$	$\{c4\}$	$\{c2, c3\}$	Priority
$\{c1\}$	1	2	6	0.58
$\{c4\}$	$\frac{1}{2}$	1	4	0.32
$\{c2, c3\}$	$\frac{1}{6}$	$\frac{1}{4}$	1	0.1

Next, we propose to model the alternative score by means of conditional bba. After constructing the pair-wise comparison matrices, the priorities vectors regarding each criterion are shown in Table reftab1. For example, the alternative $\{p\}$ given $c1$ can be represented by $m^{\Theta}[c1](\{p\}) = 0.806$, which means that we know the belief about $\{p\}$ regarding the criterion $c1$.

Table 2. Priorities values

$c1$	Priority	$c2$	Priority	$c3$	Priority	$c4$	Priority
$\{p\}$	0.806	$\{p\}$	0.4	$\{r\}$	0.889	$\{f\}$	0.606
$\{p, r, f\}$	0.194	$\{r, f\}$	0.405	$\{p, r, f\}$	0.111	$\{p, r, f\}$	0.394
		$\{p, r, f\}$	0.191				

According to our approach, the next step is to standardize the criterion and the alternative frames of discernment. For the criterion level, we suggest to apply the extension procedure. Hence, Equation 7 is used and the resulting bba's is summarized in Table 3.

After normalizing the criteria's bba, the next step is to transform the conditional belief into joint distribution. Indeed, we suggest to compute the ballooning extension using Equation 8 (see Table 4).

As explained before, once the ballooning extensions are obtained, we can apply the conjunctive rule. The result of this combination will be a unique bba representing the belief in groups of alternatives based on the combined evidence from the decisions matrices.

Table 3. Vacuous extension of bba

bbm	Vacuous extension	Values
$m^{\Omega}(\{c1\})$	$\{(p, c1), (r, c1), (f, c1)\}$	0.58
$m^{\Omega}(\{c4\})$	$\{(p, c4), (r, c4), (f, c4)\}$	0.32
$m^{\Omega}(\{c2, c3\})$	$\{(p, c2), (r, c2), (f, c2), (p, c3), (r, c3), (f, c3)\}$	0.1

Table 4. Ballooning extension of conditional bba

conditional bbm	Ballooning extension	Values
$m^{\Theta}[c1](\{p\})$	$\{(p, c1), (p, c2), (p, c3), (p, c4), (r, c2),$ $(r, c3), (r, c4), (f, c2), (f, c3), (f, c4)\}$	0.806
$m^{\Theta}[c1](\{p, r, f\})$	$\{(p, c1), (p, c2), (p, c3), (p, c4),$ $(r, c1), (r, c2), (r, c3), (r, c4), (f, c1), (f, c2), (f, c3), (f, c4)\}$	0.194

Then, we propose to apply Equation 9, to combine the obtained bba with the criterion weights (bba) as exposed in Table 5.

Table 5. The obtained bba: $m^{\Theta\times\Omega}$

$\{(p, c1), (f, c1), (r, c1)\}$	0.362
$\{(p, c1)\}$	0.315
$\{(p, c4), (f, c4), (r, c4)\}$	0.1302
$\{(f, c1)\}$	0.0064
$\{(p, c2), (f, c2), (r, c2), (p, c3), (f, c3), (r, c3)\}$	0.008
$\{(r, c2), (r, c3), (f, c2), (p, c2)\}$	0.0664
$\emptyset$	0.112

Next, to choose the best alternatives, we must define our beliefs over the frame of alternatives. The solution is then to marginalize on Θ using the Equation 10, and we obtain the following distribution: $m^{\Theta\times\Omega\downarrow\Theta}(\{p, r, f\}) = 0.5666$, $m^{\Theta\times\Omega\downarrow\Theta}(\{p\}) = 0.315$, $m^{\Theta\times\Omega\downarrow\Theta}(\{f\}) = 0.0064$ and $m^{\Theta\times\Omega\downarrow\Theta}(\emptyset) = 0.112$.

Finally, the pignistic probabilities can be computed, and we get:
$BetP(p) = 0.567$, $BetP(r) = 0.220$ and $BetP(f) = 0.213$.

As a consequence, the alternative "Peugeot" is the recommended car since it has the highest values.

5 Conclusion

This paper provides a new MCDM method that combines the analytic hierarchy process with the belief function theory. We have first introduced imperfection in the criterion and alternative levels, in order to allow the decision maker to easily express his assessments and also to correctly represent his preferences. In addition, we have shown that to correctly represent the expert's opinion, our approach investigates some ways to define the influences of the criteria on the evaluation of alternatives. Moreover, we have noticed that when applying our proposed approach, the number of comparisons is usually inferior to standard AHP because instead of using single elements we have used subsets.

As future works, we plan to apply our approach on a real application problem and we propose to do a sensibility analysis. A comparison between our presented solution and other methods like Fuzzy AHP will be also interesting to make.

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