An Extension of the Analytic Hierarchy Process Method Under the Belief Function Framework

Amel Ennaceur[1,2], Zied Elouedi[1], and Eric Lefevre[2]

[1] LARODEC, University of Tunis, Institut Supérieur de Gestion, Tunisia
amel_naceur@yahoo.fr, zied.elouedi@gmx.fr
[2] Univ. Lille Nord of France, UArtois EA 3926 LGI2A, France
eric.lefevre@univ-artois.fr

Abstract. In this paper, an extension of the belief Analytic Hierarchy Process (AHP) method is proposed, based on the belief function framework. It takes into account the fact that the pair-wise comparison between criteria and alternatives may be uncertain and imprecise. Therefore, it introduces a new way to cope with expert judgments. Thus to express his preferences, the decision maker is allowed to use a belief assessment instead of exact ratios. The proposed extension also models the relationship between the alternative and criterion levels through conditional beliefs. Numerical examples explain in detail and illustrate the proposed approach.

1 Introduction

Analytic Hierarchy Process (AHP) method [5] is one of the widely preferred multi-criteria decision making (MCDM) methods and has successfully been applied to many practical problems. Using this approach, the decision maker models a problem as a hierarchy of criteria and alternatives. Then, the expert assesses the importance of each element at each level using a pair-wise comparison matrix, where elements are compared to each other.

Though its main purpose is to capture the expert's knowledge, the standard AHP still cannot reflect the human thinking style. It is often criticized for its use of an unbalanced scale of estimations and its inability to adequately handle the uncertainty and imprecision associated with the mapping of the decision maker's perception to a crisp number [4].

In order to model imperfect judgments, the AHP method was modified by many researchers. Under the belief functions framework, Beynon et al. have proposed a method called the DS/AHP method [1] comparing not only single alternatives but also groups of alternatives. Besides, several works has been defined by Utkin [10]. Also, Ennaceur et al. [2] [3] have developed the belief AHP approach that compares groups of criteria to subsets of alternatives. Then, they model the causality relationship between these groups of alternatives and criteria.

Taking into account the above, we propose an extension of the belief AHP method [3], a Multi-Criteria Decision Making (MCDM) method under the belief function framework. On the one hand, our proposed method takes into account the conditional relationships between alternatives and criteria. In fact, our aim is to more imitate the expert reasoning since he tries to express his preferences over the sets of alternatives regarding each criterion and not regardless of the criteria. Consequently, we try to represent the influences of the criteria on the evaluation of alternatives. On the other hand, our method takes into account the fact that the pair-wise comparison may be uncertain and imprecise. Therefore, it introduces a new way to cope with expert judgments. Thus to express his assessments, the decision maker is allowed to use subjective assessments instead of using numerical values. Then, a preference degree may be assigned to each expert's response. With our method, the expert does not require to complete all the comparison matrix. He can then derive priorities from incomplete set of judgments. Therefore, a new procedure is employed, he only selects the related linguistic variable to indicate whether a criterion or alternative was more or less important to its partner by "yes" or "no".

The proposed method uses the pair-wise comparisons with the minimal information. Therefore, using our proposed approach, we cannot get the best alternative but at least we can choose the most cautious one.

In what follows, we first present some definitions needed for belief function context. Next, we describe the belief AHP method in section 3. Then, section 4 details our new MCDM method, and gives an example to show its application. Finally, section 5 concludes the paper.

2 Belief Function Theory

2.1 Basic Concepts

The Transferable Belief Model (TBM) is a model to represent quantified belief functions [9]. Let Θ be the frame of discernment representing a finite set of elementary hypotheses related to a problem domain. We denote by 2^{Θ} the set of all the subsets of Θ [6].

The impact of a piece of evidence on the different subsets of the frame of discernment Θ is represented by the so-called basic belief assignment (bba) [6]. A bba is a function denoted by m that assigns a value in $[0,1]$ to every subset A of Θ such that:

$$\sum_{A \subseteq \Theta} m(A) = 1 \ . \tag{1}$$

The value $m(A)$, named a basic belief mass (bbm), represents the portion of belief committed exactly to the event A.

2.2 Operations on the Product Space

Vacuous Extension. This operation is useful, when the referential is changed by adding new variables. Thus, a marginal mass function m^{Θ} defined on Θ will be expressed in the frame $\Theta \times \Omega$ as follows [7]:

$$m^{\Theta\uparrow\Theta\times\Omega}(C) = m^{\Theta}(A) \quad \text{if } C = A\times\Omega, A\subseteq\Theta \ . \tag{2}$$

Marginalization. Given a mass distribution defined on $\Theta\times\Omega$, marginalization corresponds to mapping over a subset of the product space by dropping the extra coordinates. The new belief defined on Θ is obtained by [7]:

$$m^{\Theta\times\Omega\downarrow\Theta}(A) = \sum_{\{B\subseteq\Theta\times\Omega|B^{\downarrow\Theta}=A)\}} m^{\Theta\times\Omega}(B), \forall A\subseteq\Theta \ . \tag{3}$$

$B^{\downarrow\Theta}$ denotes the projection of B onto Θ.

Ballooning Extension. Let $m^{\Theta}[\omega]$ represents your beliefs on Θ conditionnally on ω a subset of Ω. To get rid of conditioning, we have to compute its ballooning extension. The ballooning extension is defined as [7]:

$$m^{\Theta}[\omega]^{\Uparrow\Theta\times\Omega}(A\times\omega\cup\Theta\times\bar{\omega}) = m^{\Theta}[\omega](A), \forall A\subseteq\Theta \ . \tag{4}$$

3 Belief AHP Method

The belief AHP method is a MCDM method that combines the AHP approach with the belief function theory [3]. This method investigates some ways to define the influences of the criteria on the evaluation of alternatives.

3.1 Identification of the Candidate Alternatives and Criteria

Let $\Omega = \{c_1, \ldots, c_m\}$ be a set of criteria, and let C_k be the notation of a subset of Ω. The groups of criteria can be defined as [2]:

$$\forall\ k, j|C_k, C_j \in 2^{\Omega}, C_k\cap C_j = \emptyset \text{ and } \cup_j C_j = \Omega \text{ (with } C_j \text{ exclusive).} \tag{5}$$

This method suggests to allow the expert to express his opinions on groups of criteria instead of single one. So, he chooses these subsets by assuming that criteria having the same degree of preference are grouped together. On the other hand and similarly to the criterion level, the decision maker compares not only pairs of single alternatives but also sets of alternatives between each other ($\Theta = \{a_1, \ldots, a_n\}$ is a set of alternatives)[2].

3.2 Pair-wise Comparisons and Preference Elicitation

After identifying the set of criteria and alternatives, the weights of each element have to be defined. The expert has to provide all the pair-wise comparisons matrices. In this study, Saaty's scale is chosen in order to evaluate the importance of pairs of grouped elements in terms of their contribution. Thus, the priority vectors are then generated using the eigenvector method.

3.3 Updating the Alternatives Priorities

Within this framework, we have $C_i \subseteq 2^{\Omega}$ and we have the criterion priority vector is regarded as a bba, denoted by m^{Ω}.

Furthermore, Belief AHP tries to model the influences of the criteria on the evaluation of alternatives by conditional belief. So, given a pair-wise comparison matrix which compares the sets of alternatives according to a specific criterion, a conditional bba can be represented by: $m^{\Theta}[c_j](A_k) = w_k, \quad \forall A_k \subseteq 2^{\Theta}$ and $c_j \in \Omega$ where $m^{\Theta}[c_j](A_k)$ means that we know the belief about A_k regarding c_j.

Then, the aggregation procedure can be represented as follows. In fact, priorities concerning criteria and groups of criteria are defined on the frame of discernment Ω, whereas the sets of alternatives are defined on Θ. The idea was to standardize the frame of discernment. First, at the criterion level, the bba that represents criteria weights is extended from Ω to $\Theta \times \Omega$:

$$m^{\Omega \uparrow \Theta \times \Omega}(B) = m^{\Omega}(C_i) \quad B = \Theta \times C_i, C_i \subseteq \Omega \ . \tag{6}$$

Second, at the alternative level, the idea was to use the deconditionalization process in order to transform the conditional belief into a new belief function. In this case, the ballooning extension technique is applied:

$$m^{\Theta}[c_j]^{\Uparrow \Theta \times \Omega}(A_k \times c_j \cup \Theta \times \bar{c_j}) = m^{\Theta}[c_j](A_k), \forall A_k \subseteq \Theta \ . \tag{7}$$

Once the frame of discernment $\Theta \times \Omega$ is formalized, the belief AHP approach proposes to combine the obtained bba with the importance of their respective criteria to measure their contribution using the conjunctive rule of combination $\bigcirc\!\!\!\!\cap$ and we get [8]:

$$m^{\Theta \times \Omega} = \left[\bigcirc\!\!\!\!\!\bigcap_{j=1,\ldots,m} m^{\Theta}[c_j]^{\Uparrow \Theta \times \Omega} \right] \bigcirc\!\!\!\!\cap\, m^{\Omega \uparrow \Theta \times \Omega} \ . \tag{8}$$

Finally, to choose the best alternatives, this method proposes to marginalize the obtained bba (in the previous step) on Θ (frame of alternatives) by transferring each mass $m^{\Theta \times \Omega}$ to its projection on Θ. Then, the pignistic probabilities [8] are used to make our choices:

$$BetP(a_j) = \sum_{A_i \subseteq \Theta} \frac{|a_j \cap A_i|}{|A_i|} \frac{m^{\Theta \times \Omega \downarrow \Theta}(A_i)}{(1 - m^{\Theta \times \Omega \downarrow \Theta}(\emptyset))}, \forall a_j \in \Theta \ . \tag{9}$$

3.4 Example

To describe this approach, we consider the problem of "purchasing a car" presented in [3]. Suppose that this problem involves four criteria: $\Omega = \{$Comfort (c_1), Style (c_2), Fuel (c_3), Quietness $(c_4)\}$, and three selected alternatives: $\Theta = \{\text{Peugeot}(p), \text{Renault}(r), \text{Ford}(f)\}$. For more details see [3].

At the criterion level, the criterion weights are expressed by a basic belief assessment (bba). We get: $m^{\Omega}(\{c_1\}) = 0.58$, $m^{\Omega}(\{c_4\}) = 0.32$ and $m^{\Omega}(\{c_2, c_3\}) = 0.1$.

Table 1. Priorities values

c_1	Priority	c_2	Priority	c_3	Priority	c_4	Priority
$\{p\}$	0.806	$\{p\}$	0.4	$\{r\}$	0.889	$\{f\}$	0.606
$\{p,r,f\}$	0.194	$\{r,f\}$	0.405	$\{p,r,f\}$	0.111	$\{p,r,f\}$	0.394
		$\{p,r,f\}$	0.195				

Next, we propose to model the alternative score by means of conditional bba (see Table 1).

According to the belief AHP approach, the next step is to standardize the criterion and the alternative frames of discernment. For the criterion level, the resulting bba's is summarized in Table 2.

Table 2. Vacuous extension of bba

bbm	Vacuous extension	Values
$m^{\Omega}(\{c_1\})$	$\{(p,c_1),(r,c_1),(f,c_1)\}$	0.58
$m^{\Omega}(\{c_4\})$	$\{(p,c_4),(r,c_4),(f,c_4)\}$	0.32
$m^{\Omega}(\{c_2,c_3\})$	$\{(p,c_2),(r,c_2),(f,c_2),(p,c_3),(r,c_3),(f,c_3)\}$	0.1

After normalizing the criteria's bba, the next step is to transform the conditional belief into joint distribution using Equation 7 (see Table 3).

Table 3. Ballooning extension of conditional bba

Conditional bbm	Ballooning extension	Values
$m^{\Theta}[c_1](\{p\})$	$\{(p,c_1),(p,c_2),(p,c_3),(p,c_4),(r,c_2),$ $(r,c_3),(r,c_4),(f,c_2),(f,c_3),(f,c_4)\}$	0.806
$m^{\Theta}[c_1](\{p,r,f\})$	$\{(p,c_1),(p,c_2),(p,c_3),(p,c_4),$ $(r,c_1),(r,c_2),(r,c_3),(r,c_4),(f,c_1),(f,c_2),(f,c_3),(f,c_4)\}$	0.194

As explained before, once the ballooning extensions are obtained, we can apply Equation 8, to combine the obtained bba with the criterion weights (bba).

Next, to choose the best alternatives, we must define the beliefs over the frame of alternatives Θ and the pignistic probabilities can be computed. We get: $BetP(p) = 0.567$, $BetP(r) = 0.213$ and $BetP(f) = 0.220$.

As a consequence, the alternative "Peugeot" is the recommended car since it has the highest values.

4 An extension of the Belief AHP Method

The Belief AHP method is an interesting tool for solving multi-criteria decision problems. It provides the expert the possibility to select only some subsets of alternatives and groups of criteria.

However, this approach suffers from some weaknesses. In fact, in reality, the elicitation of preferences may be rather difficult since expert would not be able to efficiently express any kind of preference degree between the available alternatives and criteria. Therefore, the belief AHP method is extended to handle the presented problems.

4.1 Belief Pair-wise Comparison

Under this approach, a new elicitation procedure is introduced. Thus to model his assessments, the decision maker has to express his opinions qualitatively. He indicated whether a criterion (or alternative) was more or less important to its partner by "yes" or "no". Moreover, we accept that the expert may define uncertain or even unknown assessments. Indeed, we assume that each subset of criteria is described by a basic belief assignment defined on the possible responses. For instance, in a problem of purchasing a car, the following type of subjective judgments was frequently used: "the comfort criterion is evaluated to be more important than style with a confidence degree of 0.8". In fact, the decision maker responses to the question "is comfort criterion important regarding the style criterion?". Thus, the answer was: comfort criterion is more preferable than style criterion and 0.8 is referred to the degree of belief. Then, to compute the criteria weight, we describe a new pair-wise comparison procedure where the following steps must be respected:

1. The first step is to model the pair-wise comparison matrix. Let d_{ij} is the entry from the i^{th} column of pair-wise comparison matrix (d_{ij} represents the different bbm's of the identified bba).

$$\text{If } \; m_j^{\Omega_{C_i}}(.) = d_{ij}, \text{ then } m_i^{\Omega_{C_j}}(.) = \bar{m}_j^{\Omega_{C_i}}(.) = d_{ij} \tag{10}$$

   where $m_j^{\Omega_{C_i}}$ represents the importance of C_i with respect to the subset of criteria C_j (for simplicity, we denote the subset of criteria by j instead of C_j), $i \neq j$, and $\bar{m}$ is the negation of m. The negation $\bar{m}$ of a bba m is defined as $\bar{m}(A) = m(\bar{A}), \forall A \subset \Omega$.
   As regarding the previous example, if we have "the comfort criterion (C) is evaluated to be more important than style criterion (S) with a confidence degree of 0.8", that is $m_S^{\Omega_C}(\{yes\}) = 0.8$, then we can say that "the style criterion is evaluated to be less important than comfort criterion with a confidence degree of 0.8": $m_C^{\Omega_S}(\{no\}) = 0.8$.
2. Once the pair-wise comparison matrix is completed, our objective is then to obtain the priority of each subset of criteria. The idea is to combine the obtained bba using the conjunctive rule of combination [8] ($(m_1 \textcircled{\cap} m_2)(A) = \sum_{B,C \subseteq \Theta, B \cap C = A} m_1(B) m_2(C)$).
   Indeed, this function is chosen since we can regard each subset of criteria as a distinct source of information which provides distinct pieces of evidence. We will get the following bba:

$$m^{\Omega_{C_i}} = \textcircled{\cap} m_j^{\Omega_{C_i}}, \text{where } j = \{1, \ldots, k\} \tag{11}$$

At this stage, we want to know which criterion is the most important. In fact, the obtained bba measures the confidence degree assigned to a specific criterion regarding the overall criteria. However, these obtained bba represents the belief over all possible answers (yes or no). The idea is then to standardize all the frames of discernment. Obviously, we propose to use the concept of refinement operations [6], which allows to establish relationships between different frames of discernment in order to express beliefs on anyone of them. The objective consists in obtaining one frame of discernment Ω from the set Ω_{C_k} by splitting some or all of its events:

$$m^{\Omega_{C_k}\uparrow\Omega}(\rho_k(\omega)) = m^{\Omega_{C_k}}(\omega) \qquad \forall\omega \subseteq \Omega_{C_k} \tag{12}$$

where the mapping ρ_k from Ω_{C_k} to Ω is a refinement, and $\rho_k(\{yes\}) = \{C_k\}$ and $\rho_k(\{no\}) = \overline{\{C_k\}}$.

3. Finally, the obtained bba $m^{\Omega_{C_k}\uparrow\Omega}$ can be combined using the conjunctive rule of combination in order to get m^{Ω}.

The similar process is repeated to get the alternatives priorities $m^{\Theta}[c_k](A_i)$ representing the opinions-beliefs of the expert about his preferences regarding the set of alternatives.

Then, the vacuous extension is used at the criterion level and the ballooning extension is assumed at the alternative level in order to standardize the frame of discernment. So, the vacuous extension is used to extend the frame of criteria to the frame of alternatives and the ballooning is applied for the deconditioning process. After that, these obtained bba can be combined. Next, the marginalization technique is applied by transferring each mass to its projection on Θ. The final priority is then computed using the pignistic probabilities to make our choice.

4.2 Illustrative Example

Let us consider the previous example (see Section 3.5). After identifying the subsets of criteria and alternatives, the pair-wice comparison matrices should be constructed.

Computing the Criteria Weights. After collecting the expert beliefs, we have generated the following associated belief functions (see Table 4).

From Table 4, the expert may say that $\{c_1\}$ is evaluated to be more important than $\{c_4\}$ with a confidence degree of 0.4. That means, 0.4 of beliefs are exactly committed to the criterion $\{c_1\}$ is more important than $\{c_4\}$, whereas 0.6 is assigned to the whole frame of discernment (ignorance).

Then, the next step consists in combining the bba's corresponding to each criterion using the Equation 11. The obtained bba is reported in Table 5.

Subsequently, we proceed now with the standardization of our frame of discernment. By applying the Equation 12, we get for example: $m^{\Omega_{\{c_1\}}\uparrow\Omega}(\{c_1\}) =$

Table 4. The weights preferences assigned to the criteria according to the expert's opinion

	$\{c_1\}$	$\{c_4\}$	$\Omega_1 = \{c_2, c_3\}$
$\{c_1\}$	$m_{\{c_1\}}^{\Omega_{\{c_1\}}}(\Omega_{\{c_1\}}) = 1$	$m_{\{c_4\}}^{\Omega_{\{c_1\}}}(\{yes\}) = 0.4$ $m_{\{c_4\}}^{\Omega_{\{c_1\}}}(\Omega_{\{c_1\}}) = 0.6$	$m_{\Omega_1}^{\Omega_{\{c_1\}}}(\{yes\}) = 0.9$ $m_{\Omega_1}^{\Omega_{\{c_1\}}}(\Omega_{\{c_1\}}) = 0.1$
$\{c_4\}$	$m_{\{c_1\}}^{\Omega_{\{c_4\}}}(\{no\}) = 0.4$ $m_{\{c_1\}}^{\Omega_{\{c_4\}}}(\Omega_{\{c_4\}}) = 0.6$	$m_{\{c_4\}}^{\Omega_{\{c_4\}}}(\Omega_{\{c_4\}}) = 1$	$m_{\Omega_1}^{\Omega\{c_4\}}(\{no\}) = 0.3$ $m_{\Omega_1}^{\Omega_{\{c_4\}}}(\Omega_{\{c_4\}}) = 0.7$
$\Omega_1 = \{c_2, c_3\}$	$m_{\{c_1\}}^{\Omega_{\Omega_1}}(\{no\}) = 0.9$ $m_{\{c_1\}}^{\Omega_{\Omega_1}}(\Omega_{\Omega_1}) = 0.1$	$m_{\{c_4\}}^{\Omega_{\Omega_1}}(\{yes\}) = 0.3$ $m_{\{c_4\}}^{\Omega_{\Omega_1}}(\Omega_{\Omega_1}) = 0.7$	$m_{\Omega_1}^{\Omega_{\Omega_1}}(\Omega_{\Omega_1}) = 1$

Table 5. Belief pair-wise matrix: Partial combination

	$\{c_1\}$	$\{c_4\}$	$\Omega_1 = \{c_2, c_3\}$
Weight	$m^{\Omega_{\{c_1\}}}(\{yes\}) = 0.94$ $m^{\Omega_{\{c_1\}}}(\Omega_{\{c_1\}}) = 0.06$	$m^{\Omega_{\{c_4\}}}(\{no\}) = 0.58$ $m^{\Omega_{\{c_4\}}}(\Omega_{\{c_4\}}) = 0.42$	$m^{\Omega_{\Omega_1}}(\{yes\}) = 0.03$ $m^{\Omega_{\Omega_1}}(\{no\}) = 0.63$ $m^{\Omega_{\Omega_1}}(\emptyset) = 0.27$ $m^{\Omega_{\Omega_1}}(\Omega_{\Omega_1}) = 0.07$

Table 6. Belief pair-wise matrix: Refinement

	$\{c_1\}$	$\{c_4\}$	$\Omega_1 = \{c_2, c_3\}$
Weight	$m_{\{c_1\}}^{\Omega}(\{c_1\}) = 0.94$ $m_{\{c_1\}}^{\Omega}(\Omega) = 0.06$	$m_{\{c_4\}}^{\Omega}(\{c_1, c_2, c_3\}) = 0.58$ $m_{\{c_4\}}^{\Omega}(\Omega) = 0.42$	$m_{\{c_2,c_3\}}^{\Omega}(\{c_2, c_3\}) = 0.03$ $m_{\{c_2,c_3\}}^{\Omega}(\{c_1, c_4\}) = 0.63$ $m_{\{c_2,c_3\}}^{\Omega}(\emptyset) = 0.27$ $m_{\{c_2,c_3\}}^{\Omega}(\Omega) = 0.07$

$m^{\Omega_{\{c_1\}}}(\{yes\})$. To simplify, we can note by $m_{\{c_1\}}^{\Omega}$ the bba $m^{\Omega_{\{c_1\}}\uparrow\Omega}$. These bba's are presented in Table 6.

At this stage, the obtained bba's can be combined using the conjunctive rule of combination. We get: $m^{\Omega}(\emptyset) = 0.2982$, $m^{\Omega}(\{c_1\}) = 0.6799$, $m^{\Omega}(\{c_2, c_3\}) = 0.0018$, $m^{\Omega}(\{c_1, c_2, c_3\}) = 0.0024$, $m^{\Omega}(\{c_1, c_4\}) = 0.0159$ and $m^{\Omega}(\Omega) = 0.0018$.

Computing the Alternatives Priorities. Like the criterion level, the judgments between decision alternatives over different criteria are dealt within an identical manner. For example, to evaluate the alternatives according to the criterion c_1 we get Table 7.

As in the criterion level, for the subset of alternatives $\{p\}$, we use Equation 11 in order to combine the obtained bba: $m^{\Theta_{\{p\}}}[c_1] = m_{\{p\}}^{\Theta_{\{p\}}}[c_1] \textcircled{\cap} m_{\{r,f\}}^{\Theta_{\{p\}}}[c_1]$ ($m^{\Theta_{\{p\}}}[c_1](\{yes\}) = 0.95$ and $m^{\Theta_{\{p\}}}[c_1](\{\Theta_{\{p\}}\}) = 0.05$). Then, a similar process is repeated for the rest of alternatives, and we get $m^{\Theta_{\{r,f\}}}[c_1]$ ($m^{\Theta_{\{r,f\}}}[c_1](\{no\}) = 0.95$ and $m^{\Theta_{\{r,f\}}}[c_1](\Theta_{\{r,f\}}) = 0.05$).

Subsequently, we proceed now with the standardization of our frame of discernment. By applying Equation 12, we get the following: $m^{\Theta_{\{p\}}\uparrow\Theta}[c_1](\{p\}) =$

Table 7. Belief pair-wise matrix regarding c_1 criterion

c_1	$\{p\}$	$\{r, f\}$
$\{p\}$	$m^{\Theta_{\{p\}}}_{\{p\}}[c_1](\Theta_{\{p\}}) = 1$	$m^{\Theta_{\{p\}}}_{\{r,f\}}[c_1](\{yes\}) = 0.95$ $m^{\Theta_{\{p\}}}_{\{r,f\}}[c_1](\Theta_{\{p\}}) = 0.05$
$\{r, f\}$	$m^{\Theta_{\{r,f\}}}_{\{p\}}[c_1](\{no\}) = 0.95$ $m^{\Theta_{\{r,f\}}}_{\{p\}}[c_1](\Theta_{\{r,f\}}) = 0.05$	$m^{\Theta_{\{r,f\}}}_{\{r,f\}}[c_1](\Theta_{\{r,f\}}) = 1$

0.95 and $m^{\Theta_{\{p\}}\uparrow\Theta}[c_1](\Theta) = 0.05$. Also, $m^{\Theta_{\{r,f\}}\uparrow\Theta}[c_1](\{p\}) = 0.95$ and $m^{\Theta_{\{r,f\}}\uparrow\Theta}[c_1](\Theta) = 0.05$.

Finally, the obtained bba's can be directly combined using the conjunctive rule of combination. For simplicity, we denote $m^{\Theta_{\{p\}}\uparrow\Theta}[c_1]$ by $m^{\Theta}[c_1]$, we get: $m^{\Theta}[c_1](\{p\}) = 0.9975$ and $m^{\Theta}[c_1](\{\Theta\}) = 0.0025$.

Then, as shown in the previous step, the computation procedure is repeated for the rest of comparison matrices.

Updating the Alternatives Priorities. As shown in the previous example, at the criterion level, the vacuous extension is used to standardize the frame of discernment $m^{\Omega\uparrow\Theta\times\Omega}$. At the alternative level, the ballooning extension is applied $m^{\Theta}[c_j]^{\Uparrow\Theta\times\Omega}$. Then, the obtained bba can be directly combined by using Equation 8 as exposed in Table 8.

Table 8. The obtained bba: $m^{\Theta\times\Omega}$

$m^{\Theta\times\Omega}$	bbm	$m^{\Theta\times\Omega}$	bbm
$\{(p, c_1), (f, c_1), (r, c_1)\}$	0.28	$\{(p, c_1), (f, c_1)\}$	0.16
$\{(p, c_1)\}$	0.008	$\{(r, c_2), (r, c_3), (f, c_2), (p, c_2)\}$	0.03
$\{(f, c_4)\}$	0.0016	$\{(p, c_4), (f, c_4), (r, c_4)\}$	0.11
$\{(p, c_2), (f, c_2), (r, c_2), (p, c_3), (f, c_3)\}$	0.007	$\emptyset$	0.4034

To choose the best alternatives, we must define our beliefs over the frame of alternatives. As a result, the obtained bba is marginalized on Θ, we obtain the following distribution: $m^{\Theta\times\Omega\downarrow\Theta}(\{p, r, f\}) = 0.427$, $m^{\Theta\times\Omega\downarrow\Theta}(\{p\}) = 0.008$, $m^{\Theta\times\Omega\downarrow\Theta}(\{f\}) = 0.0016$, $m^{\Theta\times\Omega\downarrow\Theta}(\{p, f\}) = 0.16$ and $m^{\Theta\times\Omega\downarrow\Theta}(\emptyset) = 0.4034$.

We can now calculate the overall performance for each alternative and determine its corresponding ranking by computing the pignistic probabilities:

$BetP(p) = 0.3863$, $BetP(r) = 0.3752$ and $BetP(f) = 0.2385$.

As a consequence, the alternative "Peugeot" is the recommended car since it has the highest values. The alternative r may also be chosen since it has a value close to p. For the sake of comparison, we have obtained the same best alternative as in the previous example. This would give the expert reasonable assurance in decision making. Our objective is then not to obtain the best alternative but to identify the most cautious one since it is defined with less information.

5 Conclusion

In this paper, the proposed method has extended the belief AHP model into more uncertain environment. Indeed, our approach develops a new pair-wise comparison technique in order to facilitate the elicitation process and to handle the problem of uncertainty. It leads to more simple comparison procedure without eliciting additional information. In fact, experts do not need to provide precise comparison judgments. They select only some subsets of alternatives in accordance with a certain criterion, and groups of criteria. Then, the proposed method models the imprecise judgments based on an appropriate mathematical framework of the belief function theory.

References

1. Beynon, M., Curry, B., Morgan, P.: The Dempster-Shafer theory of evidence: An alternative approach to multicriteria decision modelling. OMEGA 28(1), 37–50 (2000)
2. Ennaceur, A., Elouedi, Z., Lefevre, E.: Handling partial preferences in the belief AHP method: Application to life cycle assessment. Proceedings of the International Conference of the Italian Association for Artificial Intelligence, LNAI 6934, Springer-Verlag pp. 396–401 (2011)
3. Ennaceur, A., Elouedi, Z., Lefevre, E.: Reasoning under uncertainty in the AHP method using the belief function theory. Proceedings of the International Conference on Information Processing and Management of Uncertainty in Knowledge-Based Systems (IPMU) pp. 373–383 (2012)
4. Holder, R.D.: Some comments on Saaty's AHP. Management Science 41, 1091–1095 (1995)
5. Saaty, T.: A scaling method for priorities in hierarchical structures. Journal of Mathematical Psychology 15, 234–281 (1977)
6. Shafer, G.: A Mathematical Theory of Evidence. Princeton University Press (1976)
7. Smets, P.: Belief functions: the disjunctive rule of combination and the generalized bayesian theorem. International Journal of Approximate Reasoning 9 9, 1–35 (1993)
8. Smets, P.: The combination of evidence in the Transferable Belief Model. IEEE Pattern analysis and Machine Intelligence 12, 447–458 (1990)
9. Smets, P., Kennes, R.: The Transferable Belief Model. Artificial Intelligence 66, 191–234 (1994)
10. Utkin, L.V.: A new ranking procedure by incomplete pairwise comparisons using preference subsets. Intelligent Data Analysis 13(2), 229–241 (2009)