# Qualitative AHP Models under the Belief Function Framework

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***Abstract*—This paper investigates a multi-criteria decision making method in an uncertain environment, where the uncertainty is represented using the belief function framework. In this context, we suggest a novel methodology that tackles the challenge of introducing uncertainty in the expert evaluations. Therefore, the Analytic Hierarchy Process with qualitative belief function framework is adopted to get numeric representation of qualitative assessment.**

**In this work, we will also focus on two AHP extensions under qualitative AHP. Besides, we intend to describe some comparisons on the standard AHP and the presented models to judge their accuracy. We use also a simulation approach to compare the results of the different models based on different matrices dimensions.**

***Keywords*-AHP; Belief function theory; Decision making; Uncertainty; Preference relations;**

## I. Introduction

Uncertainty is a source of complexity in decision making. There are various forms that may arise in multi-criteria decision making (MCDM) [1], in particular the Analytic Hierarchy Process (AHP) [2] [3], from impression to lack of knowledge or ignorance. At one level, there is an uncertainty about alternatives that appear in the identification of the candidate ones. At another level, there is an uncertainty about the ability of the selected criteria to adequately represent the objective that the decision maker tries to achieve. In addition, imperfection may also be in the evaluation process. So, variability in all these factors has the potential to affect the ranking of alternatives of a MCDM problem.

Therefore, our objective, through this research, is to handle uncertainty while expressing the decision maker judgments and not forcing the expert to give deterministic answers. Moreover, it is also recognized that human assessment on qualitative criteria is always subjective and thus imprecise. Also, as shown in [4] and [5], the AHP scale was criticized since the user cannot be consistent. Sometimes, the decision maker may say that $A$ is twice as important as $B$, $A$ is 3 times as important as $C$ and $B$ is 1.5 times as important as $C$, yet he is constrained to make the last judgment 1 or 2. In addition, the decision maker might find difficulty to distinguish among them and tell for example whether one alternative is 6 or 7 times more important than another. Furthermore, the AHP method cannot cope with the fact that alternative $A$ is 25 times more important than alternative $C$. Expert would not be able to efficiently express any kind of preference degree between the available alternatives and criteria. As a result, the scale is further incomplete and unnecessarily restricting because of the arbitrary cut-off at 9 for the maximum allowable ratio of weights.

All these criticisms have been discussed in the literature and some solutions for them have been developed. To take judgmental uncertainty into account, alternative methods such as applications of the fuzzy theory are developed for AHP [6] [7]. Different scaling methods have also been provided [8] [9].

Consequently, our problem through this work is how to quantify the linguistic choices selected by the decision maker during the evaluation of the pair-wise comparisons under the belief function framework? Besides, is it necessary to decompose even more the different levels of the Saaty scale?

To solve the problems presented above, and to facilitate the pair-wise comparison process, a new MCDM method under uncertainty that eliminates some of the drawbacks of the existing prioritization methods, is proposed. A natural way to cope with uncertain judgments is to express the comparison ratios as a belief function, which incorporates the imperfection of the human thinking. Indeed, preferential assessments are used in order to express the decision maker's subjective assessments instead of using numerical values. Within our method, the expert does not require to complete all the comparison matrices; he can then find priorities from incomplete set of judgments. Therefore, a new procedure is employed to derive crisp priorities from qualitative judgments corresponding to each level.

In what follows, we first present some definitions needed for belief function context. Next, we describe the qualitative AHP methods in section 3. Then, section 4 details the evaluation algorithm. Finally, section 5 concludes the paper.

## II. Belief Function Theory

### A. Basic Concepts

The Transferable Belief Model (TBM) is a model to represent quantified belief functions [10]. Let $\Theta$ be the frame of discernment representing a finite set of elementary hypotheses related to a problem domain. We denote by $2^\Theta$ the set of all the subsets of $\Theta$ [11].

The impact of a piece of evidence on the different subsets of the frame of discernment $\Theta$ is represented by the so-called basic belief assignment (bba), denoted by $m$ [11] such that:

$$\sum_{A \subseteq \Theta} m(A) = 1 \tag{1}$$

The value $m(A)$, named a basic belief mass (bbm), represents the portion of belief committed exactly to the event $A$. The events having positive bbm's are called focal elements. Let $\mathcal{F}(m) \subseteq 2^\Theta$ be the set of focal elements of the bba $m$.

Associated with $m$ is the belief function is defined for $A \subseteq \Theta$ and $A \neq \emptyset$ as:

$$bel(A) = \sum_{\emptyset \neq B \subseteq A} m(B) \text{ and } bel(\emptyset) = 0 \tag{2}$$

The degree of belief $bel(A)$ given to a subset $A$ of the frame $\Theta$ is defined as the sum of all the basic belief masses given to subsets that support $A$ without supporting its negation.

### B. Uncertainty Measures

In the case of the belief function framework, the bba is defined on an extension of the powerset: $2^\Theta$ and not only on $\Theta$. In the powerset, each element is not equivalent in terms of precision. Indeed, $\theta_i \subset \Theta$ ($i \in \{1, 2\}$) is more precise than $\theta_1 \cup \theta_2 \subseteq \Theta$.

In order to try to quantify this imprecision, different uncertainty measures have been defined, such as [12]:

$$H(m) = \sum_{A \in \mathcal{F}(m)} m(A) \log_2\left(\frac{|A|}{m(A)}\right). \tag{3}$$

### C. Discounting

The technique of discounting allows us to take in consideration the reliability of the information source that generates the bba $m$. Let $\beta = 1 - \alpha$ be the degree of reliability ($\alpha \in [0, 1]$) assigned to a particular belief function. If the source is not fully reliable, the bba it generates is "discounted" into a new less informative bba denoted ${}^{\alpha}m$ [13]:

$${}^{\alpha}m(A) = (1 - \alpha)m(A), \forall A \subset \Theta \tag{4}$$

$${}^{\alpha}m(\Theta) = \alpha + (1 - \alpha)m(\Theta) \tag{5}$$

### D. Decision Making

The TBM considers that holding beliefs and making decision are distinct processes. Hence, it proposes a two level model [14]: The credal level where beliefs are entertained and represented by belief functions, and the pignistic level where beliefs are used to make decisions and represented by probability functions called the pignistic probabilities, denoted *BetP* [14]:

$$BetP(A) = \sum_{B \subseteq \Theta} \frac{|A \cap B|}{|B|} \frac{m(B)}{(1 - m(\emptyset))}, \forall A \in \Theta \tag{6}$$

## III. AHP Method under the Qualitative Belief Function Framework

This section is dedicated to the presentation of our new AHP under uncertainty. Indeed, we introduce the basic stages needed to ensure the ranking of alternatives in an uncertain environment based on the belief function framework. Our model has the same features as standard AHP such as hierarchical levels and pair-wise comparisons. At first, we will briefly describe Saaty's approach. Then, we will present the computational steps of our proposed model.

### A. AHP Method

The Analytic Hierarchy Process (AHP) is a multi-criteria decision-making approach and was introduced by Saaty [2] [3]. AHP organizes the basic rationality by breaking down a problem into its smaller constituent parts. By decomposing the problem, the decision-maker can focus on a limited number of items at the same time. The AHP is carried out in two phases: the design of the hierarchy and the evaluation of the components in the hierarchy [3]. It involves building a hierarchy of decision elements and then making comparisons between each possible pair (as a matrix).

In a pair-wise comparison, the decision maker examines two alternatives by considering one criterion and indicates a preference. These comparisons are made using a preference scale, which assigns numerical values to different levels of preference. The standard preference scale used for AHP is $1 - 9$ scale which lies between "equal importance" to "extreme importance" where sometimes different evaluation scales can be used such as 1 to 5. In the pair-wise comparison matrix, the value 9 indicates that one factor is extremely more important than the other, and the value 1/9 indicates that one factor is extremely less important than the other, and the value 1 indicates equal importance. Therefore, if the importance of one factor with respect to a second is given, then the importance of the second factor with respect to the first is the reciprocal. Ratio scale and the use of verbal comparisons are used for weighting of quantifiable and non-quantifiable elements.

### B. Qualitative AHP Method

In this work, we propose a revised version of the AHP model. We demonstrate that standard AHP is thought to be a robust way to solve determined decision making problem [15]. However, it neglects the uncertainty caused by subjective preference of decision maker in criteria and alternative scoring. Its pair-wise comparison value seems not strong enough to cover most decision makers' options. Accordingly, in the proposed methodology, the expert is allowed to use preference relations only. Thus, to express his assessments, the decision maker has to express his opinions qualitatively, based on knowledge and experience that he provides in response to a given question rather than direct quantitative information. He only selects the related linguistic variable using preference modeling. The preference relations may be: a preference relation ($\succ$), an indifference relation ($\sim$), a weak preference relation ($\succeq$), or an unknown relation ($-$). Our main aim is then to combine the existing elicitatation technique [15] [16] with the AHP method to propose the qualitative AHP.

To present the qualitative AHP method, we introduce its different construction steps, described as follows:

1) Model the problem as a hierarchy containing the decision goal, the sets of alternatives $\Theta = \{a_1, ..., a_m\}$ for reaching it, and the sets of criteria $\Omega = \{c_1, \ldots, c_n\}$ for evaluating the sets of alternatives.
2) Establish priorities among the elements of the hierarchy by making a series of judgments based on pair-wise comparisons of the elements using only preference relations.
3) For each pair-wise comparison matrix, transform preference relations into numerical values using the belief function theory. Therefore, Ennaceur et al. model [16] is applied to convert these preferences into constraints of an optimization problem whose resolution, according to some uncertainty measures (UM). This model allows the generation of the least informative or the most uncertain belief functions. It can then be determined by the resolution of an optimization problem. For instance, if we use the preference relations matrix relative to the criterion level we get:

$$\begin{gathered} MaxH(m) = m(\{c_1\}) * log_2(|c_1|/m(\{c_1\}))+ \\ m(\{c_2\})log_2(|c_2|/m(\{c_2\})) + ...+ \\ m(\{c_n\}) * log_2(|c_n|/m(\{c_n\}))+ \\ m(\Omega) * log_2(|\Omega|/m(\Omega)); \\ s.t. \\ bel(\{c_i\}) - bel(\{c_j\}) \geq \gamma \quad \forall(c_i, c_j),\, c_i \succ c_j \\ bel(\{c_i\}) - bel(\{c_j\}) \leq \gamma \quad \forall(c_i, c_j),\, c_i \succeq c_j \\ bel(\{c_i\}) - bel(\{c_j\}) \geq \varepsilon \quad \forall(c_i, c_j),\, c_i \succeq c_j \\ bel(\{c_i\}) - bel(\{c_j\}) \geq -\varepsilon \quad \forall(c_i, c_j),\, c_i \sim c_j \\ bel(\{c_i\}) - bel(\{c_j\}) \leq \varepsilon \quad \forall(c_i, c_j),\, c_i \sim c_j \\ ... \\ \sum_{c_i \in \mathcal{F}(m)} m(c_i) = 1, m(A) \geq 0, \forall A \subseteq \Omega; m(\emptyset) = 0. \end{gathered}$$

Where $H$ is the uncertainty measure since it has a unique maximum. Besides, it takes into account the non-specificity and quantifies the conflict presented in the body of evidence (measure of total uncertainty). Besides, the preference relations are transformed into constraints as follows: the first constraint of the model is derived from the preference relation. The second and third constraints model the weak preference relation. The fourth and fifth constraints correspond to the indifference relation. Each preference relation must be translated into one of the presented constraint.
$\varepsilon$ and $\gamma$ are a constant specified by the expert before beginning the optimization process.
4) Assume that the priorities of criteria and alternatives are described by a basic belief assignment defined on the possible responses. Thus, the criterion bba is denoted by $m^{\Omega}$ and the alternative bba, regarding $_{c_k}$, by $m^{\Theta}_{c_k}$.
5) At the criterion level, the obtained bba is transformed into measure of reliability. If we have $c_i$ as a criterion, then we get $\beta_i$ its corresponding measure of reliability:

$$\beta_i = \frac{m^{\Omega}(\{c_i\})}{max_k m^{\Omega}(\{c_k\})} \tag{7}$$

6) Synthesize the overall judgment, that is updating the sets of alternatives priorities with the importance of their corresponding criteria. The obtained bbas are discounted such as:

$$^{\alpha}m^{\Theta}_{c_k}(a_j) = \beta_k . m^{\Theta}_{c_k}(a_j),\ \forall a_j \subset \Theta \tag{8}$$

$$^{\alpha}m^{\Theta}_{c_k}(\Theta) = (1 - \beta_k) + \beta_k . m^{\Theta}_{c_k}(\Theta) \tag{9}$$

where $m^{\Theta}_{c_k}$ the relative bba for the subset $a_j$ (obtained in the previous step), $\beta_k$ its corresponding measure of reliability, and we denote $\alpha_k = 1 - \beta_k$.
7) Combine the overall bba's to get a single representation by using the conjunctive rule ( $m^{\Theta} = \bigcirc\!\!\!\!\cap\, ^{\alpha}m^{\Theta}_{c_k}$).
8) Come to a final decision based on the the pignistic transformation to find the best alternatives. We compute the pignistic probabilities to choose the best alternatives (Equation 6).

### C. Introducing Dependency under Qualitative AHP Method

One of basic assumptions of AHP technique is that all the elements in the same hierarchy are totally independent. However, this assumption is hard to be satisfied due to ambiguousness and complexity of questions. In addition to this problem, the evaluating elements include dependent properties.

Under the AHP approach, criteria are assumed independent of the alternatives. However, paired comparisons imply dependence of a different kind. In fact, the importance assigned to an alternative depends on the evaluated criterion. This dependency is not according to structure, because we usually try to respect AHP axioms, but according to function. Thus, in this Section, our main objective is to handle dependency between the alternative and criterion levels under the qualitative AHP methodology. Besides, we propose the latter by using preference relation in order to translate the expert's assessments.

The developed method has the same steps as qualitative AHP. Nevertheless, we suggest a new aggregation procedure [17]:

1) Select and define the evaluative criteria and the alternatives.

2) Calculate the weights of the criteria and the score of alternatives. After the construction of the hierarchy, the different priority weights of each criterion and alternative must be calculated. First, the expert compared the criteria with respect to the main goal; then, the expert compared the alternatives with respect to each criterion. Instead of applying Saaty's scale, the decision maker uses the preference relations to evaluate the elements of the hierarchy as described in the previous subsection.
After computing the priorities vectors, we obtain $m^{\Omega}$, which means that we know the belief about $c_j$ in the frame $\Omega$ (where $c_j$ is a criterion). The same process is repeated at the alternative level, we get a conditional bba $m^{\Theta}[c_j](A_k)$, which represents the belief about $A_k$ regarding $c_j$.
3) Calculate the evaluation result and synthesize the solution by aggregating all the obtained bba. At this step, the priority weights of each main criteria and alternative must be combined. However, the priority concerning criteria is defined on the frame of discernment $\Omega$, whereas the sets of alternatives are defined on another frame $\Theta$. In order to solve this problem, we propose to standardize our frame of discernment.
First, at the alternative level, the idea was to use the deconditionalization process in order to transform the conditional belief into a new belief function. In this case, the ballooning extension concept is applied [18]:

$$m^{\Theta}[c_j]^{\Uparrow\Theta\times\Omega}(A_i\times c_j\cup\Theta\times\bar{c_j}) = m^{\Theta}[c_j](A_i), \forall A_i \subseteq \Theta. \quad (10)$$

4) At this stage, our objective is then to redefine the bba that represents criteria weights obtained at the criterion level (Step 2). Indeed, we propose to extend this bba from $\Omega$ to $\Theta \times \Omega$:

$$m^{\Omega\uparrow\Theta\times\Omega}(B) = m^{\Omega}(c_i) \quad B = \Theta \times c_i, c_i \subseteq \Omega. \quad (11)$$

5) We now combine $m[c_j]^{\Theta\Uparrow\Theta\times\Omega}$ and $m^{\Omega\uparrow\Theta\times\Omega}$ all the obtained bba to measure their contribution. That is, we will apply the conjunctive rule of combination:

$$m^{\Theta\times\Omega} = m[c_j]^{\Theta\Uparrow\Theta\times\Omega} \textcircled{\cap} m^{\Omega\uparrow\Theta\times\Omega} \quad (12)$$

6) To this end and after getting the joint bba, a decision under uncertainty must be defined. In the sequel, the pignistic probabilities are used. However, our obtained beliefs are defined on the product space $\Theta \times \Omega$. To solve this problem, we propose to marginalize this bba on $\Theta$ (frame of alternatives) by transferring each mass $m^{\Theta\times\Omega}$ to its projection on $\Theta$. Then, the pignistic probabilities is computed to choose the best alternatives:

$$BetP(a_j) = \sum_{A_i \subseteq \Theta} \frac{|a_j \cap A_i|}{|A_i|} \frac{m^{\Theta\times\Omega\downarrow\Theta}(A_i)}{(1 - m^{\Theta\times\Omega\downarrow\Theta}(\emptyset))}, \forall a_j \in \Theta. \quad (13)$$

## IV. Simulation Algorithm

The main objective of this research is to study the performance of the proposed approaches using random data. To generate reliable data for a numerical analysis in AHP, simulation has been extensively used in prior research [19] [20]. The experiment is based on the following steps:

1) We generate a random matrix for the decision performance and another one to represent the weight of each decision criteria. Based on these two matrices, the overall scores and ranks of the decision alternatives are calculated. These steps are usual steps in the Weighted Sum Model (WSM) method. A method is accurate in MCDM problems should also be accurate in single dimensional problems. Therefore, we use the WSM method, since in single-dimensional environment, it yields the most reasonable results. Hence, Triantaphyllou et al. [19] [20] compare the obtained results using WSM by those obtained by other MCDM methods.
Besides, WSM is the simplest and still the widest used MCDM method. In this method, each criterion is given a weight and the sum of all weights must be 1. Each alternative is assessed with regard to every criterion. The overall or composite performance score of an alternative is given by the equation:

$$P_i = \sum_j v_{ij} * \omega_j \quad (14)$$

where $P_i$ is the priority of each alternative, $\omega_j$ is the weight of each criterion and $v_{ij}$ is the score of each alternative regarding each criterion.
2) From the performance matrix, we generated pair-wise comparison matrices of different alternatives that are compared to each criterion.
3) We apply the suggested method to compute the overall priorities and to rank alternatives.
4) We compare the obtained result with the ranking of the WSM method.

*1) Example:* Let's demonstrate the evaluation procedure using the same example used in [20]. We consider 3 alternatives $a_1$, $a_2$ and $a_3$ and three criteria $c_1$, $c_2$ and $c_3$. The decision making problem is described using the matrix presented in Table I.

This example has been solved using the WSM and AHP in [20]. Applying the WSM, it can be shown that the alternative $a_1$ is the best one. However, AHP turns out that the alternative $a_2$ is the best one. Obviously, this contradicts with the conclusion derived using the WSM.

Table I
DECISION MATRIX

| | Criteria | | |
|---|---|---|---|
| | $c_1$ | $c_2$ | $c_3$ |
| Alt. | $\frac{8}{13}$ | $\frac{2}{13}$ | $\frac{3}{13}$ |
| $a_1$ | 1 | 9 | 9 |
| $a_2$ | 5 | 2 | 2 |
| $a_3$ | 1 | 5 | 9 |

Table II
THE PREFERENCE RELATIONS MATRICES

| $c_1$ | $a_1$ | $a_2$ | $a_3$ |
|---|---|---|---|
| $a_1$ | - | $\prec$ | $\sim$ |
| $a_2$ | - | - | $\succ$ |
| $a_3$ | - | - | - |

| $c_2$ | $a_1$ | $a_2$ | $a_3$ |
|---|---|---|---|
| $a_1$ | - | $\succeq$ | $\sim$ |
| $a_2$ | - | - | $\sim$ |
| $a_3$ | - | - | - |

| $c_3$ | $a_1$ | $a_2$ | $a_3$ |
|---|---|---|---|
| $a_1$ | - | $\succeq$ | $\sim$ |
| $a_2$ | - | - | $\sim$ |
| $a_3$ | - | - | - |

Now, let us model this example using qualitative AHP. If the decision maker knew the actual data shown in the original crisp decision matrix, then the qualitative matrix of the actual pair-wise comparisons would be obtained as follows:

- "Equal importance" is equivalent to indifference relation.
- "Somewhat more important" and "Much more important" are equivalent to weak preference relation.
- "Very much more important" and "Absolutely more important" are equivalent to strong preference relation.

As a result, the pair-wise comparison matrix of the three alternatives in terms of each criterion are illustrated in Table II.

Next, an optimization model is used to transform preference relation into constraints and to generate quantitative information from these qualitative assessments. Finally, we obtain the following order: $a_1 \succ a_3 \succ a_2$. Obviously, this is in contradiction with the results derived when the AHP method was applied at the beginning of this illustrative example. However, we have obtained encouraging results since it can be observed that the ranking order of the alternatives as derived by the WSM and the uncertain AHP is the same.

In order to gain a deeper understanding, a computational study was undertaken. The data were random numbers from the interval $[1, 9]$ (in order to be compatible with the numerical properties of the Saaty scale). In these test problems, the number of alternatives was equal to the following values: $3, 4, 5, 6, 7, 8, 9$, and $10$. Similarly, the number of criteria was equal to $3, 4, 5, 6, 7, 8, 9$, and $10$. Psychological experiments have shown that individuals cannot simultaneously compare more than seven objects (plus or minus two) [21]. Therefore, we choose that the number of criteria and alternatives in the analysis should not exceed $10$. Thus, a total of $(8 \times 8)$ different cases were examined with $100$ replications (in order to derive statistically significant results) per each case. Each random problem was solved using the original, the qualitative and conditional AHP methods. The test problems were treated as the previous illustrative example. Any ranking irregularity was recorded.

Let us reiterate the immediate goal of this Section: We wish to check the accuracy of our proposed models by comparing them to WSM method.

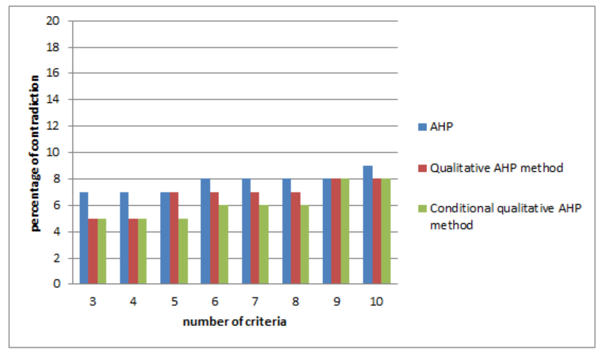


Figure 1. Percentage of contradiction (%) based on 3 alternatives

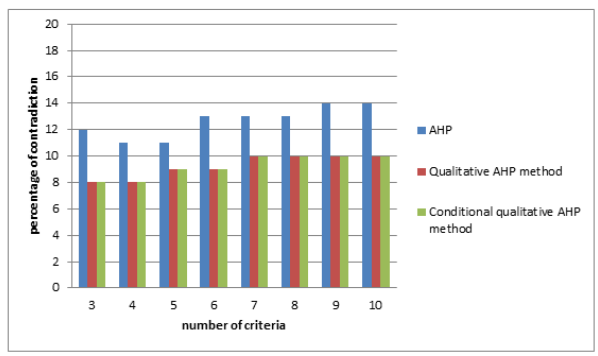


Figure 2. Percentage of contradiction (%) based on 10 alternatives

From Figures 1 and 2, our qualitative AHP achieves a satisfactory percentage of contradiction when it is compared regarding a few number of alternatives and criteria. Indeed, we have to point out that our two methods, qualitative AHP and conditional qualitative AHP, give the lowest percentage of contradiction in almost the cases. This can be explained by the appropriate use of the preference relations to model expert assessments.

The qualitative AHP methods perform much better than standard AHP on the whole. For instance, applying qualitative AHP, conditional qualitative AHP and standard AHP to 3 alternatives and $4$ criteria the percentage of contradiction is set $5\%$, $5\%$, and $7\%$, respectively.

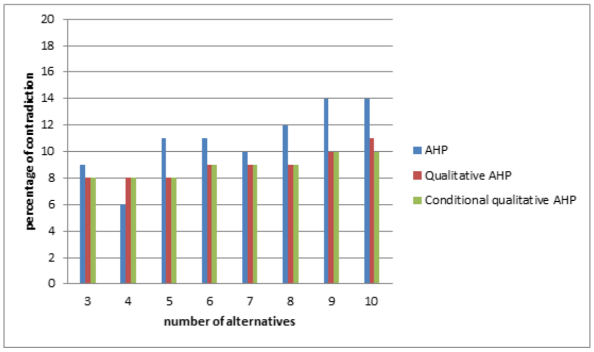


Figure 3. Percentage of contradiction (%) based on 10 criteria

From the experimental results in Figure 3, we obtain a similar observations as before. This experiment further validates the satisfactory results obtained by our method in most cases in terms of percentage of contradiction. Although, the number of alternatives and/or criteria increases, the qualitative AHP methods give the best results.

In summary, these results show that qualitative AHP methods display good performance. This is explained by the fact that our models use a more convenient elicitation technique to model the preference assessments.

## V. Conclusion

In this paper, we have formulated qualitative AHP methods in an environment characterized by imperfection. Our approaches deal with qualitative reasoning to model the uncertainty related to expert's assessment. The advantage of these newly proposed models is their ability to represent the decision maker's preferences without using numerical values. The expert is then allowed to freely express his judgments using belief preferences relations.

Some interesting future works have to be mentioned. Namely, in some situations, the decision maker may be a group or an organization. Therefore, an application of the proposed method to the group decision making situations can then be done.

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