An association algorithm for tracking multiple moving objects

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Abstract—In the field of intelligent vehicle and in order to raise the road safety, we are trying to improve the driver’s assistance and more particularly obstacles detection systems. The objects are detected with sensors laid out on a car, and then, they are tracked with an association algorithm. We have modified M. Rombaut association algorithm in order to have a better decision when the data reliability decrease or if one sensor fails. This paper presents M. Rombaut association algorithm and the modification we have made on it. Then, a comparison between the two association algorithms for a decreasing data reliability is made thanks to simulated data. Finally we have tested the robustness of the two algorithms with data coming from a CCD camera.

I. INTRODUCTION

Our laboratory is integrated into a Federative Structure named GRAISyHM (Groupement de Recherche en Automatisation Intégrée et Système Homme Machine du Nord Pas de Calais). Its aim is to structure and develop the research in Automatics in all the laboratories in the region Nord Pas de Calais.

We are currently working on the project RaViOLi (Radar Vision Orientable et Lidar). This is a regional project dedicated to long range detection and tracking of obstacles. It is based on data fusion, using the Evidence theory, of a stereovision system, a radar and a lidar.

There are also some European projects that are working on driver’s assistance in order to raise the road safety: Carsense, Prométheus [1], Argo . . . .

This paper deals with the modification of M. Rombaut tracking algorithm in order to improve tracking of objects detected by various sensors when the data reliability decrease or if the system has to work with only one sensor. A comparison is made between the two algorithms using synthetic and real data.

The fusion we use in this algorithm is made thanks to the Evidence theory. This paper will not present this theory as there are many paper that already describe it like [2], [3], [4], [5].

In section II, we describe the method for the masses creation. Then, section III shows M. Rombaut association algorithm and the modification we have made on it. Finally, the results we obtain for both algorithms, for simulated and real data, are in section IV.

II. CREATION OF MASSES

In case of an intelligent vehicle, objects are detected by means of sensors laid out on a vehicle. From these sensors that are used for RaViOLi, we know the kind of information we will have to work with which is expressed in (distance;angle) : (ρ; Θ). In addition, we also know the dimensions of the object.

The aim is to follow vehicles. Therefore, we must be able to find them from one moment to another. We use all the information we have to create several masses sets indicating the relations that exist between perceived objects at time $t$ and those known from time $t - 1$. We will note $X_i$ the Perceived objects ($i = 1 : NbP$) at time $t$ by the sensors and $Y_j$ the Known objects ($j = 1 : NbK$). The known objects are perceived objects from the previous sample time that were stored in a memory.

In a first time, we have to define the frame of discernment on which we will work. We have chosen two hypotheses : $Ω = \{ (X_i,RY_j) ; (X_i,RY_j) \}$. Either the perceived object is the same as the known one (in relation : $(X_i,RY_j)$), or it is not (in relation : $(X_i,RY_j)$) [6].

The first step consists in creating belief functions using the information. We build a different masses set for each usable information of each sensor.

Mathematical equations of the masses set, that allow to determine whether objects are in relation or not, is based on a negative exponential introduced by Denoeux [7] :

\[
\begin{align*}
    m(X_i,RY_j) &= α_0 \cdot \exp(-ε_{i,j}) \\
    m(X_i,RY_j) &= α_0 \cdot (1 - \exp(-ε_{i,j})) \\
    m(Ω_{i,j}) &= 1 - α_0
\end{align*}
\]

with $ε_{i,j}$ the variation between two informations (distances, angles, or speed) from perceived objects $X_i$ and known $Y_j$, $α_0$ is a coefficient that characterizes the sensor reliability. There is then one masses set for the distance (Fig. 1) and one for the angle for each sensor.

Next step consists in combining masses set of distance and angle for each sensor, and then to combine between the various sensors. To make the data fusion, we use the evidence theory with the conjonctive operator from Dempster and normalisation.

At the end of those two fusion steps, we get as masses set as the number of possible relation between objects. Now we have to decide which are the recognized objects, those that appeared and those that disappeared. That is what is described in the next section.
III. OBJECTS ASSOCIATION

Taking the decision when associating objects is done thanks to a fusion algorithm which was developed by M. Rombaut [8]. This algorithm was improved by D. Gruyer [9].

A. Mathematical formulation

The algorithm is based on the calculation of some masses. We compute the relation masses for a given object \( i \) to all the objects \( j \). These masses are noted \( m_{i,j} \). We proceed next to the operation that consists in computing the relation masses of each object \( j \) to all other \( i \). These masses are noted \( m_{j,i} \).

The mathematical formulation adopted for \( m_{i,j} \) and the \( m_{j,i} \) are:

\[
m_{i,j}(X_i R Y_j) = K_{i,j} \cdot m_{i,j}(X_i R Y_j) \cdot \prod_{k \neq j}^{N_{i,k}} (1 - m_{i,k}(X_i R Y_k))
\]

\[
m_{i,j}(X_i R *) = K_{i,j} \cdot \prod_{j=1}^{N_{i,k}} m_{i,j}(X_i R Y_j)
\]

\[
m_{i,j}(\Omega_{i,j}) = K_{i,j} \cdot \left[ \prod_{j=1}^{N_{i,k}} (m_{i,j}(\Omega_{i,j}) + m_{i,j}(X_i R Y_j)) - \prod_{j=1}^{N_{i,k}} m_{i,j}(X_i R Y_j) \right]
\]

\[
K_{i,j} = \frac{1}{\prod_{j=1}^{N_{i,k}} (1 - m_{i,j}(X_i R Y_j)) \cdot (1 + \sum_{k \neq j}^{N_{i,k}} m_{i,k}(X_i R Y_k))}
\]

\[
m_{j,i}(Y_j R X_i) = K_{j,i} \cdot m_{j,i}(Y_j R X_i) \cdot \prod_{k \neq i}^{N_{j,k}} (1 - m_{j,k}(Y_j R X_k))
\]

\[
m_{j,i}(Y_j R *) = K_{j,i} \cdot \prod_{i=1}^{N_{j,k}} m_{j,i}(Y_j R X_i)
\]

\[
m_{j,i}(\Omega_{i,j}) = K_{j,i} \cdot \left[ \prod_{i=1}^{N_{j,k}} (m_{j,i}(\Omega_{i,j}) + m_{j,i}(Y_j R X_i)) - \prod_{i=1}^{N_{j,k}} m_{j,i}(Y_j R X_i) \right]
\]

\[
K_{j,i} = \frac{1}{\prod_{i=1}^{N_{j,k}} (1 - m_{j,i}(Y_j R X_i)) \cdot (1 + \sum_{k \neq i}^{N_{j,k}} m_{j,k}(Y_j R X_k))}
\]

We can notice that the non relation masses are not computed. However, these informations are not necessary for the decision: deciding that \( X_i \) is not in relation with \( Y_j \) does not give any information about the other associations.

We can also see two new hypotheses \( m_{i,j}(X_i R *) \) and \( m_{j,i}(Y_j R *) \). In these hypotheses, * represents the fact that one object is in relation with none of the other. Thus if a perceived object \( X_i \) is not associated, it is a new object and if a known object is not associated, it has disappeared (masked by another object, out of range, ...). The decision is taken according to the two masses set \( m_{i,j} \) and \( m_{j,i} \). We choose the couples that have the maximum of credibility in the two masses set.

B. New formulation

When using these equations [10], we have discovered that in some situations, they were not working well. Indeed, the masses \( m_{i,j}(X_i R Y_j) \), are added on \( m_{i,j}(\Omega_{i,j}) \) and \( m_{j,i}(\Omega_{i,j}) \). If it does not matter for the decision because deciding that \( X_i \) is not in relation with \( Y_j \) does not give any information about the good relation, the way in which the formulas are made adds these masses on \( \Omega \) and can lead to choose a bad decision. For example with one perceived and two known giving the following masses sets:

\[
\begin{align*}
m_{1,1}(X_1 R Y_1) &= 0.2 \\
m_{1,2}(X_1 R Y_2) &= 0.45 \\
m_{1,1}(\Omega_{1,1}) &= 0.35 \\
m_{1,2}(\Omega_{1,2}) &= 0.4
\end{align*}
\]

We have:

\[
M_{i,j} = \begin{bmatrix}
X_1 \\
Y_1 & 0.121 \\
Y_2 & 0.396 \\
* & 0.073 \\
\Omega_{i,j} & 0.41
\end{bmatrix}
\]

From the first matrix \( m_{i,j} \), we can deduce that we do not know the relation on \( X_1 : m_{1,1}(\Omega_{1,1}) = 0.41 \) while seeing that \( m_{1,1}(X_1 R Y_2) = 0.396 \). From the second matrix, we can decide that \( Y_1 \) disappears and that \( X_1 \) is in relation with \( Y_2 \). So there is a problem to decide the couples. To solve it, we have decided to modify the equations that compute the masses \( m_{i,j} \) and \( m_{j,i} \). This modification consists in increasing the masses of relation \( (X_i R Y_j) \) and the mass on star with the non relation masses \( (X_i R Y_k) (k \neq j) \) that do not contradict the relation hypothesis. The new \( m_{i,j} \) and the \( m_{j,i} \) are defined like this:
In the terms of relation and star, coefficients appear:

\[
\begin{align*}
\text{m}_{i,j}(X_1, RY_j) &= \frac{\text{N}_{\text{K}}}{\text{N}_{\text{K}} + 1} \cdot \text{m}_{i,j}(X_1, RY_j) \cdot \prod_{k=1}^{\text{N}_{\text{P}}} \left(1 - \text{m}_{i,k}(X_1, RY_k)\right) \\
&+ \frac{\text{N}_{\text{K}}}{\text{N}_{\text{K}} + 1} \cdot \text{m}_{i,k}(X_1, RY_k) \cdot \prod_{p=1}^{\text{N}_{\text{P}}} \text{m}_{i,p}(\Omega_{\text{p}}) + \\
&\sum_{k=1}^{\text{N}_{\text{K}}} \sum_{p=1}^{\text{N}_{\text{P}}} \frac{1}{\text{N}_{\text{K}} - 1} \cdot \text{m}_{i,k}(X_1, RY_k) \cdot \text{m}_{i,j}(X_1, RY_j) \\
&\cdot \prod_{k=1}^{\text{N}_{\text{K}}} \text{m}_{i,p}(\Omega_{\text{p}}) + \cdots + \frac{1}{2} \cdot \text{m}_{i,j}(\Omega_{\text{i,j}}). \\
\text{m}_{i,j}(X_1, R\Theta) &= \frac{\text{N}_{\text{K}}}{\text{N}_{\text{K}} + 1} \cdot \text{m}_{i,j}(X_1, R\Theta) \\
&= \frac{1}{\sum_{k=1}^{\text{N}_{\text{K}}}} \prod_{k=1}^{\text{N}_{\text{K}}} \left(1 - \text{m}_{i,k}(X_1, R\Theta)\right) \\
&+ \frac{1}{\sum_{k=1}^{\text{N}_{\text{K}}}} \cdot \text{m}_{i,k}(X_1, R\Theta) \cdot \prod_{p=1}^{\text{N}_{\text{P}}} \text{m}_{i,p}(\Omega_{\text{p}}) + \\
&\sum_{k=1}^{\text{N}_{\text{K}}} \sum_{p=1}^{\text{N}_{\text{P}}} \frac{1}{\sum_{k=1}^{\text{N}_{\text{K}}} - 1} \cdot \text{m}_{i,k}(X_1, R\Theta) \cdot \text{m}_{i,j}(X_1, R\Theta) \\
&\cdot \prod_{k=1}^{\text{N}_{\text{K}}} \text{m}_{i,p}(\Omega_{\text{p}}) + \cdots + \frac{1}{2} \cdot \text{m}_{i,j}(\Omega_{\text{i,j}}). \\
\text{m}_{i,j}(\Omega_{\text{i,j}}) &= \frac{\text{N}_{\text{K}}}{\text{N}_{\text{K}} + 1} \cdot \text{m}_{i,j}(\Omega_{\text{i,j}}) \\
&= \frac{1}{\sum_{i=1}^{\text{N}_{\text{K}}}} \prod_{i=1}^{\text{N}_{\text{K}}} \left(1 - \text{m}_{i,j}(\Omega_{\text{i,j}})\right) \\
&+ \frac{1}{\sum_{i=1}^{\text{N}_{\text{K}}}} \cdot \text{m}_{i,j}(\Omega_{\text{i,j}}) \cdot \prod_{p=1}^{\text{N}_{\text{P}}} \text{m}_{i,p}(\Omega_{\text{p}}) + \\
&\sum_{i=1}^{\text{N}_{\text{K}}} \sum_{p=1}^{\text{N}_{\text{P}}} \frac{1}{\sum_{i=1}^{\text{N}_{\text{K}}} - 1} \cdot \text{m}_{i,j}(\Omega_{\text{i,j}}) \cdot \text{m}_{i,p}(\Omega_{\text{p}}) + \cdots + \frac{1}{2} \cdot \text{m}_{i,j}(\Omega_{\text{i,j}}).
\end{align*}
\]

In the terms of relation and star, coefficients appear: \((\frac{1}{\sum_{k=1}^{\text{N}_{\text{K}}}}, \ldots, \frac{1}{2})\) for \(m_{i,k}\), and \((\frac{1}{\sum_{k=1}^{\text{N}_{\text{K}}}}, \ldots, \frac{1}{2})\) for \(m_{i,j}\). They correspond to an equal distribution of the mass when there are one or more masses on \(\Omega\) multiplied with the non relation masses. Indeed, with one perceived object and two known, if we multiply \(m_{1,2}(X_1RY_2)\) by \(m_{1,1}(\Omega_{1,1})\), it means that we are sure that \(X_1\) is not in relation with \(Y_2\), so as the framework is exhaustive and closed, the result goes on \(m_{1,1}(X_1RY_1)\) and \(m_{1,1}(X_1R*)\) (multiplied by \(\frac{1}{2}\)). In case of three known objects, if we have two masses on \(\Omega\) in the product \((m_{1,1}(X_1RY_1) \cdot m_{1,2}(\Omega_{1,2}) \cdot m_{1,3}(\Omega_{1,3}))\), we cannot decide where the mass should go, so we put \(\frac{1}{3}\) to the first relation \((m_{1,1}(X_1RY_2))\), \(\frac{1}{3}\) to the second \((m_{1,1}(X_1RY_3))\) and \(\frac{1}{3}\) to the mass on star \((m_{1,1}(X_1R*))\), and so on till we divide by \(\text{N}_{\text{K}}\) (or \(\text{N}_{\text{P}}\)). Considering again the example, we have:

\[
\begin{align*}
M_{i,j} &= \left[\begin{array}{ccc}
X_1 & * & \Omega_{i,j} \\
Y_1 & 0.15 \\
Y_2 & 0.495 \\
* & 0.201 \\
\Omega_{i,j} & 0.154
\end{array}\right] \\
M_{i,j} &= \left[\begin{array}{ccc}
X_1 & * & \Omega_{i,j} \\
Y_1 & 0.2 & 0.45 & 0.35 \\
Y_2 & 0.45 & 0.15 & 0.4
\end{array}\right]
\end{align*}
\]

This time, there is no ambiguity about the best decision which is \((X_1RY_2)\) and \(Y_1\) disappears. Next section shows a comparison between the two formulas using synthetic and real data.

IV. EXPERIMENTAL RESULTS

The aim of this part is to validate the association formulas we have made thanks to results obtained from simulated or real data. The first part illustrates the use of the formulas\(^1\) with synthetic data and a decreasing data reliability. The second part shows the results with real data.

A. Synthetic data

We have taken two objects side by side with the same speed and distant from 1m from each other. Those two vehicles move away progressively from the sensor (from 20 to 100m with a constant speed of 5m.s\(^{-1}\)) which leads to the fall of the reliability coefficient (from 0.9 at 20m to 0.3 at 100m). Fig. 2 represents the evolution of the decision reliability in function of the sensor reliability for M. Rombaut’s algorithm and the one we presented. We consider that the decision is not completely reliable when the variation between the mass of good decision and another one is lower than a threshold we have set to 0.1 in this case.

When the reliability is greater than 0.69, the two algorithms give the same results. However, our algorithm is able to give a reliable decision till a data reliability of 0.49. For M. Rombaut algorithm, the decision is not reliable below 0.69 and is false if the data reliability falls below 0.63.

When real tests could be made, it would be interesting to plot the percentage of good decisions of the two algorithms with various reliability from several scenarios.

Fig. 3 shows the evolution of the non specificity in function of the data reliability. As we can see, our formulas

\(^1\)In order to make easier the computational stage, in this part, \(i\) ranges from 0 to \(\text{N}_{\text{K}} - 1\), and \(j\) from 0 to \(\text{N}_{\text{K}} - 1\)
The fact that we have used two different interpolation methods leads us to two sources of information. So we can say that we have two sensors.

As we are not specialized in image processing, the measures we have extracted were very noisy. Indeed, at 70m, we can have for the same object from an image to the next a variation of 20m. In angle, it can reach 0.02rd for a measure of 0.01rd.

First, we have extracted the measures from only one sensor of one object. This object is followed during 260 images (almost 10s) and is located at almost 70m from the sensor. Figure (Fig. 4) shows the evolution of the data reliability for the perceived object.

We have plotted the comparison between the two association algorithms for the $M_{0*}$ (Fig. 5), the $M_{*,0}$ (Fig. 6) and the global decision using the those two informations (Fig. 7).

In Fig. 5 and 6, we can see that with the new formulas, we always have the relation mass ($(X_{0}R_{y_{0}})$ and $(Y_{0}R_{x_{0}})$ and the mass on star ($(X_{0}R_{*})$ and $(Y_{0}R_{*})$) equal or higher than with M.Rombaut formulas. With the new formulas, the mass on $\Omega$ is always under the mass we obtain with M. Rombaut formulas. So the decisions on $M_{0*}$ and $M_{*,0}$ are different for the two algorithms.

We have plotted the comparison between the two association algorithms for the $M_{0*}$ (Fig. 8) and the $M_{*,0}$ (Fig. 9) for the object. The comparison between two sensors (Sec. IV-B). The data reliability for this object has already been plotted (Fig. 4).

The comparison between the two association algorithms for the $M_{0*}$ is plotted in Fig. 8 and Fig. 9 for the $M_{*,0}$. The

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**Fig. 2.** Evolution of the decision reliability in function of the data reliability.

**Fig. 3.** Evolution of the non specificity in function of the reliability.

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**Fig. 4.** Evolution of the data reliability.

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**Fig. 5.** Evolution of the decision reliability in function of the data reliability.

**Fig. 6.** Evolution of the non specificity in function of the reliability.
global decision using the those two informations is plotted in Fig. 10. In Fig. 8 and 9, we can see that with the new formulas, we always have the relation mass ($(X_0 R Y_0)$ and $(Y_0 R X_0)$) and the mass on star ($(X_0 R *)$ and $(Y_0 R *)$) equal or higher than with M. Rombaut formulas but this time, thanks to the sensor fusion, there is less variation between the two algorithms. With the new formulas, the mass on $\Omega$ is still under the mass we obtain with M. Rombaut formulas but once again, the variation is less than with one sensor.

In order to test the robustness of our algorithm, between images 175 and 225, one sensor fails. During this time, we can see that our algorithm is still working fine.

This is confirmed with the global decision (Fig. 10). The decisions are not very different when the two sensors operate. But, when one sensor fails (between 175 and 225), we can see that our algorithm still track the objet most of time. When it is not tracked, our algorithm decide that it is a new
object (mass on star) so that is not a real problem. Whereas with M. Rombaut algorithm we have a lot of decision on \( \Omega \) and that involves we do not have any information about the object.

![Fig. 10. Comparison of the global decision (using \( M_0 \), and \( M_0 \)) on association for the two algorithms. Decision plotted in the bottom corresponds to M. Rombaut algorithm and in the upper for the Modified formulas.](image)

**V. Conclusion and Future Work**

We have presented in this article an improvement of an association algorithm. This algorithm is used to track moving vehicles. We have changed the distribution of mass between the hypotheses in order to reduce the number of ambiguous and bad decisions. This algorithm was tested on synthetic and real noisy data. We obtain better results with the modified formulas especially when the reliability is weak and when there is only one sensor working. We have reduced the number of non decisions and made easier the choice of the good decision while increasing the gap between the mass for the best hypothesis and the others.

We can still improve the detection and tracking of objects while adding to this algorithm a predicting filter [12], [13], [14]. We can thus predict future associations. This information could be used to confirm at this time the associations that are made or to reduce the ambiguous decisions on some associations. It can also be used as a new sensor that we can put in the stage of data fusion.

**References**