Object association in the TBM framework, application to vehicle driving aid

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Abstract

The problem tackled in this paper deals with obstacle tracking in the context of vehicle driving aid, especially the association step, which consists in associating perceived objects with known objects detected at a previous time. A contribution in the modeling of this association problem in the belief function framework is introduced. By interpreting belief functions as weighted opinions according to the Transferable Belief Model semantics, pieces of information regarding the association of known objects and perceived objects can be expressed in a common global space of association to be combined by the conjunctive rule of combination, and a decision making process using the pignistic transformation can be made. This approach is validated on real data.

Keywords. Obstacle tracking, association step, belief functions, Transferable Belief Model.

1 Introduction

In obstacle tracking, the association step consists in establishing a correlation between tracks (known objects) and targets (perceived objects) from information usually provided by different sensors or captors. Such a mapping can be even more complex depending on the number of targets and tracks, as well as the quality of the provided information. Introduced by Dempster [5] and Shafer [21], belief functions constitute a suitable framework for the representation and manipulation of imperfect information. Thus, next to architectures based on Bayesian probabilistic framework [2, 3], Rombaut [18, 19] develops a first modeling based on belief functions. In this model, information regarding the association of couples (known objects, perceived objects) is represented by belief functions, which are combined using, for simplicity reasons, an adapted combination introduced by Rombaut. In [12] this latter model is developed by using a decision-making system based on belief matrices and the application of a coupling algorithm.

In this paper, a modeling of this association step problem is introduced in the Smets' semantic approach of belief functions: the Transferable Belief Model (TBM) [24], a subjectivist and non-probabilistic interpretation of the Dempster-Shafer theory of belief function. In particular, it is shown that TBM classical tool like the conjunctive combination rule and the pignistic decision-making can be implemented and tested in a real time application, these experimental results demonstrating the effectiveness of this approach as compared to Rombaut's combination rule.

Let us note that the works presented here reexpress and extend in the Transferable Belief Model a former model presented by some of the authors in [13]. Likewise, the association problem described here has many similarities with the works undertaken by Ristic and Smets in [17].

This paper is organized as follows. The TBM basic concepts we need are recalled in Section 2. An association algorithm based on belief functions is introduced in Section 3 and discussed in particular with the other approaches in Section 4. Then, experimental results on real data are presented in Section 5. Finally, Section 6 concludes this paper.

2 Transferable Belief Model (TBM): basic concepts

The Transferable Belief Model (TBM) is a model of uncertain reasoning and decision-making based on two levels [10, 24]:

- the credal level, where available pieces of information are represented by belief functions, and manipulated;
- the pignistic or decision level, where belief functions are transformed into probability measures

when a decision has to be made, and the expected utility is maximized.

2.1 Representing information with belief functions

2.1.1 Belief functions

The knowledge held by an agent is represented by the allocation of a finite mass of belief to subsets of the universe of discourse.

Let $\Omega = \{\omega_1, \omega_2, \ldots, \omega_N\}$, called the frame of discernment, be a finite set composed of all possible answers to a given question Q of interest. The beliefs held by a rational agent Ag regarding the answer to question Q can be quantified by a *belief mass function* or *basic belief assignment (BBA)* $m^{\Omega}_{Ag} : 2^{\Omega} \to [0, 1]$ s.t.:

$$\sum_{A \subseteq \Omega} m^{\Omega}_{Ag}(A) = 1 \ . \tag{1}$$

The quantity $m^{\Omega}_{Ag}(A)$ represents the part of the unit mass allocated to the hypothesis that the answer to question Q is in the subset A of Ω. When there is no ambiguity, the notation m^{Ω}_{Ag} will be simplified as follows m^{Ω} or m.

- A subset A of Ω such that $m(A) > 0$ is called a *focal set* of m.
- A BBA m with only one focal set A is called a *categorical BBA* and is denoted m_A; then $m_A(A) = 1$.
- Total ignorance is represented by the BBA m_{Ω} called the *vacuous BBA*.
- A *normal BBA* m satisfies the condition $m(\emptyset) = 0$.
- Let A be a subset of Ω, the cardinality of A, denoted $|A|$, is the number of elements of Ω in A; if $|A| = 1$, A is said to be a *singleton*.

The belief and plausibility functions associated with a BBA m are defined, respectively, as:

$$bel(A) = \sum_{\emptyset \neq B \subseteq A} m(B) \qquad \forall A \subseteq \Omega \ , \tag{2}$$

$$pl(A) = \sum_{B \cap A \neq \emptyset} m(B) \qquad \forall A \subseteq \Omega \ . \tag{3}$$

Functions m, bel and pl are in one-to-one correspondence, and thus constitute different forms of the same information.

2.1.2 Refinements and Coarsenings

When applying the TBM to a real-world application, the determination of the frame of discernment Ω, which defines the set of states on which beliefs will be expressed, is a crucial step. As noticed by Shafer [21, chapter 6], the degree of granularity of Ω is always, to some extent, a matter of convention, as any element of Ω representing a given state can always be split into several alternatives. Hence, it is fundamental to examine how a belief function defined on a frame may be expressed in a finer or, conversely, in a coarser frame. The concepts of refinement and coarsening can be defined as follows.

Let Θ and Ω denote two frames of discernment. A mapping $\rho : 2^{\Theta} \to 2^{\Omega}$ is called a *refining* of Θ (Figure 5) if it verifies the following properties:

1. The set $\{\rho(\{\theta\}), \theta \in \Theta\} \subseteq 2^{\Omega}$ is a partition of Ω, and
2. For all $A \subseteq \Theta$:

$$\rho(A) = \bigcup_{\theta \in A} \rho(\{\theta\}). \tag{4}$$

Θ is then called a coarsening of Ω, and Ω is called a refinement of Θ.

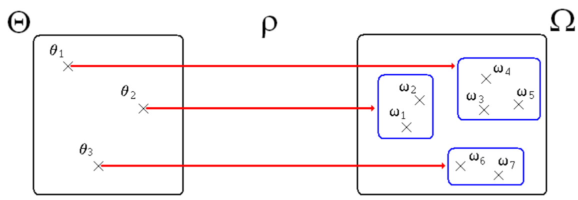

Figure 1: Illustration of a coarsening Θ of Ω associated with a refining ρ of Θ.

2.2 Manipulating information with belief functions

2.2.1 Vacuous extension

The *vacuous extension* operation allows one to convey a belief mass function m^{Θ}, expressing a state of belief on Θ, to a finer frame Ω, a refinement of Θ. Stemming from the *least committed principle* [22], this operation is denoted with an arrow pointing up, and is defined by:

$$m^{\Theta \uparrow \Omega}(\rho(A)) = m^{\Theta}(A), \quad \forall A \subseteq \Theta \ , \tag{5}$$

where ρ is the refining of Θ in Ω.

2.2.2 Combining beliefs

Two BBAs m_1 and m_2, induced by distinct and reliable sources of information, can be combined using the *conjunctive rule of combination* (CRC), also called *unnormalized Dempster's rule of combination*, defined for all $A \subseteq \Omega$ by:

$$m_1 \textcircled{\cap} m_2(A) = \sum_{B \cap C = A} m_1(B) m_2(C) \ . \qquad (6)$$

The normalization hypothesis ($m(\emptyset) = 0$) can be recovered with the following normalization step:

$$m_1 \oplus m_2(A) = \begin{cases} \frac{m_1 \textcircled{\cap}_2(A)}{1 - m_1 \textcircled{\cap}_2(\emptyset)} & \text{if } \emptyset \neq A \subseteq \Omega \ , \\ 0 & \text{otherwise.} \end{cases} \qquad (7)$$

This latter rule of combination is called *Dempster's rule of combination.*

2.3 Decision-making level

When a decision has to be made regarding the answer to question Q, some *rationally principles* [4] justify the strategy consisting in choosing the decision d among a set of possible decisions $\mathcal{D}$, which minimizes the *expected risk* defined by:

$$R(d) = \sum_{\omega \in \Omega} c(d, \omega) P^{\Omega}(\{\omega\}), \qquad (8)$$

where $P^{\Omega} : 2^{\Omega} \to [0, 1]$ is a probability measure and $c : \mathcal{D} \times \Omega \to \mathbb{R}$ a cost function, $c(d, \omega)$ representing the cost to decide d while the truth is ω.

At this level, the mass function m^{Ω} representing the available information regarding the answer to question Q belonging to Ω (resulting in practice from a fusion process) has then to be transformed in a probability measure. A solution [7] consists in computing the *pignistic probability* [23] defined by:

$$BetP^{\Omega}(\{\omega\}) = \sum_{\{A \subseteq \Omega, \omega \in A\}} \frac{m(A)}{|A| \ (1 - m(\emptyset))}, \quad \forall \omega \in \Omega. \qquad (9)$$

The chosen decision is then the one that minimizes the pignistic risk defined by:

$$R_{Bet}(d) = \sum_{\omega \in \Omega} c(d, \omega) BetP^{\Omega}(\{\omega\}) \ . \qquad (10)$$

In the case of 0-1 costs with $\mathcal{D} = \Omega$, which means that $c(\omega_i, \omega_j) = 1$ if $i = j$, 0 otherwise, choosing the decision d which minimises the pignistic risk (10) is equivalent to choose the decision d which maximizes the pignistic probability (9).

An other case consists in choosing 0-1 costs with $\mathcal{D} = \Omega \cup \{d_0\}$, where d_0, called *rejection decision* [7], consists in refusing to make a decision belonging to $\mathcal{D} \setminus \{d_0\}$ when the risk is judged too high. By denoting $c_0 = c(d_0, \omega_i) \ \forall i \in \{1, \dots, N\}$, minimizing the pignistic risk (10) is equivalent to choose the decision:

- d_0 if $\max_{i=1,\dots,N} BetP(\{\omega_i\}) < 1 - c_0$,
- ω_j if $BetP(\{\omega_j\}) = \max_{i=1,\dots,N} BetP(\{\omega_i\}) \geq 1 - c_0$.

The cost c_0 is called the *rejection cost.*

3 Object association algorithm

3.1 Representing information with belief functions

The first step when building belief functions is to define the universe of discourse.

Let us consider the following notations:

- X_i: designate a perceived object at time t, $i \in I = \{1, \dots, N\}$, N being the number of perceived objects at time t;
- Y_j: designate a known object at previous time $t - 1$, $j \in J = \{1, \dots, M\}$, M being the number of known objects at time $t - 1$;
- $\star$: a proposition meaning "no object".

The association process objective consists in finding the best possible association between a set of perceived objects $\{X_1, X_2, \dots, X_N, ^\star\}$ and a set of known objects $\{Y_1, Y_2, \dots, Y_M, ^\star\}$, under the following constraints:

- each perceived object X_i is associated with at most one known object;
- each known object Y_j is associated with at most one perceived object;
- proposition $\star$ can be associated to any objects.

Frames of discernment involved in this application are then the followings:

- $\Omega_{i,j} = \{y_{i,j}, n_{i,j}\}$, containing the two possible answers (yes or no) to the question $Q_{i,j}$: "Is the perceived object X_i associated with the known object Y_j?";

- $\Omega_{X_i} = \{Y_1, Y_2, \ldots, Y_M, ^\star\}$, containing the set of possible answers to the question Q_{X_i} : "Who is associated with the perceived object X_i?", proposition * meaning that X_i has appeared;

- $\Omega_{Y_j} = \{X_1, X_2, \ldots, X_N, ^\star\}$, containing the set of possible answers to the question Q_{Y_j} : "Who is associated with the known object Y_j?", proposition * meaning that Y_j has disappeared or is hidden.

Let us remark that $\Omega_{Y_j} = \Omega_{Y_k}$, for all $j, k \in J$, and $\Omega_{X_i} = \Omega_{X_\ell}$, for all $i, \ell \in I$. Thus, Ω_{X_i} (respectively Ω_{Y_j}) can be denoted Ω_X $\forall i$ (respectively Ω_Y $\forall j$). At last, when there is no ambiguity, the frames elements will be simplified as follows :

- $\Omega_{X_i} = J \cup \{\star\} = \{1, \ldots, M, \star\}$,
- $\Omega_{Y_j} = I \cup \{\star\} = \{1, \ldots, N, \star\}$.

In the domain of intelligent vehicles, sensors or measures generally provide information regarding the association between each perceived object X_i and each known object Y_j [18, 19, 12, 11]. More precisely, initial information is represented by belief mass functions $m^{\Omega_{i,j}}$ on frames $\Omega_{i,j}$, $i \in I$, $j \in J$:

- the mass allocated to $\{y_{i,j}\}$ expresses information on the fact that X_i is associated with Y_j;

- the mass allocated to $\{n_{i,j}\}$ expresses information on the fact that X_i is not associated with Y_j;

- the mass allocated to $\Omega_{i,j} = \{y_{i,j}, n_{i,j}\}$ expresses the ignorance regarding the association of X_i and Y_j.

$N \times M$ belief mass functions $m^{\Omega_{i,j}}$ have been defined regarding the association of each object (perceived objects X_i, known objects Y_j). These pieces of information have then to be fused to determine:

- Where do perceived objects X_i come from?
- What are known objects Y_j become?

3.2 Expressing pieces of information in a common frame

To answer these questions, the $N \times M$ belief mass functions can be combined when expressed on two possible common frames: Ω_X and Ω_Y. Frames Ω_{X_i} and Ω_{Y_j} being refinements of $\Omega_{i,j}$, each information $m^{\Omega_{i,j}}$ can be expressed either on Ω_{X_i} or on Ω_{Y_j} by the vacuous extension operation (5):

$$m^{\Omega_{i,j}\uparrow\Omega_{X_i}}(\rho_{i,j}(A)) = m^{\Omega_{i,j}}(A), \quad \forall A \subseteq \Omega_{i,j}\ , \quad (11)$$

where $\rho_{i,j}$ is the refining of $\Omega_{i,j}$ on Ω_{X_i} illustrated in Figure 2, and defined by $\rho_{i,j}(\{y_{i,j}\}) = \{j\}$ and $\rho_{i,j}(\{n_{i,j}\}) = \overline{\{j\}}$.

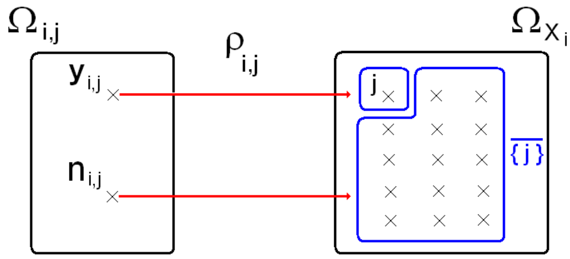

Figure 2: Refining $\rho_{i,j}$ allowing one to transport the information $m^{\Omega_{i,j}}$ on Ω_{X_i}.

Thus, for all $(i, j) \in I \times J$:

$$\begin{cases} m^{\Omega_{i,j}\uparrow\Omega_{X_i}}(\{j\}) & = m^{\Omega_{i,j}}(\{y_{i,j}\}) \\ m^{\Omega_{i,j}\uparrow\Omega_{X_i}}(\overline{\{j\}}) & = m^{\Omega_{i,j}}(\{n_{i,j}\}) \\ m^{\Omega_{i,j}\uparrow\Omega_{X_i}}(\Omega_{X_i}) & = m^{\Omega_{i,j}}(\Omega_{i,j}) \end{cases} \quad (12)$$

In the same manner, it is also possible to vacuously extend $m^{\Omega_{i,j}}$ on Ω_{Y_j}:

$$\begin{cases} m^{\Omega_{i,j}\uparrow\Omega_{Y_j}}(\{i\}) & = m^{\Omega_{i,j}}(\{y_{i,j}\}) \\ m^{\Omega_{i,j}\uparrow\Omega_{Y_j}}(\overline{\{i\}}) & = m^{\Omega_{i,j}}(\{n_{i,j}\}) \\ m^{\Omega_{i,j}\uparrow\Omega_{Y_j}}(\Omega_{Y_j}) & = m^{\Omega_{i,j}}(\Omega_{i,j}) \end{cases} \quad (13)$$

In the following of this paper, $m^{\Omega_{i,j}\uparrow\Omega_{X_i}}$ (respectively $m^{\Omega_{i,j}\uparrow\Omega_{Y_j}}$) is denoted $m_j^{\Omega_{X_i}}$ (respectively $m_i^{\Omega_{Y_j}}$).

3.3 Combining belief mass functions

At this level:

- for each $i \in I = \{1, \ldots, N\}$, M belief mass functions $m_j^{\Omega_{X_i}}$ have been created regarding the association of each object X_i toward the Y_j, the focal elements of each one being $\{j\}$, $\overline{\{j\}}$, and Ω_{X_i}.

- for each $j \in J = \{1, \ldots, M\}$, N belief mass functions $m_i^{\Omega_{Y_j}}$ have been created regarding the association of each object Y_j toward the X_i, the focal elements of each one being $\{i\}$, $\overline{\{i\}}$, et Ω_{Y_j}.

The M belief mass functions $m_j^{\Omega_{X_i}}$, considered as distinct and reliable, are combined using the conjunctive rule of combination (6).

Let us denote $m^{\Omega_{X_i}}$ the resulting mass function:

$$m^{\Omega_{X_i}} = \textcircled{\cap}_{j \in J}\, m_j^{\Omega_{X_i}} \ . \quad (14)$$

For all $k \in J$:

$$m^{\Omega_{X_i}}(\{k\}) = \sum_{\cap A_j = \{k\}} \prod_{j \in J} m_j^{\Omega_{X_i}}(A_j) \ , \quad (15)$$

where, for all $j \in J$, $A_j = \{j\}$, $\overline{\{j\}}$, or Ω_{X_i}.

But:

$$\begin{aligned}\cap_{j\in J} A_j = \{k\} \quad &\Leftrightarrow \quad A_k = \{k\} \text{ and } (A_j = \overline{\{j\}} \text{ or} \\ &\qquad A_j = \Omega_{X_i},\ \forall j \in J \setminus \{k\}), \\ &\Leftrightarrow \quad A_k = \{k\} \text{ and} \\ &\qquad A_j \neq \{j\},\ \forall j \in J \setminus \{k\} \ .\end{aligned}$$

Thus, for all $k \in J$:

$$m^{\Omega_{X_i}}(\{k\}) = m_k^{\Omega_{X_i}}(\{k\}) \prod_{\substack{j=1 \\ j \neq k}}^{M} (1 - m_j^{\Omega_{X_i}}(\{j\})) \ . \quad (16)$$

Similarly, for all $K \subseteq J$:

$$\begin{aligned}m^{\Omega_{X_i}}(\overline{K}) \quad &= \quad \sum_{\cap A_j = \overline{K}} \prod_{j \in J} m_j^{\Omega_{X_i}}(A_j) \ , \\ &= \quad \prod_{j \in K} m_j^{\Omega_{X_i}}(\overline{\{j\}}) \prod_{j \in \overline{K}} m_j^{\Omega_{X_i}}(\Omega_{X_i}) \ .\end{aligned}$$

In particular:

$$\begin{aligned}m^{\Omega_{X_i}}(\{\star\}) \quad &= \quad m^{\Omega_{X_i}}(\overline{J}) = \prod_{j \in J} m_j^{\Omega_{X_i}}(\overline{\{j\}}) \ , \\ m^{\Omega_{X_i}}(\Omega_{X_i}) \quad &= \quad m^{\Omega_{X_i}}(\overline{\emptyset}) = \prod_{j \in J} m_j^{\Omega_{X_i}}(\Omega_{X_i}) \ .\end{aligned}$$

At last:

$$\begin{aligned}m^{\Omega_{X_i}}(\emptyset) \quad &= \quad \sum_{\cap A_j = \emptyset} \prod_{j \in J} m_j^{\Omega_{X_i}}(A_j) \ , \quad (17) \\ &= \quad \sum_{\substack{j,k \in J \\ j \neq k}} m_j^{\Omega_{X_i}}(\{j\})\, m_k^{\Omega_{X_i}}(\{k\}). \quad (18)\end{aligned}$$

In the same manner, the N belief mass functions $m_i^{\Omega_{Y_j}}$ can also be conjunctively combined to result in a mass function $m^{\Omega_{Y_j}}$.

Example 1 *Let us consider one perceived object X_1 and two known objects Y_1 and Y_2 s.t.:*

$$\left\{\begin{array}{ll} m^{\Omega_{1,1}}(\{y_{1,1}\}) & = .2 \\ m^{\Omega_{1,1}}(\{n_{1,1}\}) & = .45 \\ m^{\Omega_{1,1}}(\Omega_{1,1}) & = .35 \end{array}\right. \qquad \left\{\begin{array}{ll} m^{\Omega_{1,2}}(\{y_{1,2}\}) & = .45 \\ m^{\Omega_{1,2}}(\{n_{1,2}\}) & = .15 \\ m^{\Omega_{1,2}}(\Omega_{1,2}) & = .4 \end{array}\right. \quad (19)$$

By expressing this information on Ω_{X_1} (X_1 point of view: with which known object, the perceived object X_1 is associated? In other words: Where does X_1 come from?), it is obtained:

$$\left\{\begin{array}{ll} m_1^{\Omega_{X_1}}(\{1\}) & = .2 \\ m_1^{\Omega_{X_1}}(\overline{\{1\}}) & = .45 \\ m_1^{\Omega_{X_1}}(\Omega_{X_1}) & = .35 \end{array}\right. \qquad \left\{\begin{array}{ll} m_2^{\Omega_{X_1}}(\{2\}) & = .45 \\ m_2^{\Omega_{X_1}}(\overline{\{2\}}) & = .15 \\ m_2^{\Omega_{X_1}}(\Omega_{X_1}) & = .4 \end{array}\right. \quad (20)$$

The conjunctive combination of $m_1^{\Omega_{X_1}}$ and $m_2^{\Omega_{X_1}}$ provides the following result:

$$\begin{array}{llll} m^{\Omega_{X_1}}(\{1\}) & = .2 \times (1 - .45) & = .2 \times .55 & = .11 \\ m^{\Omega_{X_1}}(\{2\}) & = .45 \times (1 - .2) & = .45 \times .8 & = .36 \\ m^{\Omega_{X_1}}(\overline{\{1\}}) & = m^{\Omega_{X_1}}(\{2, \star\}) & = .45 \times .4 & = .18 \\ m^{\Omega_{X_1}}(\overline{\{2\}}) & = m^{\Omega_{X_1}}(\{1, \star\}) & = .15 \times .35 & = .05 \\ m^{\Omega_{X_1}}(\overline{\{1,2\}}) & = m^{\Omega_{X_1}}(\{\star\}) & = .45 \times .15 & = .07 \\ m^{\Omega_{X_1}}(\Omega_{X_1}) & = m^{\Omega_{X_1}}(\{1,2,\star\}) & = .35 \times .4 & = .14 \\ m^{\Omega_{X_1}}(\emptyset) & = .2 \times .45 & = .09 \ . & \end{array} \quad (21)$$

3.4 Decision-making

The pignistic probability $BetP^{\Omega_{X_i}}$ (9) computed from $m^{\Omega_{X_i}}$ is defined for all $\omega \in \Omega_{X_i}$ by:

$$BetP^{\Omega_{X_i}}(\{\omega\}) = \sum_{\{A \subseteq \Omega_{X_i}, \omega \in A\}} \frac{m^{\Omega_{X_i}}(A)}{|A|\,(1 - m^{\Omega_{X_i}}(\emptyset))}. \quad (22)$$

Then, for all $k \in J$:

$$\begin{aligned}BetP^{\Omega_{X_i}}(\{k\}) = \mathcal{K}_1 \Bigg[& m_k^{\Omega_{X_i}}(\{k\}) \prod_{\substack{j=1 \\ j \neq k}}^{M} (1 - m_j^{\Omega_{X_i}}(\{j\})) \\ & + \sum_{\substack{k \in \overline{K} \\ K \subseteq J}} \frac{1}{|\overline{K}|} \prod_{j \in K} m_j^{\Omega_{X_i}}(\overline{\{j\}}) \prod_{j \in \overline{K}} m_j^{\Omega_{X_i}}(\Omega_{X_i}) \Bigg] \ ,\end{aligned} \quad (23)$$

where

$$\mathcal{K}_1 = \frac{1}{1 - m^{\Omega_{X_i}}(\emptyset)} = \frac{1}{1 - \sum_{\substack{j,k \in J \\ j \neq k}} m_j^{\Omega_{X_i}}(\{j\})\, m_k^{\Omega_{X_i}}(\{k\})} \ . \quad (24)$$

At last:

$$BetP^{\Omega_{X_i}}(\{\star\}) = \mathcal{K}_1 \sum_{K \subseteq J} \frac{1}{|\overline{K}|} \prod_{j \in K} m_j^{\Omega_{X_i}}(\overline{\{j\}}) \prod_{j \in \overline{K}} m_j^{\Omega_{X_i}}(\Omega_{X_i}) \ . \quad (25)$$

Once the pignistic probabilities $BetP^{\Omega_{X_i}}$ computed for each $i \in I$, the chosen decision is the one that maximizes the pignistic probability associated to the joint law $BetP^{\Omega_{X_1}\times\ldots\times\Omega_{X_N}}$ which verifies the constraints expressed in Section 3.1.

Similarly, an equivalently justified solution consists in computing the decision from the Y_j points of view, by maximizing the pignistic probability $BetP^{\Omega_{Y_1}\times\ldots\times\Omega_{Y_M}}$.

Example 2 (Example 1 continued) *Let us consider again one perceived object X_1 and two known objects Y_1 and Y_2 with the information represented by the BBAs $m^{\Omega_{1,1}}$ and $m^{\Omega_{1,2}}$ defined by Equation 19.*

From X_1 point of view, the conjunctive combination of $m_1^{\Omega_{X_1}}$ and $m_2^{\Omega_{X_1}}$ has been detailed in Example 1. The pignistic probability $BetP^{\Omega_{X_1}}$ regarding the association of X_1 is then given by:

A	$\emptyset$	$\{1\}$	$\{2\}$	$\{\star\}$	$\{1,\star\}$	$\{2,\star\}$	$\{1,2,\star\}$
$m^{\Omega_{X_1}}(A)$	.09	.11	.36	.07	.05	.18	.14
$BetP^{\Omega_{X_1}}(A)$		.20	.55	.25	.45	.80	1

Conclusion from X_1 point of view:

1. *The singleton maximizing $BetP^{\Omega_{X_1}}$ is $\{2\}$, so X_1 is associated with Y_2;*
2. *knowing that Y_1 is not associated, Y_1 has disappeared (or is hidden).*

On the other hand, it is also possible to express the available information on Ω_{Y_1} and Ω_{Y_2}:

$$\begin{cases} m_1^{\Omega_{Y_1}}(\{1\}) & = .2 \\ m_1^{\Omega_{Y_1}}(\overline{\{1\}}) & = .45 \\ m_1^{\Omega_{Y_1}}(\Omega_{Y_1}) & = .35 \end{cases} \qquad \begin{cases} m_1^{\Omega_{Y_2}}(\{1\}) & = .45 \\ m_1^{\Omega_{Y_2}}(\overline{\{1\}}) & = .15 \\ m_1^{\Omega_{Y_2}}(\Omega_{Y_2}) & = .4 \end{cases}$$

As there is only one perceived object X_1, no combination is necessary:

A	$\emptyset$	$\{1\}$	$\{\star\}$	$\{1,\star\}$
$m^{\Omega_{Y_1}}(A)$		.2	.45	.35
$BetP^{\Omega_{Y_1}}(A)$		.375	.625	1
$m^{\Omega_{Y_2}}(A)$		.45	.15	.4
$BetP^{\Omega_{Y_2}}(A)$		.65	.35	1

From the association constraints (Section 3.1), the known objects (Y_1, Y_2) can be associated to $(1,\star)$, $(\star,1)$, or $(\star,\star)$. As:

- $BetP^{\Omega_{Y_1}\times\Omega_{Y_2}}(\{1,\star\}) = .375\times .35 = .131$;
- $BetP^{\Omega_{Y_1}\times\Omega_{Y_2}}(\{\star,1\}) = .625\times .65 = .406$;
- $BetP^{\Omega_{Y_1}\times\Omega_{Y_2}}(\{\star,\star\}) = .625\times .35 = .219$,

then $BetP^{\Omega_{Y_1}\times\Omega_{Y_2}}$ reaches its "valid" maximum in $\{\star,1\}$, so (Y_1,Y_2) is associated with $(\star,1)$; in other words, Y_1 has disappeared and Y_2 is associated with X_1.

In the previous example, the decision coming from X_1 and the decision coming from the Y_j are the same. Unfortunately, as illustrated by the following example, the decision providing by the criteria of maximizing the joint pignistic probability can be different depending on which point of view (perceived objects X_i or known objects Y_j) it is computed.

Let us also remark that the introduction of a rejection decision, as presented in Section 2.3, can also imply a different decision according to the X_i or Y_j points of view. For instance, by choosing c_0 equal to 0.5 in the previous Example 2, from X_1 the same decision is made as $BetP^{\Omega_{X_1}}(\{2\}) \geq 1-c_0$, however as $BetP^{\Omega_{Y_1}\times\Omega_{Y_2}}(\{\star,1\}) < 1-c_0$, the decision made according to the Y_j is d_0 (a rejection).

Example 3 *Let us considered one perceived object X_1, and two known objects Y_1 and Y_2, s.t.:*

$$\begin{cases} m^{\Omega_{1,1}}(\{y_{1,1}\}) & = .5 \\ m^{\Omega_{1,1}}(\{n_{1,1}\}) & = 0 \\ m^{\Omega_{1,1}}(\Omega_{1,1}) & = .5 \end{cases} \qquad \begin{cases} m^{\Omega_{1,2}}(\{y_{1,2}\}) & = .7 \\ m^{\Omega_{1,2}}(\{n_{1,2}\}) & = .3 \\ m^{\Omega_{1,2}}(\Omega_{1,2}) & = 0\ . \end{cases}$$

By expressing the beliefs on the frames Ω_{X_i}:

$$\begin{cases} m_1^{\Omega_{X_1}}(\{1\}) & = .5 \\ m_1^{\Omega_{X_1}}(\overline{\{1\}}) & = 0 \\ m_1^{\Omega_{X_1}}(\Omega_{X_1}) & = .5 \end{cases} \qquad \begin{cases} m_2^{\Omega_{X_1}}(\{2\}) & = .7 \\ m_2^{\Omega_{X_1}}(\overline{\{2\}}) & = .3 \\ m_2^{\Omega_{X_1}}(\Omega_{X_1}) & = 0\ , \end{cases}$$

the following results are obtained:

A	$\emptyset$	$\{1\}$	$\{2\}$	$\{\star\}$
$m^{\Omega_{X_1}}(A)$	.35	.15	.35	0
$BetP^{\Omega_{X_1}}(A)$		.35	.54	.11

A	$\{1,\star\}$	$\{2,\star\}$	$\{1,2,\star\}$
$m^{\Omega_{X_1}}(A)$	.15	0	0
$BetP^{\Omega_{X_1}}(A)$	.46	.65	1

Then from object X_1 point of view:

- *X_1 is associated with Y_2,*
- *Y_1 has disappeared.*

From Y_1 and Y_2 points of view:

$$\begin{cases} m_1^{\Omega_{Y_1}}(\{1\}) & = .5 \\ m_1^{\Omega_{Y_1}}(\overline{\{1\}}) & = 0 \\ m_1^{\Omega_{Y_1}}(\Omega_{Y_1}) & = .5 \end{cases} \qquad \begin{cases} m_1^{\Omega_{Y_2}}(\{1\}) & = .7 \\ m_1^{\Omega_{Y_2}}(\overline{\{1\}}) & = .3 \\ m_1^{\Omega_{Y_2}}(\Omega_{Y_2}) & = 0\ . \end{cases} \tag{26}$$

So:

A	$\{1\}$	$\{\star\}$
$BetP^{\Omega_{Y_1}}$	.75	.25
$BetP^{\Omega_{Y_2}}$	.70	.30

(27)

As $.75 \times .3 > .7 \times .25$*,* $BetP^{\Omega_{Y_1} \times \Omega_{Y_2}}$ *reaches its valid maximum in* $\{1, \star\}$*, which implies that:*

- Y_1 *is associated with* X_1*,*
- Y_2 *has disappeared.*

This decision is then different from the previous one.

Works are currently undertaken by the authors to investigate properties input BBAs $m^{\Omega_{i,j}}$ should verify to not encounter this problem. A conjecture to be proved, is that if BBAs $m^{\Omega_{i,j}}$ are simple BBAs, which means BBAs that have two focal elements: the universe $\Omega_{i,j}$ and an other one element, then no conflicting decision appears. In other words, BBAs $m^{\Omega_{i,j}}$ should not assign masses to both propositions $\{y_{i,j}\}$ and $\{n_{i,j}\}$.

Until something better turns up, a practical solution consists in choosing a decision by favoring either the perceived objects or the known objects. However, to relativize this problem, it is shown on a particular application described in Section 5, that conflicting decisions can happen in very few cases, less than 1% in this instance.

4 Discussion

4.1 What's new in comparison to Rombaut and Gruyer's approaches?

The approach presented in this paper differs mainly from Rombaut and Gruyer's approaches [18, 12] by regarding two points:

1. the combination of BBAs $m_j^{\Omega_{X_i}} = m^{\Omega_{i,j} \uparrow \Omega_{X_i}}$ and $m_i^{\Omega_{Y_j}} = m^{\Omega_{i,j} \uparrow \Omega_{Y_j}}$;
2. the decision-making process.

In both Rombaut's approach [18] and Gruyer's approach [12], BBAs $m_j^{\Omega_{X_i}}$ and $m_i^{\Omega_{Y_j}}$ are not classically conjunctively combined with (14). To simplify the combination and to make it computationally efficient, it is proposed to allocate masses only on singletons and the universe. Thus the following mergers are proposed, $\forall i \in I$:

$$\begin{array}{rcl} m_{Rombaut}^{\Omega_{X_i}}(\{\emptyset\}) & = & m^{\Omega_{X_i}}(\{\emptyset\}) \\ m_{Rombaut}^{\Omega_{X_i}}(\{k\}) & = & m^{\Omega_{X_i}}(\{k\}), \quad \forall k \in J, \\ m_{Rombaut}^{\Omega_{X_i}}(\{\star\}) & = & m^{\Omega_{X_i}}(\{\star\}) \\ m_{Rombaut}^{\Omega_{X_i}}(\Omega_{X_i}) & = & \prod_{j \in J} (m_j^{\Omega_{X_i}}(\Omega_{X_i}) + m_j^{\Omega_{X_i}}(\overline{\{j\}})) \\ & & - \prod_{j \in J} m_j^{\Omega_{X_i}}(\overline{\{j\}}). \end{array} \quad (28)$$

In [12], the authors suggest a decision-making system based on BBAs $m^{\Omega_{X_i}}$ and $m^{\Omega_{Y_j}}$ whose focal elements, thanks to Rombaut's combination, are either a singleton or the universe. In outline:

- An association matrix $N \times M$ is built such that each of its elements (i, j) is equal to the product $m^{\Omega_{X_i}}(\{j\}) \times m^{\Omega_{Y_j}}(\{i\})$. Each row i is then associated with a perceived object X_i, and each column j is associated with a known object Y_j.
- If necessary, fictive objects are added to make the latter matrix squared.
- A coupling algorithm, the Hungarian algorithm, is then applied to this matrix, this latter algorithm providing an optimal decision regarding the sum of the beliefs.
- A final treatment deals with the objects appearance.

In the examples presented in [18] and [12], the model presented in this paper and Gruyer's approach lead to the same results.

4.2 About Ristic and Smets' approach

The problem tackled by Ristic and Smets in [17] is somewhat different from the association problem described in this paper. Ristic and Smets consider a given volume of interest containing an unknown number of objects. While sensors we consider give information regarding the associations of each object detected at a time step t, with previous objects detected at a previous time step $t-1$, Ristic and Smets's sensors provide information regarding the class of each object they have detected in the scene, for instance helicopter, airplane, ... The "association problem" they try to solve consists then in determining the number of objects as well as the class of each one. Besides, the appearance and disappearance of objects do not take directly part of their problem. The application of Ristic and Smets' works to our problem is consequently not straightforward.

However, some technical points of this model should be taken into account and investigated.

Following [8], the authors remark that the mass given to the empty set, after conjunctively combining two BBAs expressing themselves on the class of two different objects is equal to the belief that these two objects do not belong to the same class, an idea already present in [1] (multi-sensor fusion for submarine detection) and in [20] (intelligence clustering).

At last, the criteria the authors maximize is based on the plausibility of each possible associations. As justified in [23], the pignistic transformation has been chosen to make the decision in this paper. A first investigation in the direction of the plausibility consists in using the plausibility-probability transformation [16].

5 Results on real data

In this section, the approach presented in this paper (Section 3) is compared to the approach of Rombaut and Gruyer on real data coming from a DV camera placed behind the windshield of a car. This DV camera has a CCD sensor, a 720×576 pixels resolution, an angle ranging from -0.5 to $+0.5$ radians (i.e. approximately $\pm 30°$), and works at 25 images per second ($\Delta_t = .04s$), a filmed image example being presented in Figure 3.

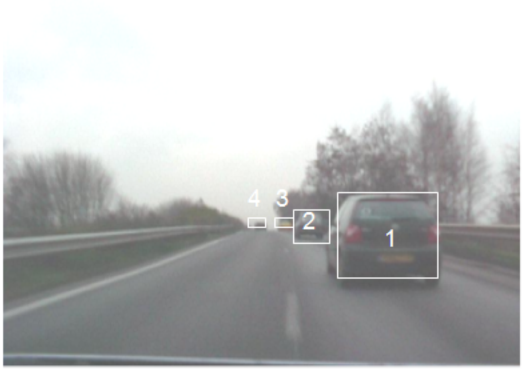

Figure 3: Four vehicles in a selected filmed image.

The video sequence allowing one to compare the two approaches includes 3250 images corresponding to a 130-second playing time. Images contain 1 to 8 objects. During the sequence, 75 distinct objects were manually identified as illustrated in Figure 3, the number of associations to realize being equal to 6800. The ground truth is known, which allows one to compute the good recognition rate of each approach during this sequence.

Distance and angle criteria allow the creation of two belief functions denoted $m^{\Omega_{i,j}}_{distance}$ and $m^{\Omega_{i,j}}_{angle}$, regarding the association between each perceived object X_i and known object Y_j.

The distance was estimated as a function of the height and the width in pixels of the object observed in the scene thanks to an interpolation method illustrated in Figure 4.

On the other hand, the angle between two objects is computed from the gravity center of the perceived object in the image (Figure 3).

The measurements provided are very noisy. For instance, there can be a variation of $20m$ for the same object from an image to the next one. Likewise, angle variations can be as high as 100%, from $0.01rd$ to $0.02rd$ for two consecutive measurements of the same object.

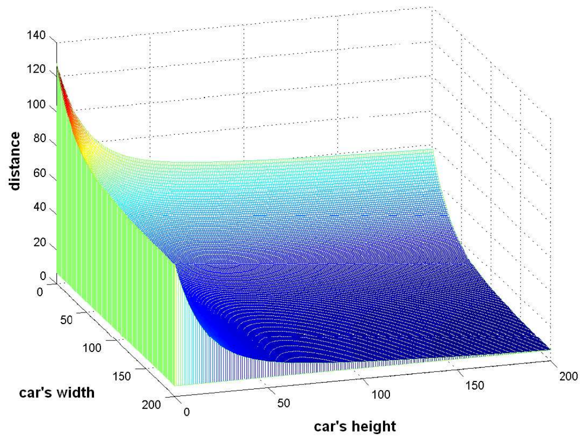

Figure 4: Interpolation function giving the distance in meter depending on the height and the width in pixels of the object in the scene.

In this application, masses are fixed in the following way:

$$\begin{cases} m^{\Omega_{i,j}}(\{y_{i,j}\}) & = \beta\ \phi_{i,j}(e_{i,j}) \\ m^{\Omega_{i,j}}(\{n_{i,j}\}) & = \beta\ (1 - \phi_{i,j}(e_{i,j})) \\ m^{\Omega_{i,j}}(\Omega_{i,j}) & = 1 - \beta \end{cases} \qquad (29)$$

where:

- $0 < \beta < 1$ is a constant representing the degree of reliability of the source of information (cf the discounting operation [21, page 252], and [14, 15] for other correction mechanisms).

- $\phi_{i,j}(.)$ is a monotone decreasing function s.t. $\phi_i(0) = 1$ and $\lim_{e\to\infty} \phi_i(e) = 0$;
- $e_{i,j}$ is the dissimilarity measure between the perceived object X_i and the known object Y_j, which means the difference of distance and the difference of angle in this application.

The function $\phi_{i,j}$ is chosen as follows [6]:

$$\phi_{i,j}(e_{i,j}) = exp^{(-(e_{i,j})^2)}. \quad (30)$$

Constant β being fixed at 0.9, these two belief mass functions are combined thanks to the Dempster's rule of combination to obtain a mass function $m^{\Omega_{i,j}}$:

$$m^{\Omega_{i,j}} = m^{\Omega_{i,j}}_{distance} \oplus m^{\Omega_{i,j}}_{angle} \quad \forall i \in I, \forall j \in J\ . \quad (31)$$

The association model presented in Section 3 only need one BBA expressing the information regarding the association between object X_i and object Y_j. In this application, we are lucky enough to have two information sources. Thus these two pieces of information are firstly combined using a well justified rule for the combination of two distinct sources. The choice to combine theses sources at this step, and the choice of the rule have been left for further study.

In Figure 5, the good recognition rate of the two approaches presented in this paper obtained in this video sequence is represented as a function of the rejection cost (Section 2.3). It can be observed that as soon as the rejection cost becomes greater than 0, the good recognition rates obtained with the conjunctive combination are greater than those obtained with Rombaut's combination, which is recalled to be also used in Gruyer's approach.

Let us note that the decisions have been computed on the basis of the perceived objects. As mentioned in Section 3.4, these decisions are not necessary identical with those computed from the known objects point of view. However, as illustrated in Figure 6, this conflicting decision rate remains very low in this application (from 0% to less than 1% depending on the rejection cost). Let us also recall that, as illustrated at the end of Example 2, the introduction of a rejection cost enhances the appearance of conflicting decisions.

6 Conclusion and prospects

In this paper, a modeling of the association step problem in obstacle tracking in the belief function framework has been presented. In particular, it has been shown how tools from the theory of belief functions such as the vacuous extension, the conjunctive combination rule and the pignistic transformation can

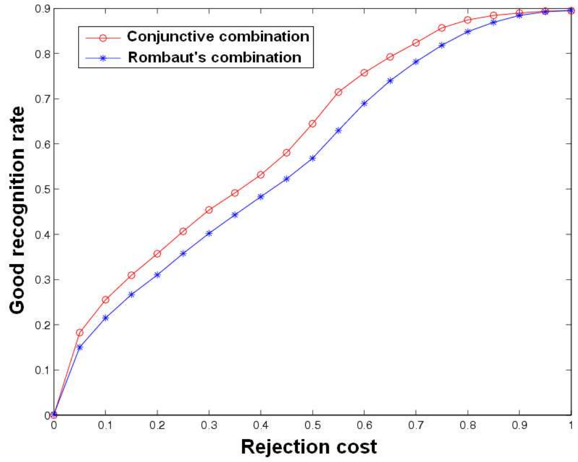

Figure 5: Good recognition rate in function of the rejection cost.

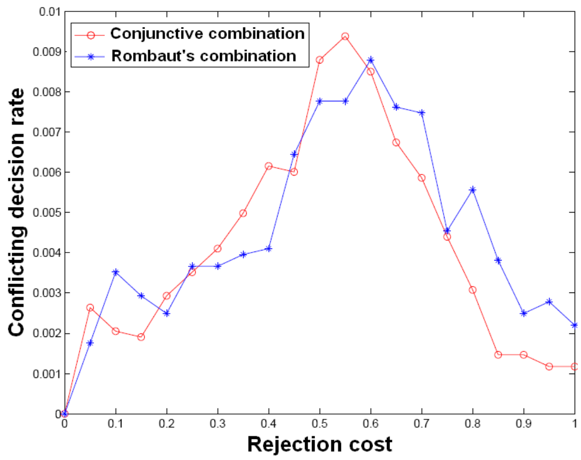

Figure 6: Conflicting decision rate in function of the rejection cost.

be applied. Validated on real data, this approach can perform better good recognition rates than Rombaut's initial approach as soon as a rejection cost is introduced.

Concerning the prospects, even if it concerns a reduce number of cases, a more convincing solution has to be brought regarding the resolution of the possible conflicting decisions between the perceived and known objects points view. This points is currently under investigation.

The decomposition of the BBAs [9] expressing the beliefs regarding the associations between known objects and perceived objects could also be studied in order to use a more adapted rule.

Subsequently, this approach should be enhanced by introducing information coming from the tracking of vehicles at time steps preceding the current analysis.

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