Introducing Incomparability in Modeling Qualitative Belief Functions

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Abstract. This paper investigates a new model for generating belief functions from qualitative preferences. Our approach consists in constructing appropriate quantitative information from incomplete preferences relations. It is able to combine preferences despite the presence of incompleteness and incomparability in their preference orderings. The originality of our model is to provide additional interpretation values to the existing methods based on strict preferences and indifferences only.

1 Introduction

When solving problems dealing with belief function theory, expert is usually required to provide precise numerical values, for determining the portion of belief committed exactly to an event in a particular domain. However, when handling with such situation, the main difficulty is how to quantify these numeric values, therefore linguistic assessments could be used instead. So, the expert is then asked to express his opinions qualitatively, based on knowledge and experience that he provides in response to a given question rather than direct quantitative information.

However, in some cases, the decision maker may be unable to express his opinions due to his lack of knowledge. He is then forced to provide incomplete or even erroneous information. Obviously, rejecting this difficulty in eliciting the expert preference is not a good practice.

Besides, in preference modeling, expert may express preferences among a pair of alternatives in distinct ways: either an expert has a strict preference of one alternative compared to the other, or is indifferent between both alternatives. However, these two interpretations are possible because we made the assumption of complete and sure information. They do not cover all possible situations a decision maker can be faced with. Consider now the situations in which the expert has symmetrically lack of information and also excess of information in the sense that he has contradictory inputs. He may then introduce two new situations: incompleteness and incomparability. The intuition is that the expert cannot compare apple and cheese because they are too different. For instance, he may consider that alternatives may be incomparable because the expert does not wish very dissimilar alternatives to be compared. Incompleteness, on the other hand, represents simply an absence of knowledge about the preference of certain pairs of alternatives. It arises when we have not fully elicited an expert's preferences or when expert do not has the full information.

To deal with such situations, a more realistic solution should be proposed, that is able to efficiently imitate the expert reasoning using belief function theory. In this paper, our main aim is then to elaborate on how may be incomparability and incompleteness represented in the belief function framework as understood in the Transferable Belief Model (TBM) [11]. Some researchers have already dealt with this problem and generate associated quantitative belief functions like [1] [4] [7] [12]. However, these approaches are only based on strict preference and indifference relations.

In this work, we focused on Ben Yaghlane et al. [1] [2] approach in order to construct quantitative belief functions from qualitative preferences by transforming these opinions into basic belief assessment. This approach is chosen since it does not require that the expert supplies the preference relations between all pairs of propositions. In fact, it allows the generation of belief functions using incomplete qualitative preference relations. Besides, it has been noted that this method handles the issue of inconsistency in the pair-wise comparisons. In this research, we propose a general model for constructing belief functions, which takes into account all information contained within the expert. With this approach focused on incompleteness and incomparability, the originality of our model is to allow the expert to easily express his preferences and to provide a convenience framework for constructing quantitative belief functions from qualitative assessments. Thus, we present a model that is able to combine the expert assessments despite the presence of incompleteness and incomparability in their preference orderings.

In order to do this, the paper is set out as follows: Section 2 provides a brief description of basics of belief function theory. In section 3, we describe the preference modeling approach. Then, Section 4 reviews some existing approaches. Next in Section 5, our suggested solution will be approached. Section 6 presents an example to illustrate our method. Finally, Section 7 ends this work.

2 Belief Function Theory

In this section, we briefly review the main concepts underlying the belief function theory as interpreted by the Transferable Belief Model (TBM). Details can be found in [9] [11].

2.1 Basic Concepts

The TBM is a model to represent quantified belief functions [11]. Let Θ be the frame of discernment representing a finite set of elementary hypotheses related to a problem domain. We denote by 2^{Θ} the set of all the subsets of Θ [9].

The impact of a piece of evidence on the different subsets of the frame of discernment Θ is represented by the so-called basic belief assignment (bba), called initially by Shafer, basic probability assignment [9].

$$\sum_{A \subseteq \Theta} m(A) = 1 \tag{1}$$

The value $m(A)$, named a basic belief mass (bbm), represents the portion of belief committed exactly to the event A. The events having positive bbm's are called focal elements. Let $\mathcal{F}(m) \subseteq 2^{\Theta}$ be the set of focal elements of the bba m.

Associated with m is the belief function is defined for $A \subseteq \Theta$ and $A \neq \emptyset$ as:

$$bel(A) = \sum_{\emptyset \neq B \subseteq A} m(B) \text{ and } bel(\emptyset) = 0 \tag{2}$$

The degree of belief $bel(A)$ given to a subset A of the frame Θ is defined as the sum of all the basic belief masses given to subsets that support A without supporting its negation.

The plausibility function pl expresses the maximum amount of specific support that could be given to a proposition A in Θ. It measures the degree of belief committed to the propositions compatible with A. $pl(A)$ is then obtained by summing the bbm's given to the subsets B such that $B \cap A \neq \emptyset$ [9]:

$$pl(A) = \sum_{B \cap A \neq \emptyset} m(B), \ \ \forall A \subseteq \Theta \tag{3}$$

2.2 Decision Making

The TBM considers that holding beliefs and making decision are distinct processes. Hence, it proposes a two level model:

- The credal level where beliefs are entertained and represented by belief functions.
- The pignistic level where beliefs are used to make decisions and represented by probability functions called the pignistic probabilities, denoted *BetP* [10]:

$$BetP(A) = \sum_{B \subseteq \Theta} \frac{|A \cap B|}{|B|} \frac{m(B)}{(1 - m(\emptyset))}, \forall A \in \Theta \tag{4}$$

2.3 Uncertainty Measures

In this section, we introduce some uncertainty measures defined in the belief function theory, which quantify the information content or the degree of uncertainty of a piece of information [5] [6].

Nonspecificity Measures. The nonspecificity measure is introduced by Dubois and Prade in order to measure the nonspecificity of a normal bba by a function N defined as [5] [6]:

$$N(m) = \sum_{A \in \mathcal{F}(m)} m(A) \log_2 |A| \tag{5}$$

The bba m is all the most imprecise (least informative) that $N(m)$ is large. The minimum ($N(m) = 0$) is obtained when m is a Bayesian bba (focal elements are singletons) and the maximum ($N(m) = \log_2 |A|$) is reached when m is a vacuous bba ($m(\Theta) = 1$).

Conflict Measures. Conflict measures are a generalization of the Shannon's entropy and they were expressed as follows [5] [6]:

$$conflict(m) = - \sum_{A \in \mathcal{F}(m)} m(A) \log_2 f(A) \tag{6}$$

where f is, respectively, pl, bel or $BetP$ and the conflict measures are called, respectively, Dissonance (E), Confusion (C) and Discord (D).

Composite Measures. Different measures have been defined by Pal, Bezdek and Hemasinha [5] [6] such as:

$$H(m) = \sum_{A \in \mathcal{F}(m)} m(A) \log_2 (\frac{|A|}{m(A)}) \tag{7}$$

$$EP(m) = - \sum_{\omega \in \Omega} BetP(\omega) \log_2 BetP(\omega) \tag{8}$$

The interesting feature of $H(m)$ is that it has a unique maximum.

3 Preference Relations

The preference structure is a basic step of preference modeling. Given two alternatives, decision maker defines three binary relations: preference (P :$\succ$), indifference (I :$\sim$) and incomparability (J :?) [8].

A preference structure is a basic concept of preference modeling. In a classical preference structure, a decision maker makes three decisions for any pair (a, b) from the set A of all alternatives. His decision defines a triplet P, I, J of crisp binary relations on A:

1. a is prefered to b ($(a, b) \in P$) iff $(a \succ b) \wedge \neg(b \succ a)$
2. a is indifferent to b ($(a, b) \in I$) iff $(a \succ b) \wedge (b \succ a)$
3. a is incomparable to b ($(a, b) \in J$) iff $\neg(a \succ b) \wedge \neg(b \succ a)$

However, P, I and J must satisfy some rather basic additional conditions. For instance, any couple of alternatives belongs to exactly one of the relations P, P^t (the transpose of P), I or J. More formally, a preference structure is defined as follows.

1. I is reflexive and J is irreflexive;
2. P is asymmetrical;
3. I and J are symmetrical;
4. $P \cap I = \emptyset$, $P \cap J = \emptyset$ and $J \cap I = \emptyset$
5. $P \cup P^t \cup I \cup J = A^2$

Property (1) means that the user is always indifferent between a and a, and that a can always be compared to itself; (2) is the property that a user cannot prefer a to b and b to a at the same time; (3) means that when a user is indifferent between a and b, he is equally so to b and a , and that when a and b are incomparable, so are b and a. Property (4) states that a pair (a, b) cannot belong to two of the relations P, I and J at the same time. Finally, (5) is the property that a pair (a, b) always belongs to one of the relations P , P^t, I or J, and to no other. Note that the asymmetry of P implies the irreflexivity of P.

4 Qualitative Belief Functions Methods

The problem of eliciting qualitatively expert opinions and generating basic belief assignments have been addressed by many researchers [1] [4] [7] [12]. In this section, we provide an overview of some existing approaches.

4.1 Wong and Lingras' Method

Wong and Lingras [12] proposed a method for generating quantitative belief functions from qualitative preference assessments. So, given a pair of propositions, experts may express which of the propositions is more likely to be true. Thus, they defined two binary relations preference $\succ$ and indifference $\sim$ defined on 2^Θ such as:

$$a \succ b \text{ is equivalent to } bel(a) > bel(b) \tag{9}$$

$$a \sim b \text{ is equivalent to } bel(a) = bel(b) \tag{10}$$

where $a, b \in 2^\Theta$.

This approach is based on two steps. The first one consists in considering that all the propositions that appear in the preference relations are potential focal elements. However, some propositions are eliminated according to the following condition: if $a \sim b$ for some $a \subset b$, then a is not a focal element.

After that, the basic belief assignment is generated using the two presented Equations 9 and 10. This formulation has multiple belief functions that are consistent with the input qualitative information, and so their procedure only generates one of them.

It should be noted that Wong and Lingras' approach do not address the issue of inconsistency in the pair-wise comparisons. For example, the expert could specify the apparently inconsistent preference relationships: $bel(a) > bel(b)$, $bel(a) > bel(c)$, and $bel(c) > bel(a)$.

4.2 Bryson et al.' Method

Qualitative discrimination process (QDP), a model for generating belief functions from qualitative preferences, was presented by Bryson, et al. [4].

This method is based on the following steps. First, using this QDA approach, each proposition is assigned into a Broad category bucket, then to a corresponding Intermediate bucket, and finally to a corresponding Narrow category bucket. The qualitative scoring is done using a table where each Broad category is a linguistic quantifier in the sense of Parsons [7]. He considers that linguistic quantifiers could provide a useful approach to representing beliefs vaguely, and that mass and hence bba should be represented using numeric intervals. Then, in step 2, the previous table is used to identify and remove non focal propositions. For each superset proposition, determine if the expert is indifferent in his strength of belief, in the truthfulness of the given proposition and any of its subset propositions in the same or lower Narrow category bucket.

Step 3 is called "imprecise pair-wise comparisons" because the expert is required to provide numeric intervals to express his beliefs on the relative truthfulness of the propositions. In step 4, the consistency of the belief information provided by the expert is checked. Then, the belief function is generated in step 5 by providing a bba interval for each focal element. Finally, in step 6, the expert examines the generated belief functions and stops the QDP if it is acceptable, otherwise the process is repeated.

4.3 Ben Yaghlane et al.'s Method

Ben Yaghlane et al. proposed a method for generating optimized belief functions from qualitative preferences [1].

So giving two alternatives, an expert can usually express which of the propositions is more likely to be true, thus they used two binary preference relations: the preference and the indifference relations. The objective of this method is then to convert these preferences into constraints of an optimization problem whose resolution, according to some uncertainty measures (UM) (nonspecificity measures, conflict measures, composite measures), allows the generation of the least informative or the most uncertain belief functions defined as follows:

$$a \succ b \Rightarrow bel(a) - bel(b) \geq \varepsilon \tag{11}$$

$$a \sim b \Rightarrow |bel(a) - bel(b)| \leq \varepsilon \tag{12}$$

where ε is considered to be the smallest gap that the expert may discern between the degrees of belief in two propositions A and B. Note that ε is a constant specified by the expert before beginning the optimization process.

Ben Yaghlane et al. proposed a method that requires that propositions be represented in terms of focal elements, and they assume that Θ (where Θ is the frame of discernment) should always be considered as a potential focal element. Then, a mono-objective technique was used to solve such constrained optimization problem:

$$\begin{array}{c} Max_m UM(m) \\ s.t. \\ bel(a) - bel(b) \geq \varepsilon \\ bel(a) - bel(b) \leq \varepsilon \\ bel(a) - bel(b) \geq -\varepsilon \\ \sum_{a \in \mathcal{F}(m)} m(a) = 1, m(a) \geq 0, \forall a \subseteq \Theta; m(\emptyset) = 0 \end{array} \tag{13}$$

where the first, second and third constraints are derived from Eqs 11 and 12, representing the quantitative constraints corresponding to the qualitative preference relations.

Furthermore, the proposed method addresses the problem of inconsistency. In fact, if the preference relations are consistent, then the optimization problem is feasible. Otherwise no solutions will be found. Thus, the expert may be guided to reformulate his preferences.

An extension of the proposed solution is also presented. In fact, the authors suggested to use the goal programming, a multiobjective method, in order to take into account simultaneously several objectives in the formulation of the problem. So, the idea behind the use of this method is to be able to integrate additional information about the belief functions to be generated.

It should be noted that this method does not address the issue of incomparability in the pair-wise comparisons. In fact, this proposed method treats incomparability as incompleteness. However, we believe that this interpretation is not appropriate. If an expert is unable to compare two alternatives then this situation should be reflected in the preference relation not as an incomplete situation, but with an entry for that particular pair of alternatives. So, in the following section, we present our method that deal with this problem.

5 Modelling Belief Functions using Qualitative Preferences

As presented above, representing efficiently the expert preferences is a crucial task in elaborating the necessary data for a considered problem. Therefore, we propose a realistic solution that is able to efficiently imitate the expert reasoning. In fact, our main aim is then to elaborate on how may be incomparability and incompleteness represented in qualitative belief functions. The solution we suggest is then a qualitative model for constructing belief functions from elicited expert opinions when dealing with qualitative preference relations. In this section, we start by identifying sufficient conditions of introducing these imperfect preferences. Then, we consider the computational procedure.

5.1 Incompleteness in the Belief Function Theory

Incomparability and incompleteness represent very different concepts. In this subsection, we try also to differentiate incomplete preferences from incomparable ones. This situation is illustrated by complete ignorance, missing information, lack of knowledge or an ongoing preference elicitation process. Incompleteness represents then simply an absence of knowledge about the relationship between these pairs of alternatives.

Given such considerations, it may perhaps be useful at times to take incomplete order as the primitive of analysis. Besides, expert is freely allowed to assign this belief to any pairs of alternatives. In other words, a partial order allows some relations between pairs of alternatives to be unknown.

Example. Given three alternatives $\Theta = \{a, b, c\}$, an incomplete order can be for example: $(a \succ c, b?c)$ or $(c \succ a, a \sim b)$, where some relations between pairs of alternatives are unknown.

5.2 Incomparability in the Belief Function Theory

A missing value in a linguistic preference relation is not always equivalent to a lack of preference of one alternative over another. A missing value can also be the result of the incapacity of an expert to compare one alternative over another because they are too different. In such cases, the expert may not put his opinion forward about certain aspects of the problem, he would not be able to efficiently express his preference between two or more of the available alternative. As a result, he may find some of them to be incomparable and thus has an incomplete preference ordering, i.e., he neither prefers one alternative over the other nor finds them equally as good. Therefore, it would be of great importance to provide the expert with tools that allow them to efficiently model his preferences.

In order to model this situation, we first consider how to represent the incomparability relation. In fact, our problem here is that incomparability is expressed entirely in terms of negations:

$$a?b \text{ iff } \neg(a \succ b) \wedge \neg(b \succ a) \tag{14}$$

By definition, a couple of alternatives (a, b) belongs to the incomparability relation J if and only if the expert is unable to compare a and b. Furthermore, it is hard to see what kind of behavior could correspond to Equation 14. If neither a nor b is chosen, then the expert may not be able to tell which alternative is better, since not aPb, not bPa and not aIb. In other terms, we apply incomparability when the preference profiles of two alternatives are severely conflicting.

The question now is how to formalize this situation in the belief function framework. In order to build this new preference relation, we may accept that there exist positive reasons which support the relation $\neg(a \succ b)$ and also there exist sufficient negative information to establish the relation $(a \succ b)$. These two assumptions can properly model the contradictory information. Besides, we can surely establish that "a is preferred to b" as there are not sufficient reasons supporting the opposite and there are sufficient information against it, while we can also surely establish that "b is preferred to a" for the same reasons. Therefore, a and b are in conflicting position. On the other hand, and based on the belief function framework and as defined by Boujelben et al. [3], the incomparability situation appears between two alternatives when their evaluations given by basic belief assessments differ significantly.

In the following, our objective is to represent the case of incomparability with the belief function theory. Consider two alternatives a and b, as proved in Wong et al. [12] the belief function exists since the preference relation $\succ$ satisfies the following axioms:

1. Asymmetry: $a \succ b \Rightarrow \neg(b \succ a)$
2. Negative Transitivity: $\neg(a \succ b)$ and $\neg(b \succ c) \Rightarrow \neg(a \succ c)$
3. Dominance: For all $a, b \in 2^{\Theta}, a \supseteq b \Rightarrow a \succ b$ or $a \sim b$
4. Partial monotonicity: For all $a, b, c \in 2^{\Theta}$, if $a \supset b$ and $a \cap c = \emptyset \Rightarrow (a \cup c) \succ (b \cup c)$

So, Wong et al. have justified the existing of the following relation:

$$a \succ b \Leftrightarrow bel(a) > bel(b) \tag{15}$$

In other words, Wong et al. have proved that it may exist functions other than the belief functions, which are also compatible with a preference relation such that for every $a, b \in 2^{\Theta}$.

$$a \succ b \Leftrightarrow f(a) > f(b) \tag{16}$$

if and only if the relation $\succ$ satisfies the previous axioms.

Similarly to this idea, we can prove that the plausibility function also exists since the preference relation $\succ$ satisfies the previous axioms. Besides, we can conclude that it exists a plausibility function pl: $2^{\Theta} \rightarrow [0, 1]$ such as:

$$a \succ b \Leftrightarrow pl(a) > pl(b) \tag{17}$$

To summarize, we can get the following relation:

$$a \succ b \Leftrightarrow bel(a) > bel(b) \text{ and } pl(a) > pl(b) \tag{18}$$

As we have defined previously, the incomparability situation appears between two alternatives when their preference profiles are severely conflicting. That is when their evaluations given by basic belief assessments differ significantly. We can then intuitively conclude from Equation 18 that, if a is incomparable with b, then:

$$a?b \Leftrightarrow bel(a) \geq bel(b) \text{ and } pl(a) \leq pl(b) \tag{19}$$

The first part of Equation 19 supports the assumption "a is preferred to b" however the second one supporting the opposite affirmation. Also, the second part of the Equation supports the assumption "b is preferred to a" and the first part affirms the opposite assumption.

Consequently, our purpose is then to prove the existing of the previous Equation 19 in order to correctly represent the bba relative to the incomparability relation.

Proof. According to the definition of the plausibility function, we have:

$$pl(a) = bel(\Theta) - bel(\bar{a}).$$

We start from the second part of the Equation 19, our assumption is: $pl(a) \leq pl(b)$
$\Leftrightarrow bel(\Theta) - bel(\bar{a}) \leq bel(\Theta) - bel(\bar{b})$
$\Leftrightarrow$ - $bel(\bar{a}) \leq -bel(\bar{b})$
$\Leftrightarrow bel(\bar{a}) \geq bel(\bar{b})$
Using the Equation 9, we can therefore conclude that: $\bar{a} \succ \bar{b}$*, which means that* $\neg(a \succ b)$*. This contradicts with the assumption "if* $pl(a) \leq pl(b)$ *then* $a \succ b$*". Hence, if we have* $pl(a) \leq pl(b)$*, then* $b \succ a$*. However, from the first part of the assumption, we have:* $a \succ b$*.*

As a conclusion, such representation of incomparability (Equation 18), enables us to correctly express the conflicting information produced by the alternative a and the alternative b. In fact, the first part of the Equation 19 "$bel(a) \geq bel(b)$" implies that a is preferred to b. Then, the plausibility function is used since it expresses the maximum amount of specific support that could be given to a proposition a. However, when we define the second part of the Equation 19, we propose to assume that $pl(a) \leq pl(b)$ which means that b is preferred to a. This contradicts the first assumption, and can properly express the conflicting information's produced by a and b.

5.3 Computational Procedure

Now and after modeling the incompleteness and the incomparability preferences, we propose to extend Ben Yaghlane et al. method [1]. We transform these preferences relations into constraints as presented in section 4.1. We get:

$$\begin{array}{c} Max_m UM(m) \\ s.t. \\ bel(a) - bel(b) \geq \varepsilon \\ bel(a) - bel(b) \leq \varepsilon \\ bel(a) - bel(b) \geq -\varepsilon \\ bel(a) \geq bel(b) \\ pl(a) \leq pl(b) \\ \sum_{a \in \mathcal{F}(m)} m(a) = 1; m(a) \geq 0; \forall a \subseteq \Theta; m(\emptyset) = 0 \end{array} \tag{20}$$

where the first, second and third constraints of the model are derived from the preference and indifference relations. The fourth and fifth constraints correspond to the incomparability relation. ε is a constant specified by the expert before beginning the optimization process.

A crucial step is needed before beginning the task of generating belief functions, is the identification of the candidate focal elements. Thus, as applied in the existing approaches, we may initially assume that prepositions which may appear in the preference relationships are considered as focal elements. Then, other focal elements could be appear or also eliminated. The next phase of our procedure consists of establishing the local preference relations between each pair of two alternatives. Finally, these obtained relations are transformed into constraints to obtain the quantitative belief function.

6 Example

Let us consider a problem of eliciting the weight of the candidate criteria. The problem involves six criteria: $\Omega = \{a, b, c, d, e, f\}$. The focal elements are: $F1 = \{a\}$, $F2 = \{a, b, c\}$, $F3 = \{b, e\}$, $F4 = \{e, f\}$ and $F5 = \{a, e, d\}$.

Next, the expert opinions should be elicitated. For this purpose, an interview with the expert is realized in order to model his preferences. Consequently, he has validated the following relations:

$$\begin{array}{c} F2 \succ F1 \text{ , } F1 ? F3 \text{ , } F4 \succ F1 \\ F5 \succ F1 \text{ , } F3 \sim F2 \text{ , } F5 \succ F4, \end{array}$$

After eliciting the expert preferences, the following step is to identify the candidate focal elements. So, we get:

$$\mathcal{F}(m) = \{F1, F2, F3, F4, F5, \Theta\}$$

Next, these obtained relations are transformed into optimization problem according to our proposed method. We assume that $\varepsilon = 0.01$ and the uncertainty measures is H since it has a unique maximum as defined in Equation 7.

Table 1. The obtained bba using our proposed model

Criteria	$\{a\}$	$\{a,b,c\}$	$\{b,e\}$	$\{e,f\}$	$\{a,e,d\}$	$\{b,c,d,e,f\}$	$\{a,c,d,f\}$	Θ
bba	0.069	0.079	0.069	0.079	0.119	0.198	0.194	0.193
bel	0.069	0.148	0.069	0.079	0.119	0.346	0.263	1
pl	0.535	0.069	0.46	0.047	0.312	0.391	0.391	1

Table 1 gives the results of all ordered couples on the basis of their preference relation. Besides, we are interested in obtaining their corresponding quantitative bba.

In fact, there are different ways to obtain a result by aggregating the obtaining binary relation. But, the existing approaches present the inconvenience of eliminating useful information as the incompleteness and incomparability. However, by applying our presented solution, it is easy to see that our method aggregates all the elicited data. Here, in the present example, all the incomparabilities are detected. We obtain for example $F1?F3$.

Once the preferences relations are defined, the corresponding bba (Table 1) should be constructed. We suggest to transform our problem into a constrained optimization model in order to choose the optimal solution and to get the previous result.

Now we propose to apply Ben Yaghlane et al. method. By using this model, we assume that the incomparability and the incompleteness are modeled in the same way. In other words, the relation $F1?F3$ will be eliminated and we get the following Table 2.

Table 2. The obtained bba using Ben Yaghlane et al. method

Criteria	$\{a\}$	$\{a,b,c\}$	$\{b,e\}$	$\{e,f\}$	$\{a,e,d\}$	Θ
bba	0.063	0.159	0.149	0.126	0.189	0.315
bel	0.063	0.222	0.149	0.126	0.252	1

In absence of incomparabilities, we note the couple of alternatives $\{b,c,d,e,f\}$ and $\{a,c,d,f\}$ do not appears because the incomparability relation has been assigned to other relation: the incompleteness. Observing the two obtaining results, it is possible to see that in spite of the use of two different models, we get almost the same partial order.

7 Conclusion

In this study, the incomplete linguistic preference relations are used to derive quantitative belief function. By presenting our method, a new model for constructing belief functions from elicited expert opinions has been defined, that takes into account the incomplete and even the incomparable alternatives. The originality of our model is then to provide additional interpretation values to the existing methods based on strict preferences and indifferences only.

Under this perspective the paper introduces a new method based on Ben Yaghlane et al. approach [1]. Our work makes it possible to separate incomparability from incompleteness. Then, we suggest to extend Ben Yaghlane et al. method to take into account these distinct levels of preferences. Finally, our method transforms the preference relations provided by the expert into constraints of an optimization problem.

An interesting future work is to make our method able to explore uncertainty preferences. For instance, an expert may not be able to say if a couple of alternatives (a, b) belongs to the preference or indifference relation. Besides, we propose to apply our proposed method in multi-criteria decision making field, which can be interesting in eliciting expert judgments.

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