Contextual discounting of belief functions

David Mercier[1,2], Benjamin Quost[1] and Thierry Denœux[1]

[1] Université de Technologie de Compiègne, UMR CNRS 6599 Heudiasyc, BP20529, F-60205 Compiègne Cedex, France
{dmercier, bquost, tdenoeux}@hds.utc.fr
[2] SOLYSTIC, 14 avenue Raspail, F-94257 Gentilly Cedex, France

Abstract. The Transferable Belief Model is a general framework for managing imprecise and uncertain information using belief functions. In this framework, the *discounting* operation allows to combine information provided by a source (in the form of a belief function) with metaknowledge regarding the reliability of that source, to compute a "weakened", less informative belief function. In this article, an extension of the discounting operation is proposed, allowing to make use of more detailed information regarding the reliability of the source in different contexts, a context being defined as a subset of the frame of discernment. Some properties of this *contextual discounting* operation are studied, and its relationship with classical discounted is explained.

1 Introduction

In the past years, the need to manipulate various forms of imperfect information and partial knowledge has led to study new uncertainty management frameworks. One of them, the *theory of evidence* [6] or *theory of belief functions*, has been declined into several approaches, among which the *Transferable Belief Model* (TBM) [8, 11]. This model, on which we will focus in this article, constitutes a powerful and flexible framework, well suited for information fusion [2, 5, 9].

In information fusion applications, it is usually important to take into account the reliability of the different sources in the evidence aggregation process. In the TBM, this is achieved by the discounting operation, which transforms each belief function provided by a source into a less informative one, based on a degree of confidence in the reliability of the source [6, 7]. In certain applications, however, it is possible to assess the reliability of the source in different contexts [1]. The *contextual discounting* operation presented in this paper extends the classical discounting so as to exploit such information.

This paper is organized as follows. Background material on the TBM will first be recalled in Section 2. Contextual discounting will then be introduced in Section 3, and an example will be analyzed in Section 4. Section 5 will conclude the paper.

2 The Transferable Belief Model

2.1 Basic concepts

Let x be a variable taking values in a finite set $\Omega = \{\omega_1, \ldots, \omega_K\}$, called the *frame of discernment* (or *frame*). The knowledge held by a rational agent Y, regarding the actual value ω_0 taken by x, given an evidential corpus EC, can be quantified by *basic belief assignment* (bba) $m_Y^\Omega[EC]$ defined as a function from 2^Ω to $[0,1]$ $m(A)$ verifying :

$$\sum_{A \subseteq \Omega} m_Y^\Omega[EC](A) = 1$$

When there is now ambiguity, the full notation $m_Y^\Omega[EC]$ will be simplified to m_Y^Ω, m^Ω, or even m. The vacuous bba, defined by $m(\Omega) = 1$, represents complete ignorance.

Two distinct pieces of evidence, quantified by bbas m_1 and m_2, may be combined using the *conjunctive rule of combination* (CRC) or the *disjunctive rule of combination* (DRC), defined, respectively, as :

$$m_1 \textcircled{\cap} m_2(A) = \sum_{B \cap C = A} m_1(B) m_2(C),$$
$$m_1 \textcircled{\cup} m_2(A) = \sum_{B \cup C = A} m_1(B) m_2(C), \quad \forall A \subseteq \Omega.$$

The CRC applies when both sources are known to be reliable, whereas the DRC corresponds the hypothesis that at least one of the two sources is reliable [7].

2.2 Marginalization and vacuous extension

A bba defined on a product space $\Omega \times \Theta$ may be marginalized on Ω, by transfering each mass $m^{\Omega \times \Theta}(B)$ for $B \subseteq \Omega \times \Theta$ to its projection on Ω:

$$m^{\Omega \times \Theta \downarrow \Omega}(A) = \sum_{\{B \subseteq \Omega \times \Theta \mid \text{Proj}(B \downarrow \Omega) = A\}} m^{\Omega \times \Theta}(B) \ , \forall A \subseteq \Omega \tag{1}$$

where $\text{Proj}(B \downarrow \Omega)$ denotes the projection of B onto Ω.

It is usually not possible to retrieve the original bba $m^{\Omega \times \Theta}$ from its marginalization $m^{\Omega \times \Theta \downarrow \Omega}$ on Ω. However, the *least commited bba* [7] such that its projection on Ω is $m^{\Omega \times \Theta \downarrow \Omega}$ may be computed; this *vacuous extension* of a bba m^Ω on the product space $\Omega \times \Theta$ is given by:

$$m^{\Omega \uparrow \Omega \times \Theta}(B) = \begin{cases} m^\Omega(A) & \text{if } B = A \times \Theta, \ A \subseteq \Omega \\ 0 & \text{otherwise.} \end{cases} \tag{2}$$

Marginalization and vacuous extension are both illustrated in Figure 1.

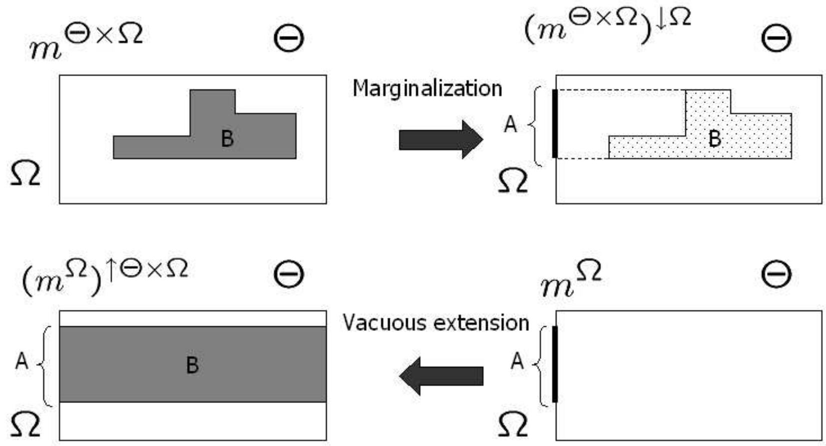

Fig. 1. Marginalization (above) and vacuous extension (below) of a bba in the case of a product space.

2.3 Conditioning and ballooning extension

Conditional beliefs represent knowledge which is valid provided that an hypothesis is satisfied. Let m be a bba, $B \subseteq \Omega$ an hypothesis and m_B such as $m_B(B) = 1$; the conditional belief function $m[B]$ is:

$$m[B] = m \textcircled{\cap} m_B. \tag{3}$$

If $m^{\Omega\times\Theta}$ is defined on the product space $\Omega \times \Theta$, and θ_0 is a subset of Θ, the conditional bba $m^{\Omega}[\theta_0]$ is defined by combining $m^{\Omega\times\Theta}$ with $m_{\theta_0}^{\Theta\uparrow\Omega\times\Theta}$, with $m_{\theta_0}^{\Theta}(\theta_0) = 1$, and marginalizing the result on Ω:

$$m^{\Omega}[\theta_0] = \left(m^{\Omega\times\Theta} \textcircled{\cap} m_{\theta_0}^{\Theta\uparrow\Omega\times\Theta}\right)^{\downarrow\Omega} \tag{4}$$

Assume now that $m^{\Omega}[\theta_0]$ represents your beliefs on Ω conditionnally on θ_0, i.e., in a context where θ_0 holds. There are usually many bbas on $\Omega \times \Theta$, whose conditioning on θ_0 yields $m^{\Omega}[\theta_0]$. Among these, the least committed one is the balloning extension defined by:

$$m^{\Omega}[\theta_0]^{\Uparrow\Omega\times\Theta}(A \times \theta_0 \cup \Omega \times \overline{\theta_0}) = m^{\Omega}[\theta_0](A), \quad \forall A \subseteq \Omega. \tag{5}$$

Conditioning and ballooning extension are both presented in Figure 2.

2.4 Discounting

Let us assume that Y receives a bba m_S^{Ω} from a source S, describing the source's beliefs regarding the actual value ω_0. Moreover, Y has some knowledge about

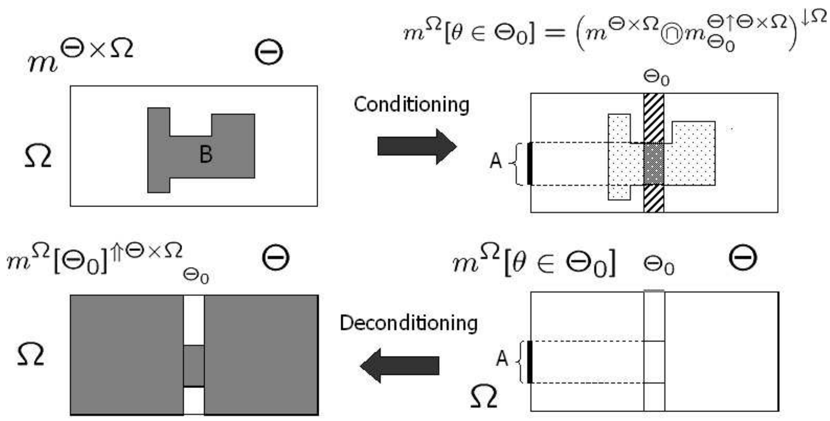

Fig. 2. Conditioning (above) and deconditioning (below) of a bba in the case of a product space.

the *reliability* of S, quantified by a bba $m_Y^{\mathcal{R}}$ on the space $\mathcal{R} = \{R, NR\}$, where R stands for "the source is reliable", and NR for "the source is not reliable" [7]. Let us assume that $m_Y^{\mathcal{R}}$ has the following form:

$$\begin{cases} m_Y^{\mathcal{R}}(\{R\}) &= 1-\alpha \\ m_Y^{\mathcal{R}}(\{R, NR\}) &= \alpha, \end{cases} \tag{6}$$

for some $\alpha \in [0,1]$.

If S is reliable, the information provided by S becomes Y's knowledge:

$$m_Y^{\Omega}[\{R\}] = m_S^{\Omega}. \tag{7}$$

If S is not reliable, the information provided by S cannot be taken into account, and Y's knowledge is vacuous:

$$m_Y^{\Omega}[\{NR\}](\Omega) = 1. \tag{8}$$

Therefore, we have two non-vacuous pieces of evidence, $m_Y^{\mathcal{R}}$ and $m_Y^{\Omega}[\{R\}]$. Assuming that they are distinct, they can be combined by vacuously extending $m_Y^{\mathcal{R}}$ to $\Omega\times\mathcal{R}$, computing the ballooning extension of $m_Y^{\Omega}[\{R\}]$ in the same space, applying the CRC, and marginalizing the result on Ω:

$$m_Y^{\Omega}[m_S^{\Omega}, m_Y^{\mathcal{R}}] = \left(m_Y^{\Omega}[\{R\}]^{\Uparrow\Omega\times\mathcal{R}} \bigcirc\!\!\!\!\cap\, m_Y^{\mathcal{R}\uparrow\Omega\times\mathcal{R}}\right)^{\downarrow\Omega}. \tag{9}$$

The resulting bba $m_Y^{\Omega}[m_S^{\Omega}, m_Y^{\mathcal{R}}]$ (where the brackets [ ] indicate the evidential corpus) only depends on m_S^{Ω} and α. Let us denote it by ${}^{\alpha}m_Y^{\Omega}$. It is equal to

$$\begin{cases} {}^{\alpha}m_Y^{\Omega}(A) = (1-\alpha)m_S^{\Omega}(A), \quad \forall A \subset \Omega, \\ {}^{\alpha}m_Y^{\Omega}(\Omega) = (1-\alpha)m_S^{\Omega}(\Omega) + \alpha. \end{cases} \tag{10}$$

This operation was called discounting by Shafer [6], who introduced it on intuitive grounds. The justification presented in this section was proposed by Smets [7].

Remark 1. If $m_Y^{\mathcal{R}}$ is Bayesian:

$$\begin{cases} m_Y^{\mathcal{R}}(\{R\}) & = 1-\alpha, \\ m_Y^{\mathcal{R}}(\{NR\}) & = \alpha, \end{cases} \tag{11}$$

the result of the discounting is the same [7].

Remark 2. We can see ${}^{\alpha}m_Y^{\Omega}$ as the disjunctive combination of m_S^{Ω} with m_0^{Ω} defined by $m_0^{\Omega}(\emptyset) = 1-\alpha$ and $m_0^{\Omega}(\Omega) = \alpha$.

Remark 3. Alternatively, ${}^{\alpha}m_Y^{\Omega}$ can be computed as

$${}^{\alpha}m_Y^{\Omega}(A) = \sum_{B\subseteq\Omega} G(A,B) m_S^{\Omega}(B) \tag{12}$$

with

$$G(A,B) = \begin{cases} 1-\alpha & \text{if } A = B \neq \Omega, \\ \alpha & \text{if } A = \Omega \text{ and } B \subset A, \\ 1 & \text{if } A = B = \Omega \\ 0 & \text{otherwise.} \end{cases} \tag{13}$$

$G(A,B)$ is equal to the fraction on m_S^{Ω} tranferred to A, for each $A \supseteq B$. The whole set of such coefficients define a generalization matrix [10].

3 Contextual discounting

3.1 Basic assumptions

Let us now assume that we have evidence regarding the reliability of S, conditionally on each $\omega_k \in \Omega$. We thus have K conditional bbas $m_Y^{\mathcal{R}}[\{\omega_k\}]$, $k = 1,\ldots,K$, instead of the single unconditional bba in (6). Assume that they are defined as

$$\begin{cases} m_Y^{\mathcal{R}}[\{\omega_k\}](\{R\}) & = \beta_k, \\ m_Y^{\mathcal{R}}[\{\omega_k\}](\{R,NR\}) & = \alpha_k, \end{cases} \tag{14}$$

with $\beta_k = 1-\alpha_k$.

Each of these bbas is conditional to a context ω_k: their combination with m_S^{Ω} will define a *contextual discounting* $m_Y^{\Omega}\left[m_S^{\Omega}, m_Y^{\mathcal{R}}[\{\omega_1\}],\ldots,m_Y^{\mathcal{R}}[\{\omega_K\}]\right]$. As the classical discounting, characterized by a scalar α, is written ${}^{\alpha}m$, the contextual discounting is defined by a vector $(\alpha_1,\ldots,\alpha_K)$, and it will be written ${}^{(\alpha)}m_Y^{\Omega}$.

3.2 Computation of ${}^{(\alpha)}m_Y^{\Omega}$

Ballooning extention and combination of the $m_Y^{\mathcal{R}}[\{\omega_k\}]$ The balloning extension of $m_Y^{\mathcal{R}}[\{\omega_k\}]$ is defined as:

$$m_Y^{\mathcal{R}\Uparrow\Omega\times\mathcal{R}}(\{\omega_k\}\times\{R\}\cup\overline{\{\omega_k\}}\times\mathcal{R})=\beta_k, \tag{15}$$

$$m_Y^{\mathcal{R}\Uparrow\Omega\times\mathcal{R}}(\Omega\times\mathcal{R})=\alpha_k. \tag{16}$$

Let $m_r^{\Omega\times\mathcal{R}}$ be the conjunctive combination of the $m_Y^{\mathcal{R}}[\{\omega_k\}]^{\Uparrow\Omega\times\mathcal{R}}$. Using the following equality, for any $k\neq l$:

$$(\{\omega_k\}\times\{R\}\cup\overline{\{\omega_k\}}\times\mathcal{R})\cap(\{\omega_l\}\times\{R\}\cup\overline{\{\omega_l\}}\times\mathcal{R})=\{\omega_k,\omega_l\}\times\{R\}\cup\overline{\{\omega_k,\omega_l\}}\times\mathcal{R},$$

we easily obtain the expression of $m_r^{\Omega\times\mathcal{R}}$ as:

$$m_r^{\Omega\times\mathcal{R}}(\overline{C}\times\{R\}\cup C\times\mathcal{R})=\begin{cases}\prod_{\omega_i\in C}\alpha_i\prod_{\omega_j\in\overline{C}}\beta_j & \text{if } C\neq\emptyset \text{ and } C\neq\Omega,\\ \prod_{\omega_i\in\Omega}\alpha_i & \text{if } C=\Omega,\\ \prod_{\omega_j\in\Omega}\beta_j & \text{if } C=\emptyset.\end{cases} \tag{17}$$

In the following, we simply note:

$$m_r^{\Omega\times\mathcal{R}}(\overline{C}\times R\cup C\times\mathcal{R})=\prod_{\omega_i\in C}\alpha_i\prod_{\omega_j\in\overline{C}}\beta_j \tag{18}$$

with the convention that a product of terms vanishes when the index set is empty.

It can be checked that the initial conditional bbas are retrieved by conditioning $m_r^{\Omega\times\mathcal{R}}$ on each ω_k:

$$m_r^{\Omega\times\mathcal{R}}[\{\omega_k\}]=\beta_k=m_Y^{\mathcal{R}}[\{\omega_k\}],\quad k=1,\ldots,K. \tag{19}$$

Combination with m_S^{Ω} The contextual discounting can be obtained from the bbas $m_Y^{\Omega}[\{R\}]^{\Uparrow\Omega\times\mathcal{R}}$ and $m_r^{\Omega\times\mathcal{R}}$:

$${}^{(\alpha)}m_Y^{\Omega}=\left(m_Y^{\Omega}[\{R\}]^{\Uparrow\Omega\times\mathcal{R}}\textcircled{\cap}\, m_r^{\Omega\times\mathcal{R}}\right)^{\downarrow\Omega} \tag{20}$$

The bbas $m_Y^{\Omega}[\{R\}]^{\Uparrow\Omega\times\mathcal{R}}$ and $m_r^{\Omega\times\mathcal{R}}$ have focal sets of the form $B\times\{R\}\cup\Omega\times\{NR\}$ and $\overline{C}\times\{R\}\cup C\times\mathcal{R}$, respectively, with $B,C\subseteq\Omega$. The intersection of two such focals sets is:

$$(\overline{C}\times\{R\}\cup C\times\mathcal{R})\cap(B\times\{R\}\cup\Omega\times\{NR\})=B\times\{R\}\cup C\times\{NR\},$$

and it can be obtained only for a particular choice of B and C. Then:

$$m_Y^{\Uparrow\Omega\times\mathcal{R}}\textcircled{\cap}\, m_r^{\Omega\times\mathcal{R}}(B\times\{R\}\cup C\times\{NR\})=\left[\prod_{\omega_i\in C}\alpha_i\prod_{\omega_j\in\overline{C}}\beta_j\right]m_S^{\Omega}(B). \tag{21}$$

Marginalizing this bba on Ω gives:

$$^{(\alpha)}m^{\Omega}(A) = \sum_{B \cup C = A} \left[\prod_{\omega_i \in C} \alpha_i \prod_{\omega_j \in \overline{C}} \beta_j \right] m_S^{\Omega}(B), \ \forall A \subseteq \Omega, \tag{22}$$

which can also be written as:

$$^{(\alpha)}m^{\Omega}(A) = \sum_{B \subseteq A} G(A,B) m_S^{\Omega}(B), \ \forall A \subseteq \Omega, \tag{23}$$

with:

$$G(A,B) = \sum_{C: B \cup C = A} \prod_{\omega_i \in C} \alpha_i \prod_{\omega_j \in \overline{C}} \beta_j \ , \ \forall B \subseteq A \subseteq \Omega. \tag{24}$$

Coefficients $G(A,B)$ for all $A, B \subseteq \Omega$ define a generalization matrix [10]: $G(A,B)$ is equal to the fraction of $m_S^{\Omega}(B)$ transferred to $^{(\alpha)}m^{\Omega}(A)$, for $A \supseteq B$.

Proposition 1. *A simpler form of the generalization matrix in (24) is*

$$G(A,B) = \prod_{\omega_i \in A \setminus B} \alpha_i \prod_{\omega_j \in \overline{A}} \beta_j \ , \ \forall B \subseteq A \subseteq \Omega, \tag{25}$$

Proof: We have

$$B \cup C = A \Leftrightarrow \exists D \subseteq B : C = A \setminus B \cup D \Leftrightarrow \exists D \subseteq B : \overline{C} = \overline{A} \cup B \setminus D,$$

and therefore:

$$\begin{aligned} G(A,B) &= \sum_{D \subseteq B} \prod_{\omega_i \in A \setminus B \cup D} \alpha_i \prod_{\omega_j \in \overline{A} \cup B \setminus D} \beta_j \\ &= \prod_{\omega_i \in A \setminus B} \alpha_i \prod_{\omega_j \in \overline{A}} \beta_j \underbrace{\sum_{D \subseteq B} \prod_{\omega_i \in B \setminus D} \beta_i \prod_{\omega_j \in D} \alpha_j}_{=1} . \end{aligned}$$

Remark 4. It can be seen from Equation (22) that $^{(\alpha)}m^{\Omega}$ is the disjunctive combination of m_S^{Ω} with a bbm m_0^{Ω} defined by $m_0^{\Omega}(C) = \prod_{\omega_i \in C} \alpha_i \prod_{\omega_j \in \overline{C}} \beta_j$, for all $C \subseteq \Omega$.

Remark 5. Contextual discounting as defined in this section does *not* generalize the classical discounting recalled in Section 2.4. In particular, the solution obtained by discounting m_S^{Ω} with rates $\alpha_i = \alpha$, $i = 1, \ldots, K$ is different, in general, from the one obtained using the classical discounting operation with a single rate α. Both classical and contextual discounting appear in fact to be two instances of a more general concept, which is introduced in the next section.

3.3 Θ-contextual discounting

Contextual discounting defined above may be generalized by assuming that the available evidence allows to assess the reliability of S in more general contexts $\theta_l \subseteq \Omega$, $l = 1, \ldots, L$, where $\theta_1, \ldots, \theta_L$ form a partition of Ω. The set $\Theta = \{\theta_1, \ldots, \theta_L\}$ then constitutes a coarsening of Ω.

In such a case, information regarding the reliability of the source takes the form of L conditional bbas

$$\begin{cases} m_Y^{\mathcal{R}}[\theta_l](\{R\}) &= \beta_l, \\ m_Y^{\mathcal{R}}[\theta_l](\{R, NR\}) &= \alpha_l, \quad l = 1, \ldots, L. \end{cases} \tag{26}$$

A similar line of reasoning as performed in Section 3.2 yields

$$m_r^{\Omega \times \mathcal{R}}(\overline{C} \times R \cup C \times \mathcal{R}) = \prod_{\theta_i : \cup_i \theta_i = C} \alpha_i \prod_{\theta_j : \cup_j \theta_j = \overline{C}} \beta_j, \tag{27}$$

which is the equivalent of (18) in the previous case, but where C now ranges in the set $\mathcal{C}$ of subsets of Ω which are the union of some θ_i's:

$$C \in \mathcal{C} = \{A \subseteq \Omega \mid \exists I \subseteq \{1, \ldots, L\}, A = \bigcup_{i \in I} \theta_i\}.$$

After marginalizing on Ω, we finally obtain:

$$\begin{aligned} {}^{(\alpha)}m^{\Omega}(A) &= \sum_{B \cup C = A} m_r^{\Omega \times \mathcal{R}}(\overline{C} \times R \cup C \times \mathcal{R}) m_S^{\Omega}(B), \ \forall A \subseteq \Omega \\ &= \sum_{B \cup C = A} \left[\prod_{\theta_i : \cup_i \theta_i = C} \alpha_i \prod_{\theta_j : \cup_j \theta_j = \overline{C}} \beta_j \right] m_S^{\Omega}(B), \ \forall A \subseteq \Omega \\ &= \sum_{B \subseteq A} G(A, B) m_S^{\Omega}(B), \ \forall A \subseteq \Omega, \end{aligned} \tag{28}$$

where $G(A, B)$ denote again the coefficients of the generalization matrix associated with the contextual discounting:

$$G(A, B) = \sum_{B \cup C = A} \prod_{\theta_i : \cup_i \theta_i = C} \alpha_i \prod_{\theta_j : \cup_j \theta_j = \overline{C}} \beta_j \ , \ \forall B \subseteq A \subseteq \Omega. \tag{29}$$

The operation defined by Equation (28) will be called Θ-contextual discounting, with discount rates $\alpha_1, \ldots, \alpha_L$. The contextual discounting defined in Section 3.2 corresponds to the special case where $\theta_i = \{\omega_i\}$, $i = 1, \ldots, L$. It will be called Ω-contextual discounting for short.

Remark 6. As before, it can be seen from (28) that ${}^{(\alpha)}m^{\Omega}$ is the disjunctive combination of m_S^{Ω} with a bba m_0^{Ω} defined by $m_0^{\Omega}(C) = \prod_{\theta_i : \cup_i \theta_i = C} \alpha_i \prod_{\theta_j : \cup_j \theta_j = \overline{C}} \beta_j$ if $C \in \mathcal{C}$, and $m_0^{\Omega}(C) = 0$ otherwise.

Remark 7. Assume that Θ is composed of a single element $\theta = \Omega$. Then, from Remark 6, ${}^{(\alpha)}m^{\Omega}$ is the disjunctive combination of m_S^{Ω} with a bba m_0^{Ω} defined by $m_0^{\Omega}(\emptyset) = 1 - \alpha$ and $m_0^{\Omega}(\Omega) = \alpha$. Hence, from Remark 2, ${}^{(\alpha)}m^{\Omega}$ is equal to the classical discounting of m_S^{Ω}: classical discounting is thus Θ-contextual discounting with $\Theta = \{\Omega\}$.

Remark 8. It can be shown that the same results are obtained if knowledge about the reliability of S is expressed as

$$\begin{cases} m_Y^{\mathcal{R}}[\theta_k](\{R\}) &= \beta_k, \\ m_Y^{\mathcal{R}}[\theta_k](\{NR\}) &= \alpha_k. \end{cases} \tag{30}$$

4 Examples

Example 1. Let $\Omega = \{\omega_1, \omega_2, \omega_3\}$, un m a bba on Ω. The Ω-contextual discounting of m with rates $(\alpha) = (\alpha_1, \alpha_2, \alpha_3)$ yields

$$\begin{aligned}
{}^{(\alpha)}m(\emptyset) &= \beta_1\beta_2\beta_3)m(\emptyset) \\
{}^{(\alpha)}m(\{\omega_1\}) &= \beta_2\beta_3[m(\omega_1) + \alpha_1 m(\emptyset)] \\
{}^{(\alpha)}m(\{\omega_2\}) &= \beta_1\beta_3[m(\omega_2) + \alpha_2 m(\emptyset)] \\
{}^{(\alpha)}m(\{\omega_3\}) &= \beta_1\beta_2[m(\omega_3) + \alpha_3 m(\emptyset)] \\
{}^{(\alpha)}m(\{\omega_1,\omega_2\}) &= \beta_3[m(\{\omega_1,\omega_2\}) + \alpha_1 m(\{\omega_2\}) + \alpha_2 m(\{\omega_1\}) + \alpha_1\alpha_2 m(\emptyset)] \\
{}^{(\alpha)}m(\{\omega_1,\omega_3\}) &= \beta_2[m(\{\omega_1,\omega_3\}) + \alpha_1 m(\{\omega_3\}) + \alpha_3 m(\{\omega_1\}) + \alpha_1\alpha_3 m(\emptyset)] \\
{}^{(\alpha)}m(\{\omega_2,\omega_3\}) &= \beta_1[m(\{\omega_2,\omega_3\}) + \alpha_2 m(\{\omega_3\}) + \alpha_3 m(\{\omega_2\}) + \alpha_2\alpha_3 m(\emptyset)] \\
{}^{(\alpha)}m(\Omega) &= m(\Omega) + \alpha_1 m(\{\omega_2,\omega_3\}) + \alpha_2 m(\{\omega_1,\omega_3\}) + \alpha_3 m(\{\omega_1,\omega_2\}) \\
&\quad + \alpha_1\alpha_2 m(\{\omega_3\}) + \alpha_2\alpha_3 m(\{\omega_1\}) + \alpha_1\alpha_3 m(\{\omega_2\}) + \\
&\quad \alpha_1\alpha_2\alpha_3 m(\emptyset).
\end{aligned}$$

The corresponding generalization matrix is show in Table 1.

Table 1. Generalization matrix associated to the Ω-contextual discounting of m.

	$\emptyset$	$\{\omega_1\}$	$\{\omega_2\}$	$\{\omega_1,\omega_2\}$	$\{\omega_3\}$	$\{\omega_1,\omega_3\}$	$\{\omega_2,\omega_3\}$	$\{\omega_1,\omega_2,\omega_3\}$
$\emptyset$	$\beta_1\beta_2\beta_3$							
$\{\omega_1\}$	$\alpha_1\beta_2\beta_3$	$\beta_2\beta_3$						
$\{\omega_2\}$	$\beta_1\alpha_2\beta_3$		$\beta_1\beta_3$					
$\{\omega_1,\omega_2\}$	$\alpha_1\alpha_2\beta_3$	$\alpha_2\beta_3$	$\alpha_1\beta_3$	β_3				
$\{\omega_3\}$	$\beta_1\beta_2\alpha_3$				$\beta_1\beta_2$			
$\{\omega_1,\omega_3\}$	$\alpha_1\beta_2\alpha_3$	$\beta_2\alpha_3$			$\alpha_1\beta_2$	β_2		
$\{\omega_2,\omega_3\}$	$\beta_1\alpha_2\alpha_3$		$\beta_1\alpha_3$		$\beta_1\alpha_2$		β_1	
$\{\omega_1,\omega_2,\omega_3\}$	$\alpha_1\alpha_2\alpha_3$	$\alpha_2\alpha_3$	$\alpha_1\alpha_3$	α_3	$\alpha_1\alpha_2$	α_2	α_1	1

With $\alpha_1 = 1$, $\alpha_2 = \alpha_3 = 0$, we obtain:

$$
\begin{aligned}
{}^{(\alpha)}m(\emptyset) = {}^{(\alpha)}m(\{\omega_2\}) = {}^{(\alpha)}m(\{\omega_3\}) = {}^{(\alpha)}m(\{\omega_2,\omega_3\}) &= 0 \\
{}^{(\alpha)}m(\{\omega_1\}) &= m(\omega_1) + m(\emptyset) \\
{}^{(\alpha)}m(\{\omega_1,\omega_2\}) &= m(\{\omega_1,\omega_2\}) + m(\{\omega_2\}) \\
{}^{(\alpha)}m(\{\omega_1,\omega_3\}) &= m(\{\omega_1,\omega_3\}) + m(\{\omega_3\}) \\
{}^{(\alpha)}m(\{\omega_1,\omega_2,\omega_3\}) &= m(\{\omega_1,\omega_2,\omega_3\}) + m(\{\omega_2,\omega_3\}).
\end{aligned}
$$

The belief given to $\{\omega_1\}$ is unchanges (the others elements are perfectly recognized). The source being reliable when identifying $\{\omega_2\}$ and $\{\omega_3\}$, the belief given to each element A containing those latter is transferred on $A \cup \{\omega_1\}$: the ability of the source to recognize this element is indeed unknown.

Example 2. Consider now the Θ-contextual discounting of m from the previous example, for $\Theta = \{\theta_1, \theta_2\}$ with $\theta_1 = \{\omega_1\}$, $\theta_2 = \{\omega_2, \omega_3\}$, associated with α_1 and α_2 respectively. The generalization matrix is shown in Table 2.

Table 2. Generalization matrix associated to the Θ-contextual discounting of m, with $\Theta = \{\{\omega_1\}, \{\omega_2, \omega_3\}\}$.

	$\emptyset$	$\{\omega_1\}$	$\{\omega_2\}$	$\{\omega_1,\omega_2\}$	$\{\omega_3\}$	$\{\omega_1,\omega_3\}$	$\{\omega_2,\omega_3\}$	$\{\omega_1,\omega_2,\omega_3\}$
$\emptyset$	$\beta_1\beta_2$							
$\{\omega_1\}$	$\alpha_1\beta_2$	β_2						
$\{\omega_2\}$			$\beta_1\beta_2$					
$\{\omega_1,\omega_2\}$			$\alpha_1\beta_2$	β_2				
$\{\omega_3\}$					$\beta_1\beta_2$			
$\{\omega_1,\omega_3\}$					$\alpha_1\beta_2$	β_2		
$\{\omega_2,\omega_3\}$	$\beta_1\alpha_2$		$\beta_1\alpha_2$		$\beta_1\alpha_2$		β_1	
$\{\omega_1,\omega_2,\omega_3\}$	$\alpha_1\alpha_2$	α_2	$\alpha_1\alpha_2$	α_2	$\alpha_1\alpha_2$	α_2	α_1	1

Remark that, with $\alpha_1 = 1$, $\alpha_2 = 0$, the result is the same as the one obtained previously, which is not true in the general case.

5 Conclusion

We defined in this article a contextual discounting. This concept allows to model accurately the reliability of a source; it is shown to generalize the classical discounting introduced by Shafer [6]. It seems to provide an adequate tool to tackle, e.g., sensor fusion applications, in which the reliability of sensors depends on the context.

It seems interesting to learn the reliability of the source from a training set, instead of having it assessed by an expert. In the case of classical discounting, an approach has already been proposed in [4], where the discounting coefficients α

for each source are computed such that they minimize a measure of discrepency between observations and sensor outputs. In the case of the contextual discounting, both the partition Θ of Ω and the set of coefficients have to be determined. This is left for future research.

References

1. Appriou, A.: Uncertain data aggregation in classification and tracking. In B. Bouchon-Meunier, editor, Aggregation and Fusion of imperfect information (1998) 231-260.
2. Bloch, I.: Fusion d'informations en traitement du signal et des images. Hermès (2003).
3. Dubois, D., Prade, H.: Possibility Theory. Plenum Press, New-York (1988).
4. Elouedi, Z., Mellouli, K., Smets, Ph.: Assessing sensor reliability for multisensor data fusion with the transferable belief model. IEEE Transactions on Systems, Man and Cybernetics **B34** (2004) 782-787.
5. Milisavljevic, N., Bloch, I., van den Broek, S., Acheroy, M.: Improving mine recognition through processing and Dempster-Shafer fusion of ground-penetrating radar data. Pattern Recognition **36** (2003) 1233-1250.
6. Shafer, G.: A mathematical theory of evidence. Princeton University Press (1976).
7. Smets, Ph.: Belief functions: the disjunctive rule of combination and the generalized bayesian theorem. International Journal of Approximate Reasoning **9** (1993) 1–35.
8. Smets, Ph.: The transferable belief model for quantified belief representation. Handbook of Defeasible Reasoning and Uncertainty Management Systems **1** (1998) 267–301.
9. Smets, Ph.: Data fusion in the transferable belief model. Proceedings of the 3rd International Conference on Information Fusion, FUSION 2000 (2000) 21–33.
10. Smets, Ph.: The application of the matrix calculus to belief functions. International Journal of Approximate Reasoning **31** (2002) 1–30.
11. Smets, Ph., Kennes, R.: The transferable belief model. Artificial Intelligence **66** (1994) 191–234.