A parameterized family of belief functions correction mechanisms.

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Abstract

To correct the information, represented by a belief function, provided by a source, different tools can be used such as discounting, de-discounting, extended discounting. In this paper, the links between these operations are explored. A new interpretation of these schemes, as well as a parameterized family of transformations encompassing all previous schemes is introduced and justified. A postal application illustrates the benefits obtained by the use of one of these new correction mechanisms.

Keywords: Dempster-Shafer theory, correction mechanisms of belief functions, discounting.

1 Introduction

Introduced by Dempster and Shafer [11], belief functions constitute one of the main frameworks for reasoning with imperfect information.

When receiving a piece of information represented by a belief function, an agent can hold some metaknowledge regarding the reliability of the source which provides the information. To correct the information according to this metaknowledge, different tools can be used:

- the *discounting operation*, introduced by Shafer in his seminal book [11], allows one to weaken information.
- the *de-discounting operation*, introduced by Denœux and Smets [2], allows one to strengthen information.
- the *extended discounting operation*, introduced by Zhu and Basir [16], allows one to weaken, strengthen or contradict information.

In this paper, the links between these operations are explored. A new interpretation of these schemes, as well as a parameterized family of transformations encompassing all previous schemes is introduced and justified. This family includes all possible transformations, expressed by a belief function, regarding the states in which the source can be when the information is supplied. This paper extends and reexpresses some results, limited to the aspects of reinforcement and discounting, introduced by the authors in [9].

Belief functions are used in different models, for instance, models based on lower and upper probabilities, like Dempster's model [1] or Hints model [6], the random sets theory [5], or the transferable belief model (TBM) developed by Smets [13, 15]. In the TBM, belief functions are interpretated as weighted opinions of an agent or a sensor. This model is adopted in this paper.

This paper is organized as follows. Background material on belief functions is recalled in Section 2. Correction mechanisms are presented in Section 3. A new interpretation of these schemes as well as a parameterized family of correction mechanisms is introduced and

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justified in Section 4. An application example, which aims at fusing decisions associated with confidence scores, is described in Section 5. Finally, Section 6 concludes this paper.

2 TBM: basic concepts

Let $\Omega = \{\omega_1, \ldots, \omega_K\}$, called the *frame of discernment*, be a finite set of possible answers to a given question Q. Information held by an agent Ag regarding the answer to question Q, given evidence EC, can be quantified by a *basic belief assignment (bba)* $m^{\Omega}_{Ag}[EC]$, defined as a function from 2^{Ω} to $[0,1]$, and verifying:

$$\sum_{A\subseteq\Omega} m^{\Omega}_{Ag}[EC](A) = 1 \ . \qquad (1)$$

When there is no ambiguity, the full notation $m^{\Omega}_{Ag}[EC]$ will be simplified to m^{Ω}_{Ag}, m^{Ω}, or even m.

The quantity $m^{\Omega}(A)$ represents the part of the unit mass allocated to the hypothesis that the answer to question Q is in the subset A of Ω.

A subset A of Ω such that $m(A) > 0$ is called a *focal element of* m. A bba m with only one focal element A is called a *categorical belief function* and is denoted m_A, then $m_A(A) = 1$. Total ignorance is represented by the bba m_{Ω}, called the *vacuous belief function*. A bba m such that $m(\emptyset) = 0$ is said to be normal.

Two bbas m_1 and m_2, induced by distinct and reliable sources of information, can be combined using the *conjunctive rule of combination* (CRC), also called *unnormalized Dempster's rule of combination*, defined for all $A \subseteq \Omega$ by:

$$m_1 \textcircled{\cap} m_2(A) = \sum_{B\cap C=A} m_1(B)m_2(C) \ . \qquad (2)$$

Marginalization and vacuous extension on a product space A bba defined on a product space $\Omega\times\Theta$ may be *marginalized* on Ω, by transferring each mass $m^{\Omega\times\Theta}(B)$ for $B \subseteq \Omega\times\Theta$ to its projection on Ω:

$$m^{\Omega\times\Theta\downarrow\Omega}(A) \ = \sum_{\substack{B\subseteq\Omega\times\Theta,\\ \mathrm{Proj}(B\downarrow\Omega)=A}} m^{\Omega\times\Theta}(B), \quad (3)$$

for all $A \subseteq \Omega$ where $\mathrm{Proj}(B \downarrow \Omega)$ denotes the projection of B onto Ω.

It is usually not possible to retrieve the original bba $m^{\Omega\times\Theta}$ from its marginal $m^{\Omega\times\Theta\downarrow\Omega}$ on Ω. However, the *least committed*, or *least informative bba* [12] such that its projection on Ω is $m^{\Omega\times\Theta\downarrow\Omega}$ may be computed. This defines the *vacuous extension* of m^{Ω} in the product space $\Omega\times\Theta$ [12], noted $m^{\Omega\uparrow\Omega\times\Theta}$, and given by:

$$m^{\Omega\uparrow\Omega\times\Theta}(B) = \begin{cases} m^{\Omega}(A) & \text{if } B = A\times\Theta, \\ & A\subseteq\Omega \\ 0 & \text{otherwise.} \end{cases} \qquad (4)$$

Conditioning and ballooning extension on a product space Conditional beliefs represent knowledge which is valid provided that an hypothesis is satisfied. Let m be a bba and $B \subseteq \Omega$ an hypothesis; the *conditional belief function* $m[B]$ is given by:

$$m[B] = m \textcircled{\cap} m_B. \qquad (5)$$

If $m^{\Omega\times\Theta}$ is defined on the product space $\Omega\times\Theta$, and θ is a subset of Θ, the conditional bba $m^{\Omega}[\theta]$ is defined by combining $m^{\Omega\times\Theta}$ with $m_{\theta}^{\Theta\uparrow\Omega\times\Theta}$, and marginalizing the result on Ω:

$$m^{\Omega}[\theta] = \left(m^{\Omega\times\Theta} \textcircled{\cap} m_{\theta}^{\Theta\uparrow\Omega\times\Theta}\right)^{\downarrow\Omega} \qquad (6)$$

Assume now that $m^{\Omega}[\theta]$ represents the agent's beliefs on Ω conditionally on θ, which means in a context where θ holds. There are usually many bbas on $\Omega\times\Theta$, whose conditioning on θ yields $m^{\Omega}[\theta]$. Among these, the least committed one is defined for all $A \subseteq \Omega$ by:

$$m^{\Omega}[\theta]^{\Uparrow\Omega\times\Theta}(A\times\theta \cup \Omega\times\overline{\theta}) = m^{\Omega}[\theta](A). \ (7)$$

This operation is referred to as the *deconditioning* or *ballooning extension* [12] of $m^{\Omega}[\theta]$ on $\Omega\times\Theta$.

3 Correction mechanisms

3.1 Discounting

When receiving a piece of information, represented by a bba m, agent Ag can have some doubt regarding the reliability of the source which has provided this bba. Such a meta-knowledge can be taken into account by using the discounting operation introduced by Shafer [11, page 252], and defined by:

$$^{\alpha}m = (1-\alpha)m + \alpha\, m_{\Omega}\ , \tag{8}$$

where $\alpha \in [0,1]$.

A discount rate α equal to 1, means that the source is not reliable and the piece of information it provides cannot be taken into account, so Ag's knowledge remains vacuous: $m^{\Omega}_{Ag} = {}^{1}m = m_{\Omega}$. On the contrary, a null discount rate indicates that the source is fully reliable and the piece of information it provides is entirely accepted: $m^{\Omega}_{Ag} = {}^{0}m = m$. In practice however, agent Ag does not know for sure whether the source is reliable or not, but he has some degree of belief expressed by:

$$\begin{cases} m^{\mathcal{R}}_{Ag}(\{R\}) & = & 1-\alpha \\ m^{\mathcal{R}}_{Ag}(\mathcal{R}) & = & \alpha, \end{cases} \tag{9}$$

where $\mathcal{R} = \{R, NR\}$, R standing for *"the source is reliable"*, and NR standing for *"the source is not reliable"*. This formalization yields to the expression (8), as demonstrated by Smets in [12].

3.2 De-Discounting

In this process, agent Ag receives a bba $^{\alpha}m$ from a source S, different from m_{Ω} and discounted with a discount rate $\alpha < 1$. If Ag knows α, then it can recompute m by reversing the discounting operation (8):

$$m_{Ag} = m = \frac{^{\alpha}m - \alpha\, m_{\Omega}}{1-\alpha}\ . \tag{10}$$

This procedure is called *de-discounting* by Denœux and Smets in [2].

If the agent receives a bba m discounted with an unknown discount rate α, agent Ag can imagine all possible values in the range $[0, m(\Omega)]$. Indeed, as shown in [2], $m(\Omega)$ is the largest value for α such that the de-discounting operation (10) leads to a bba. De-discounting m with this maximal value is called *maximal de-discounting.* The result is the *totally reinforced belief function*, noted ^{tr}m and defined as follows:

$$^{tr}m(A) = \begin{cases} \frac{m(A)}{1-m(\Omega)} & \forall A \subset \Omega, \\ 0 & \text{otherwise.} \end{cases} \tag{11}$$

The bba ^{tr}m is obtained from m by redistributing the mass $m(\Omega)$ totally and uniformly on focal elements of m.

3.3 Extended Discounting Scheme

In [16], Zhu and Basir have proposed to extend the discounting process, in order to *strengthen, discount or contradict* belief functions.

The extended discounting scheme is composed of two transformations.

The first transformation, allowing one to strengthen or weaken a source of information, is introduced by retaining the discounting equation (8), and allowing the discount rate α to be in the range $[-m(\Omega)/(1-m(\Omega)), 1]$.

- If $\alpha \in [0,1]$, this transformation is the discounting operation.
- If $\alpha \in [-m(\Omega)/(1-m(\Omega)), 0]$, this transformation is equivalent to the de-discounting equation (10) with the reparameterization $\alpha = \frac{-\alpha'}{1-\alpha'}$ with $\alpha' \in [0, m(\Omega)]$. Indeed:

$$\begin{aligned} ^{\alpha}m &= (1 - \tfrac{-\alpha'}{1-\alpha'})m + \tfrac{-\alpha'}{1-\alpha'}m_{\Omega} \\ &= \tfrac{m-\alpha' m_{\Omega}}{1-\alpha'}\ . \end{aligned} \tag{12}$$

The second transformation, allowing one to contradict a non-vacuous and normal belief function m, is defined by the following equation:

$$\begin{cases} ^{\alpha}m(\overline{A}) = (\alpha-1)m(A) & \text{if } A \subset \Omega, \\ ^{\alpha}m(\Omega) = (\alpha-1)m(\Omega) + 2 - \alpha & \text{otherwise,} \end{cases} \tag{13}$$

where $\alpha \in [1, 1 + \frac{1}{1-m(\Omega)}]$.

- If $\alpha = 1$, $^{\alpha}m = m_{\Omega}$.
- If $\alpha = 1 + \frac{1}{1-m(\Omega)}$, $^{\alpha}m = \overline{^{tr}m}$, where $\overline{m}$ denotes the negation of m [3], defined by

$\overline{m}(A) = m(\overline{A})$, $\forall A \subseteq \Omega$. In other words, after being totally reinforced, each basic belief mass $m(A)$ is transferred to its complementary. The bba m is then fully contradicted.

This scheme has been successfully applied in medical imaging [16]. However, it suffers from a lack of formal justification. Indeed, the number $(1-\alpha)$ can no longer be interpreted as a degree of belief as it can take values greater than 1 and lesser than 0.

In the following section, a new parameterized family of transformations encompassing all the schemes presented in this section, is introduced and justified.

4 A parameterized family of correction mechanisms

In the discounting operation (8), agent Ag considers that the source can be in two states: reliable or not reliable. These states can be interpreted as follows:

- if the source is reliable (state R), the information m it provides becomes Ag's knowledge. Formally, $m_{Ag}[\{R\}] = m$,
- if the source is not reliable (state NR), the information m it provides is discarded, and Ag's knowledge is vacuous: $m_{Ag}[\{NR\}] = m_\Omega$.

In this section, these hypotheses are extended in the following way. We assume that the source can be in N states R_i, $i \in [\![1, N]\!]$, whose interpretations are given by transformations m_i of m: if the source is in the state R_i then $m^\Omega_{Ag}[\{R_i\}] = m_i$.

Let $\mathcal{R} = \{R_1, \dots, R_N\}$, and let us supposed that $m^{\mathcal{R}}_{Ag}(\{R_i\}) = \nu_i$, $\forall i \in [\![1, N]\!]$, with $\sum_{i=1}^N \nu_i = 1$.

The knowledge held by agent Ag, when receiving an information m^Ω_S of a source S and possessing a metaknowledge $m^{\mathcal{R}}_{Ag}$ regarding the different states in which the source can be, can then be computed by:

- deconditioning the $m^\Omega_{Ag}[\{R_i\}]$ on the product space $\Omega \times \mathcal{R}$ using (7);
- extending vacuously $m^{\mathcal{R}}_{Ag}$ on the same product space $\Omega \times \mathcal{R}$ using (4);
- combining them using the CRC (2);
- marginalizing the result on Ω using (3).

Formally:

$$m^\Omega_{Ag}[m^\Omega_S, m^{\mathcal{R}}_{Ag}] = \left(\textcircled{\cap}_{i=1}^N m^\Omega_{Ag}[\{R_i\}]^{\Uparrow \Omega\times\mathcal{R}} \textcircled{\cap} m^{\mathcal{R}\uparrow\Omega\times\mathcal{R}}_{Ag}\right)^{\downarrow\Omega} . \quad (14)$$

Proposition 1 *The bba m^Ω_{Ag}, resulting from equation (14), only depends on m_i and ν_i, $i \in [\![1, N]\!]$. The result is noted $^{\boldsymbol{\nu}}m$, $\boldsymbol{\nu}$ denoting the vector of ν_i, and verifies:*

$$m^\Omega_{Ag} = {}^{\boldsymbol{\nu}}m = \sum_{i=1}^N \nu_i \, m_i \ . \quad (15)$$

Proof 1 $\forall i \in [\![1, N]\!]$ *and* $\forall A \subseteq \Omega$*:*

$$m^{\mathcal{R}\uparrow\Omega\times\mathcal{R}}_{Ag}(\Omega \times \{R_i\}) = \nu_i \ , \quad (16)$$

and

$$m^\Omega_{Ag}[R_i]^{\Uparrow\Omega\times\mathcal{R}}(A \times \{R_i\} \cup \Omega \times \overline{\{R_i\}}) = m_i(A) \ . \quad (17)$$

Moreover, $\forall i \in [\![1, N]\!]$ *and* $\forall A_i \subseteq \Omega$*:*

$$\cap_{i=1}^N \left(A_i \times \{R_i\} \cup \Omega \times \overline{\{R_i\}}\right) = \cup_{i=1}^N A_i \times \{R_i\} \ , \quad (18)$$

and, $\forall j \in [\![1, N]\!]$*:*

$$(\cup_{i=1}^N A_i \times \{R_i\}) \cap \Omega \times \{R_j\} = A_j \times \{R_j\} \ . \quad (19)$$

Therefore, the conjunctive combination of $m^\Omega_{Ag}[R_i]^{\Uparrow\Omega\times\mathcal{R}}$, $i \in [\![1, N]\!]$, with $m^{\mathcal{R}\uparrow\Omega\times\mathcal{R}}_{Ag}$, noted $\textcircled{\cap} m^{\Omega\times\mathcal{R}}_{Ag}$, has N focal elements such that:

$$\textcircled{\cap} m^{\Omega\times\mathcal{R}}_{Ag}(A_j \times \{R_j\}) = \nu_j \ m_j(A_j) \prod_{i\neq j} \underbrace{\sum_{A\subseteq\Omega} m_i(A)}_{=1}, \quad \forall j \in [\![1, N]\!] \ , \quad (20)$$

or, equivalently, $\forall A \subseteq \Omega$ *and* $\forall i \in [\![1, N]\!]$*:*

$$\circledcirc m_{Ag}^{\Omega\times\mathcal{R}}(A \times \{R_i\}) = \nu_i \; m_i(A). \qquad (21)$$

Then, after projection onto Ω*:*

$$m_{Ag}^{\Omega}(A) = \sum_{i=1}^{N} \nu_i \; m_i(A) \quad \forall A \subseteq \Omega. \qquad (22)$$

□

Correction mechanisms described in Section 3 are particular cases of correction mechanisms expressed by (15).

The discounting operation corresponds to the case of two states R_1 and R_2 such that $m_1 = m_\Omega$ and $m_2 = m$.

The de-discounting operation corresponds to the case of two states such that $m_1 = m$ and $m_2 = {}^{tr}m$, which means a first state where the information provided by the source is accepted, and a second state where this information is totally reinforced. With these hypotheses, ${}^{\boldsymbol{\nu}}m = \nu_1 m + \nu_2{}^{tr}m$ is a reparameterization of the de-discounting operation (10) with $\nu_1 = \frac{m(\Omega)-\alpha}{(1-\alpha)m(\Omega)}$, $\alpha \in [0, m(\Omega)]$. Indeed:

$$\begin{aligned}
&{}^{\boldsymbol{\nu}}m(A)\\
&= \tfrac{m(\Omega)-\alpha}{(1-\alpha)m(\Omega)}\; m(A) + (1 - \tfrac{m(\Omega)-\alpha}{(1-\alpha)m(\Omega)})\tfrac{m(A)}{1-m(\Omega)}\\
&= \tfrac{m(\Omega)-\alpha}{(1-\alpha)m(\Omega)}\; m(A) + \tfrac{(1-\alpha)m(\Omega)-m(\Omega)+\alpha}{(1-\alpha)m(\Omega)}\tfrac{m(A)}{1-m(\Omega)}\\
&= \tfrac{m(\Omega)-\alpha}{(1-\alpha)m(\Omega)}\; m(A) + \tfrac{\alpha\,(1-m(\Omega))}{(1-\alpha)m(\Omega)}\tfrac{m(A)}{1-m(\Omega)}\\
&= \tfrac{m(A)}{1-\alpha} \quad \forall A \subset \Omega, \text{ and}\\
&{}^{\boldsymbol{\nu}}m(\Omega)\\
&= \tfrac{m(\Omega)-\alpha}{(1-\alpha)m(\Omega)}\; m(\Omega) = \tfrac{m(\Omega)-\alpha}{1-\alpha}.
\end{aligned}$$

The first transformation of the extended discounting operation, equation (8) with $\alpha \in [-m(\Omega)/(1 - m(\Omega)), 1]$, concerning the discounting or reinforcement of the source, is a reparameterization of (15) in the particular case of two states such that $m_1 = m_\Omega$ and $m_2 = {}^{tr}m$ with $\nu_1 = (1 - \alpha)m(\Omega) + \alpha$. Indeed:

$$\begin{cases}
{}^{\boldsymbol{\nu}}m(A) &= (1-(1-\alpha)m(\Omega)-\alpha)\frac{m(A)}{1-m(\Omega)}\\
&= (1-\alpha)\; m(A) \quad \forall A \subset \Omega,\\
{}^{\boldsymbol{\nu}}m(\Omega) &= (1-\alpha)m(\Omega)+\alpha.
\end{cases}$$

At last, the second transformation of the extended discounting operation (13), allowing one to contradict a source, is also a reparameterization of (15) by considering two states such that $m_1 = \overline{{}^{tr}m}$ and $m_2 = m_\Omega$, and setting $\nu_1 = (\alpha - 1)(1 - m(\Omega))$ with $\alpha \in [1, 1 + \frac{1}{1-m(\Omega)}]$:

$$\begin{cases}
{}^{\boldsymbol{\nu}}m(\overline{A}) &= (\alpha-1)(1-m(\Omega))\frac{m(A)}{1-m(\Omega)}\\
&= (\alpha-1)\; m(A) \quad \forall A \subset \Omega,\\
{}^{\boldsymbol{\nu}}m(\Omega) &= 1-(\alpha-1)(1-m(\Omega))\\
&= 1-\alpha+\alpha\, m(\Omega)+1-m(\Omega)\\
&= (\alpha-1)m(\Omega)+2-\alpha.
\end{cases}$$

The source can be in two states: a first one where the source is considered as lying (so the contrary is correct) [14], and a second one where the information provided by the source is rejected.

All these results are recapitulated in Table 1.

Table 1: Models yielding to the correction mechanisms presented in Section 3.

Interpretations		**Operation**
$m_1 = m_\Omega$	$m_2 = m$	discounting
$m_1 = m$	$m_2 = {}^{tr}m$	de-discounting
$m_1 = m_\Omega$	$m_2 = {}^{tr}m$	extended disc. (1)
$m_1 = \overline{{}^{tr}m}$	$m_2 = m_\Omega$	extended disc. (2)

Remark 1 *The first transformation of the extended discounting operation is then a discounting of* ${}^{tr}m$*, while the second transformation is a discounting of* $\overline{{}^{tr}m}$*.*

Remark 2 *The de-discounting operation is a particular case of reinforcement process. A more informative reinforcement than* ${}^{tr}m$ *can be chosen, for instance, the "pignistic bba" defined,* $\forall \omega \in \Omega$*, by:*

$${}^{bet}m(\{\omega\}) = \sum_{\{A\subseteq\Omega, \omega\in A\}} \frac{m(A)}{(1-m(\emptyset))|A|}\;. \qquad (23)$$

Thus, an other reinforcement process is given by: ${}^{\boldsymbol{\nu}}m = \nu_1 m + \nu_2{}^{bet}m$*.*

Proposition 2 *By choosing* $m_{Ag}^{\mathcal{R}}$ *of the fol-*

lowing manner:

$$\begin{cases} m_{Ag}^{\mathcal{R}}(\{R_i\}) &= \nu_i \qquad \forall i \in [\![1, N]\!], \\ m_{Ag}^{\mathcal{R}}(\mathcal{R}) &= 1 - \sum_{i=1}^{N} \nu_i, \end{cases} \tag{24}$$

with $\sum_{i=1}^{N} \nu_i \leqslant 1$, *one more degree of freedom can be added:*

$${}^{\boldsymbol{\nu}}m = \sum_{i=1}^{N} \nu_i \, m_i + (1 - \sum_{i=1}^{N} \nu_i) m_\Omega \ . \tag{25}$$

5 An application example

In this section, an application example in the domain of postal address recognition illustrates the possible benefits obtained by using a correction mechanism.

In the present application, three postal address readers (PARs) are available, each one providing pieces of information regarding the address lying on the image of a mail. These pieces of knowledge are represented by belief functions on a frame of discernment gathering the whole postal addresses. Belief functions can then be combined in order to make a decision. This fusion scheme is represented in Figure 1. The details of this application can be found in [8], and the construction of the mass assignments is explained in [7].

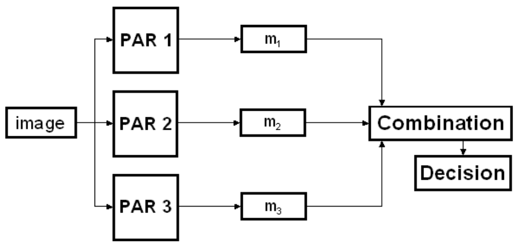

Figure 1: Fusion scheme with three PARs in the belief function framework.

An extension is exposed by considering the fact that PAR 1 and PAR 2 output an address and a confidence score regarding the town part of the address.

To visualize the real information provided by these confidence scores, scores of correct and incorrect towns output by PAR 1 for a set of postal addresses are represented in Figure 2.

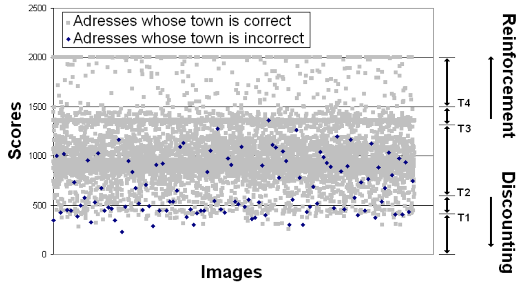

Figure 2: Confidence scores and addresses provided by PAR 1 regarding images of a learning set. A dark rhomb corresponds to an address whose town is incorrect. A clear square is associated with an address whose town is correct.

It can be observed that the greater the score is, the more important is the proportion of addresses with a correct town. Hence, this score carries useful information regarding the reliability of the town information in the output address. Similar observations were made with PAR 2. Therefore, bbas m_1 and m_2 representing the information provided by PAR 1 and PAR 2, should be corrected according to these scores. An idea consists in reinforcing the information provided by a PAR when the score is high, and, conversely, discounting it when the score is too low. For that purpose, we defined four thresholds T_1, T_2, T_3, and T_4 illustrated in Figure 2, such that information provided by the PAR is:

- totally discounted, if the score is lower than T_1;
- discounted according to the score, if the score belongs to $[T_1, T_2]$;
- kept unchanged, if the score belongs to $[T_2, T_3]$;
- reinforced according to the score, if the score belongs to $[T_3, T_4]$;
- at last, totally reinforced, if the score is greater than T_4.

Formally, this adjustment can be realized, for both PARs 1 and 2 (Figure 3), by using the correction mechanism defined by:

$$^{\boldsymbol{\nu}}m = \nu_1 \, m_\Omega + \nu_2 \, m + \nu_3 \, {}^{tr}m \ , \qquad (26)$$

where parameters ν_i are set as illustrated in Figure 4.

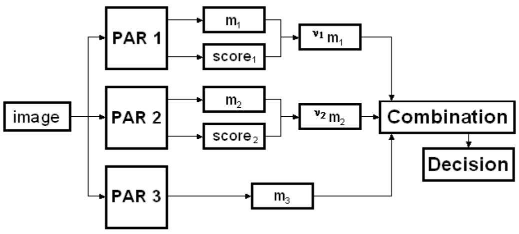

Figure 3: An extended model adjusting bbas provided by PAR 1 and PAR 2 according to supplied scores.

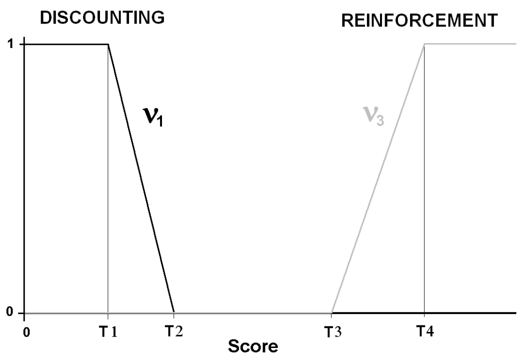

Figure 4: Correction parameters as function of the scores ($\nu_1 + \nu_2 + \nu_3 = 1$).

Performances of this combination, on a test set of mails, are represented in Figure 5. To preserve the confidentiality of PARs performances, reference values have been used when representing performance rates. Correct recognition rates, represented on the x-axis, are expressed relatively to a reference correct recognition rate, denoted R. Error rates, represented on the y-axis, are expressed relatively to a reference error rate, denoted E. The rate R has a value greater than 80%. The rate E has a value smaller than 0.1%.

In this application, different PARs being available, a combination leading to the greatest possible recognition rates, while being associated with an acceptable error rate, is expected. In this paper, the maximal tolerated

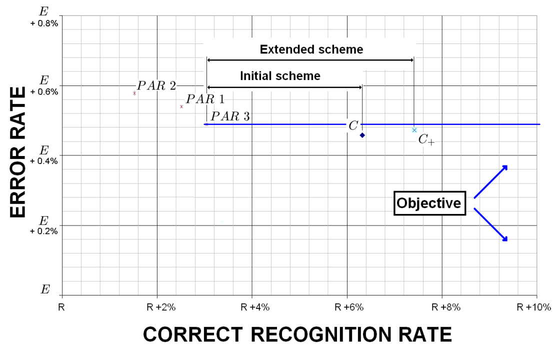

Figure 5: PARs and combination performances regarding towns written on mails.

error rate is chosen equal to the least PARs error rates.

This extended model allows us to obtain a combination point denoted C_+, which is associated with an acceptable error rate and a greater recognition rate than the previous combination point C, obtained with the model illustrated in Figure 1. Then, PARs individual performances are improved once more using the extended model based on a correction mechanism.

6 Conclusion

In this paper, links between the discounting, the de-discounting, and the extended discounting have been clarified. It has been shown that these schemes are particular cases of a general correction mechanism process. Different transformations, expressed by belief functions, can be associated to different states in which the source can be: reliable, not reliable, too cautious, lying, ...

An application example illustrates the practical interest of this family. It introduces a way to combine scores with decisions to improve the performances of the recognition.

Future works consist in contextualizing this family in the same manner as it has been done with the discounting [10]. Likewise, it would be interesting to automatically learn the coefficients ν_i from data, as it has been proposed for the classical or the contextual discounting [4, 10].

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