Towards an alarm for opposition conflict in a conjunctive combination of belief functions

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Abstract. In the framework of belief functions, information fusion is based on the construction of a unique belief function resulting from the combination of available belief functions induced from several information sources. When sources are reliable and distinct, Smets' conjunctive rule, which is equivalent to Dempster's rule of combination without the normalization process, can be considered. This rule offers interesting properties, but in return the empty set is an absorbing element: a series of conjunctive combinations tends to bring a mass equal to 1 to the empty set, making impossible the distinction between a real problem and an effect due to this absorbing effect of the empty set. Then a formalism allowing the preservation of the conflict which reflects the opposition between sources, is introduced in this paper. Based on the normalization process and on distance measures between belief functions, it is tested and compared with classic conjunctive operators on synthetic belief functions.

1 Introduction

Since more than about twenty years, the scientific community has been showing an increasing interest in information fusion [5, 16, 39]. Generally based on confidence measures including probability measure, fuzzy sets, possibility and belief measures, information fusion allows the consideration of the redundancy and the complementarity of different available pieces of information to improve the global quality of these inputs, and consequently reach a better decision-making. In the framework of belief functions [33], information fusion has been used in several fields such as multi-sensor fusion [1, 4], classification [17, 27], diagnosis [6, 31] or multi-object tracking [2, 29]. It is based on the application of an operator allowing the combination of belief functions representing different propositions or hypotheses relative to a given problem.

One classical rule is the conjunctive rule of combination. Introduced by Smets [34, 37], it is equivalent to Dempster's rule of combination [12, 33] without the normalization process. Its properties are well established as well as the hypotheses the sources must verify to be combined by the use of this rule [36, Section 3.2.2].

In a nutshell, sources must be distinct, reliable and must refer to the same object. As a consequence, this rule provides an orthogonal behaviour which is very valuable when a rapid and clear convergence on a solution is required, but in return the empty set is an absorbing element.

Smets [36, Section 6.1] supports the existence of this mass on the empty set to play an alarm role. Indeed, this conflict should not be hidden as it expresses important pieces of information which can be gathered together into two main categories:

- prerequisites for the application of the conjunctive rule are not fulfilled: two sources may not be distinct, one of the sources at least is not reliable (maybe a sensor is broken or ineffective in some unknown condition, etc), or the sources do not deal with the same object.
- the model itself suffers from a bad adequacy to the reality: the frame of discernment is not exhaustive (it is not composed of all the possible values the variable of interest can take), the choice of the frame(s) is not appropriate, etc.

On account of its absorbing effect, a series of conjunctive combinations tends then to bring a mass equal to 1 to the empty set, making impossible the distinction between a real problem and an effect due to the absorbing power of the empty set [24][36, Section 7].

Let us note that other works have been undertaken to complete this definition of the conflict. In [24], a definition of the conflict between belief functions is proposed. It is based on quantitative measures of both the mass on the empty set after a conjunctive combination of these belief functions and the distance between betting commitments of these same belief functions, the mass on the empty set being then no more sufficient to define the conflict. This behaviour is also described by Osswald et al. [25, 30] who defined the *auto-conflict* as the amount of intrinsic conflict of a belief function.

In this paper, the opposition between belief functions is quantified by a dissimilarity measure between these functions. This approach, called Combination With Adapted Conflict (CWAC), allows the mass on the empty set to keep its initial role of alarm signal.

This paper is organized as follows. A rapid overview of the basic concepts needed on belief functions is exposed in Section 2, details can be found in [33, 37]. In Section 3, the classical combinations of information in the belief function framework are detailed. The postulates and principles of our contribution are explained in Section 4. Then, tests on synthetic belief functions are presented in Section 5 showing the efficiency of the introduced formalism. Finally, Section 6 sums up our contributions and advances possible future work.

2 Belief function theory: basic concepts

2.1 Representing information

Let $\Omega = \{\omega_1, \ldots, \omega_K\}$, named the frame of discernment, be a finite non empty set including all the elementary hypotheses related to a given problem. These hypotheses are assumed to be exhaustive and mutually exclusive.

To represent the impact of a piece of evidence on the subsets of hypotheses of the frame of discernment Ω, the so-called basic belief assignment (bba) is defined as a function $m : 2^\Omega \rightarrow [0, 1]$ satisfying:

$$\sum_{A \subseteq \Omega} m(A) = 1. \tag{1}$$

The quantity $m(A)$, called a basic belief mass (bbm) or a mass for short, represents the part of belief which is exactly committed to the subset A of Ω.

Shafer [33] has initially proposed a normality condition expressed by: $m(\emptyset) = 0$. As previously exposed in the introduction of this paper, Smets proposes to keep the value $m(\emptyset)$ and to consider it as the amount of conflict between the pieces of evidence, which is also considered in this paper.

All the subsets A of Ω such that $m(A)$ is strictly positive, are called the focal elements of m.

2.2 Discounting information

A doubt on the reliability of a bba m is sometimes possible. The discounting operation [33] of m by $\alpha \in [0, 1]$, named *discount rate*, allows one to take into account this *meta knowledge* on the information m. This correction operation of m is defined by:

$$\begin{cases} m^\alpha(A) = (1-\alpha)m(A), & \forall A \subset \Omega, \\ m^\alpha(\Omega) = (1-\alpha)m(\Omega) + \alpha. \end{cases} \tag{2}$$

The coefficient $\beta = (1 - \alpha)$ represents the reliability degree of the source. If the source is not reliable, this degree β is equal to 0, the discount rate α is equal to 1, and m^α is equal to the vacuous bba m_Ω. On the contrary, if the source is reliable, the discounting rate α is null, and m will not be discounted.

2.3 Pignistic transformation

To make a decision, Smets proposes to transform beliefs to a probability measure. This latter, denoted $BetP$ [37], is called pignistic probability and is defined by:

$$BetP(\omega) = \sum_{A \subseteq \Omega, \omega \in A} \frac{1}{|A|} \frac{m(A)}{1 - m(\emptyset)}, \tag{3}$$

where $|A|$ is the cardinality of subset A. $BetP$ can be extended as a function on 2^Ω as $BetP(A) = \sum_{\omega \in A} BetP(\omega)$. Beyond the pignistic, lots of probability transforms of belief functions have been proposed [7, 8, 10].

2.4 Distance between two belief functions

Many distance measures between two bbas have been developed (e.g. [21, 22, 38]).

Tessem's distance is among those based on the pignistic transformation [3, 18, 38], it is used in several applications [3, 24]. Let m_1 and m_2 be two bbas and, respectively $BetP_{m_1}$ and $BetP_{m_2}$ their pignistic transformations. Tessem's distance is then defined as follows:

$$d_T(m_1, m_2) = \max_{A \subseteq \Omega}(|BetP_{m_1}(A) - BetP_{m_2}(A)|) \tag{4}$$

In [24], this measure is called the *distance between betting commitments* of m_1 and m_2.

Jousselme et al.'s distance is one of the most used in the framework of belief functions and satisfies useful properties such as non-negativity, non-degeneracy and symmetry. It is defined as follows:

$$d_J(m_1, m_2) = \sqrt{\frac{1}{2}(m_1 - m_2)^t \mathcal{D}(m_1 - m_2)} \tag{5}$$

where $\mathcal{D}$ is the Jaccard index defined by:

$$\mathcal{D}(A, B) = \begin{cases} 0 & \text{if} \quad A = B = \emptyset \\ \frac{|A \cap B|}{|A \cup B|} & \forall A, B \in 2^{\Omega}. \end{cases} \tag{6}$$

3 Combining different pieces of information

The objective of the combination is to synthesize a set of belief functions into a unique belief function. Two main approaches may be distinguished: conjunctive and disjunctive rules.

3.1 Conjunctive rules of combination

When sources are considered as distinct and reliable (note that they can have been adjusted according to their reliability, this adjustment being possibly realized through a discounting operation (see equation (2)) from additional information [18, 28] or by comparing the belief functions to combine with each others [23, 25, 32] by means of a distance), the combination of Demspter [12] can be classically used. This combination is noted $\oplus$ and defined, m_1 and m_2 being two bbas, by:

$$m_{\oplus}(A) = \frac{1}{1 - m_{\bigcirc\!\!\!\!\cap}(\emptyset)} m_{\bigcirc\!\!\!\!\cap}(A) \quad \forall A \neq \emptyset \qquad \text{and} \qquad m_{\oplus}(\emptyset) = 0 \tag{7}$$

with:

$$m_{\bigcirc\!\!\!\!\cap}(A) = \sum_{B \cap C = A} m_1(B) m_2(C) \qquad \forall A \subseteq \Omega. \tag{8}$$

Combination $m_{\textcircled{\cap}}$ is called the conjunctive rule of combination [37]. The value $m(\emptyset)$ is called conflict because it represents the disagreement between sources involved in the fusion. Let us note that the cautious conjunctive rule of combination developed by Denœux [13] has also a conjunctive behaviour, and it can be applied when sources are not distinct.

3.2 Disjunctive rule of combination

When one source is not reliable, and we do not know which one and an adjustment is not possible, the conjunctive combination cannot be used directly. Several combinations were then proposed like the disjunctive rule of combination [14] defined by:

$$m_{\textcircled{\cup}}(A) = \sum_{B \cup C = A} m_1(B) m_2(C) \qquad \forall A \subseteq \Omega. \tag{9}$$

This rule represents the dual rule of the conjunctive combination. It is discussed within the framework of the Generalized Bayes Theorem by Smets [35]. The universe Ω is the absorbing element of this rule. In the same spirit as the cautious rule, Denoeux [13] has proposed the bold disjunctive rule of combination, when belief functions to combine are provided by sources which are neither distinct nor reliable.

Other combination rules having intermediate behaviour between the conjunctive and the disjunctive combination have been proposed. For instance, the following rules may be mentionned: the combination of Dubois and Prade [15], the one of Delmotte et al. [11], Martin et al.'s mixed rules [26] or more recently the robust rule of combination of Florea et al. [20]. For other combination rules, it is a question of distributing the partial conflict [19, 26]. Objectives of all these rules is to distribute the conflict which arises during the fusion. This redistribution may be seen as a loss of information about a possible dysfunction.

4 Combination with adapted conflict (CWAC)

In this paper, sources are assumed to be distinct and reliable. In this context, the conflict $m(\emptyset)$ obtained during a conjunctive combination allows the decision maker to turn his attention to a possible problem related to a bad modelling, an unreliable source, etc.

However, when applying the conjunctive combination on a large number of belief functions, the conflict can take important proportions without reflecting a problem. This phenomenon is due to the absorbing effect of the empty set. On the other hand, most of the combination propositions found in the literature (see Section 3) try to redistribute this conflict and not to use it as an indicator.

Based on this analysis, we wish to develop a method which allows us to transform the value of the conflict and to adapt it in order to be a real indicator of problems, even if the number of sources to combine is important. This rule is called Combination With Adapted Conflict (CWAC). Considering that there is a serious problem when sources produce strongly different belief functions, the conflict should be kept during the fusion. On the contrary, in the case of the combination of information sources for which the bbas are equivalent, the conflict does not have to exist. To define the CWAC, a measure allowing one to distinguish similarities between bbas is necessary.

4.1 With two belief functions

First, the case of only two bbas m_1 and m_2 is studied. The notion of dissimilarity is obtained through a distance measure. This distance can be obtained by one of both measures presented in Section 2.4 and is noted $d(m_1, m_2)$[3]. The borders of d are:

- $d(m_1, m_2) = 0$: m_1 and m_2 are similar (and are thus in agreement) and their combination should not generate a conflict. In this case, the conflict will be redistributed in the same way as Dempster's rule of combination.
- $d(m_1, m_2) = 1$: m_1 and m_2 are antinomic (i.e. $m_1(\{\omega_j\}) = 1$ and $m_2(\{\omega_i\}) = 1$ with $\omega_i \neq \omega_j$). Their combination will produce a conflictual mass expressing this opposition. The conflict will be kept in the same manner as the conjunctive rule.

The CWAC is defined by an adaptive weighting between the conjunctive and Dempster's rules, making the rule acting like a conjunctive rule when the belief functions are antinomic and like Dempster's rule when belief functions are similar. Between these two extremes, a gradual evolution can be considered. The following combination rule noted $\circledcirc\!\!\!\!\leftrightarrow$ is then proposed, it is defined by:

$$m_{\circledcirc\!\!\!\leftrightarrow}(A) = \gamma_1 m_{\bigcirc\!\!\!\!\cap}(A) + \gamma_2 m_{\oplus}(A) \qquad \forall A \subseteq \Omega \tag{10}$$

with:

$$m_{\oplus}(A) = (m_1 \oplus m_2)(A) \qquad \forall A \subseteq \Omega \tag{11}$$

$$m_{\bigcirc\!\!\!\!\cap}(A) = (m_1 \bigcirc\!\!\!\!\cap\, m_2)(A) \qquad \forall A \subseteq \Omega \tag{12}$$

and with γ_1 and γ_2 are functions of the distance $d(m_1, m_2)$. These functions should satisfy the following constraints:

$$\gamma_1 = f_1(d(m_1, m_2)) \qquad \text{with} \qquad f_1(0) = 0 \quad \text{and} \quad f_1(1) = 1 \tag{13}$$

$$\gamma_2 = f_2(d(m_1, m_2)) \qquad \text{with} \qquad f_2(0) = 1 \quad \text{and} \quad f_2(1) = 0 \tag{14}$$

with $\gamma_1 + \gamma_2 = 1$. Although other functions are possible, we can take, at first, linear functions such that:

$$\gamma_1 = d(m_1, m_2) \tag{15}$$

$$\gamma_2 = 1 - d(m_1, m_2). \tag{16}$$

[3] However, other measures of dissimilarity could be used [7, 9]. Details on distance measure can be found in [22]. The aim of this article is not to compare these measures but to quantify the opposition between belief functions.

Hence, the combination can be written $\forall A \subseteq \Omega$ and $m_{\textcircled{\cap}}(\emptyset) \neq 1$:

$$m_{\textcircled{\leftrightarrow}}(A) = m_1 \textcircled{\leftrightarrow} m_2(A) = d(m_1, m_2) m_{\textcircled{\cap}}(A) + (1 - d(m_1, m_2))\, m_{\oplus}(A). \quad (17)$$

When $m_{\textcircled{\cap}}(\emptyset) = 1$, then we get $m_{\textcircled{\leftrightarrow}}(\emptyset) = 1$.

4.2 General case

The question of the generalization of this approach is natural when we have more than two information sources to fuse. Indeed, the problem settles because the distance measure used here, is defined between only two bbas. Let $m_1, \ldots, m_i \ldots, m_N$ be N bbas which have to be combined. The measure of dissimilarity between these functions, which is necessary for our proposed combination rule, may be a synthesis of the distances between these bbas. The objective is to identify if at least one of the sources is in disagreement with the others. This synthesis can be obtained by taking, for example, the maximal value of all the distances. So, the value of D can be defined as $D = \max\limits_{i,j} [d(m_i, m_j)]$ with $i \in [1, N]$ and $j \in [1, N]$. The combination rule becomes then $\forall A \subseteq \Omega$ and $m_{\textcircled{\cap}}(\emptyset) \neq 1$:

$$m_{\textcircled{\leftrightarrow}}(A) = \left(\underset{i}{\textcircled{\leftrightarrow}} m_i \right)(A) = D m_{\textcircled{\cap}}(A) + (1 - D) m_{\oplus}(A) \quad (18)$$

and

$$m_{\textcircled{\leftrightarrow}}(\emptyset) = 1 \quad when \quad m_{\textcircled{\cap}}(\emptyset) = 1 \quad (19)$$

with:

$$m_{\textcircled{\cap}}(A) = \left(\underset{i}{\textcircled{\cap}} m_i \right)(A) \quad \text{and} \quad m_{\oplus}(A) = \left(\underset{i}{\oplus} m_i \right)(A) \quad \forall i \in [1, N]. \quad (20)$$

4.3 Properties

- **Commutativity:** The combination of two mass functions m_1 and m_2 using the CWAC is commutative. Since the two basic rules composing the CWAC (the conjunctive rule and Dempster's rule) are commutative and since the CWAC is a weighted sum of these rules based on distance which is also commutative, the CWAC is commutative.
- **Associativity:** The CWAC operator is not associative. It is however possible to find operators that produce associative rules or quasi-associative.
- **Neutral element:** The neutral element of the CWAC is Ω. When combining a piece of evidence m_1 with $m(\Omega) = 1$, we have $m_1 \oplus m = m_1$ and $m_1 \textcircled{\cap} m = m_1$. The CWAC can be written: $m_{\textcircled{\leftrightarrow}}(A) = d(m_1, m) m_1(A) + (1 - d(m_1, m)) m_1(A) = m_1(A)$. Thus, the CWAC preserves the neutral impact of the $m(\Omega) = 1$.
- **Absorbing element:** From equation (19), the absorbing element of the CWAC is $\emptyset$.
- **Idempotent:** As both Dempster's rule and the conjunctive rule of combination, the CWAC operator is not idempotent.

5 Results

In this Section, the CWAC operator is compared on synthetic data with the conjunctive rule. The CWAC operator is used with two dissimilarity measures: Tessem's distance and Jousselme et al.'s distance presented in Section 2.4.

5.1 Example 1

In this first example, two sources are considered as being in agreement: they have a similar distribution of masses. These distributions and the combinations results by the operators $\oplus$, $\bigcirc\!\!\!\!\cap$ and $\ominus$ are given in Table 1. Bba $m^{J}_{\ominus}$ is obtained by the CWAC operator with Jousselme et al.'s distance and $m^{T}_{\ominus}$ is obtained by the CWAC operator with Tessem's distance. The conflict induced by the conjunctive combination is relatively important which is not the case for the proposed combination (0.363 against 0.004). Now these two bbas are considered in disagreement (Table 2). If we compare these results to those obtained in the previous test, we observe that there is only 23 % of increase of the conflict for the conjunctive combination (while the distribution of masses are radically different). Regarding our rule, the increase of the conflict is of the order of 3150% which reflects well the difference between the first test and the second one.

Table 1. Results of the fusion between two sources in agreement.

	m_1	m_2	$m_{\oplus}$	$m_{\bigcirc\!\!\!\!\cap}$	$m^{J}_{\ominus}$	$m^{T}_{\ominus}$
$\{\omega_1\}$	0.60	0.59	0.742	0.473	0.74	0.74
$\{\omega_2\}$	0.30	0.31	0.242	0.154	0.24	0.24
Ω	0.10	0.10	0.016	0.01	0.016	0.016
$\emptyset$	0	0	0	**0.363**	**0.004**	**0.004**

Table 2. Results of the fusion between two sources in disagreement.

	m_1	m_2	$m_{\oplus}$	$m_{\bigcirc\!\!\!\!\cap}$	$m^{J}_{\ominus}$	$m^{T}_{\ominus}$
$\{\omega_1\}$	0.60	0.31	0.501	0.277	0.436	0.436
$\{\omega_2\}$	0.30	0.59	0.481	0.266	0.419	0.419
Ω	0.1	0.1	0.018	0.01	0.015	0.015
$\emptyset$	0	0	0	**0.447**	**0.13**	**0.13**

5.2 Example 2

In this second example, a number N of sources is considered with N varying from 2 to 25. All the bbas are firstly chosen in agreement and are defined, with $\Omega = \{\omega_1, \omega_2, \omega_3\}$, as follows with ϵ a random value between $[-0.1; 0.1]$:

$$m(\{\omega_1\}) = 0.6+\epsilon \qquad m(\{\omega_1, \omega_2\}) = 0.15-\epsilon \qquad m(\{\omega_3\}) = 0.15 \qquad m(\Omega) = 0.1.$$

Conflict evolution for operators $\bigcirc\!\!\!\!\cap$ and $\ominus$ according to the number of sources N to combine is presented in Figure 1. The absorbing effect of the empty set can be observed: even if the bbas are in agreement the value of the conflict increases with the number of combinations. In a second time, one bba is now chosen as being in contradiction with the others. It is defined in the following way:

$$m(\{\omega_1\}) = 0.15+\epsilon \qquad m(\{\omega_1, \omega_2\}) = 0.15-\epsilon \qquad m(\{\omega_3\}) = 0.6 \qquad m(\Omega) = 0.1$$

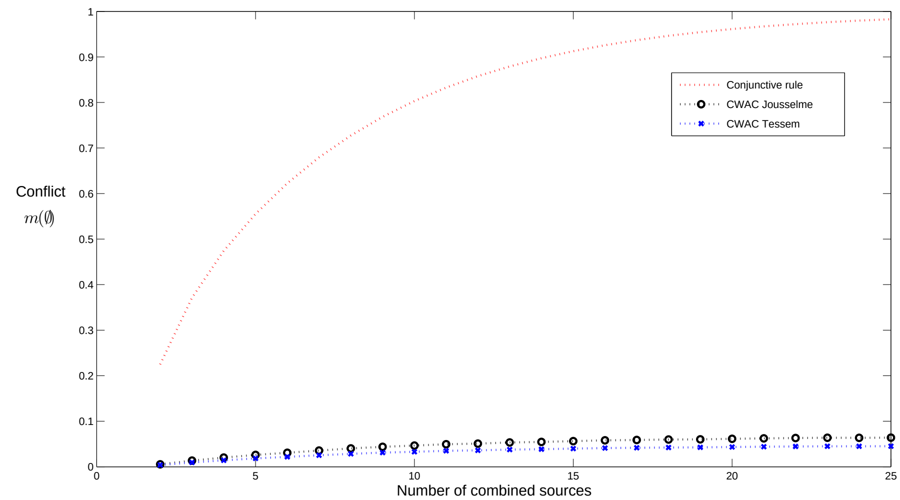

Fig. 1. Conflict evolution of the combination of N not contradictory bbas.

Figure 2 illustrates the evolution of the conflict in this configuration, the latter being compared with the previous in Figure 3.

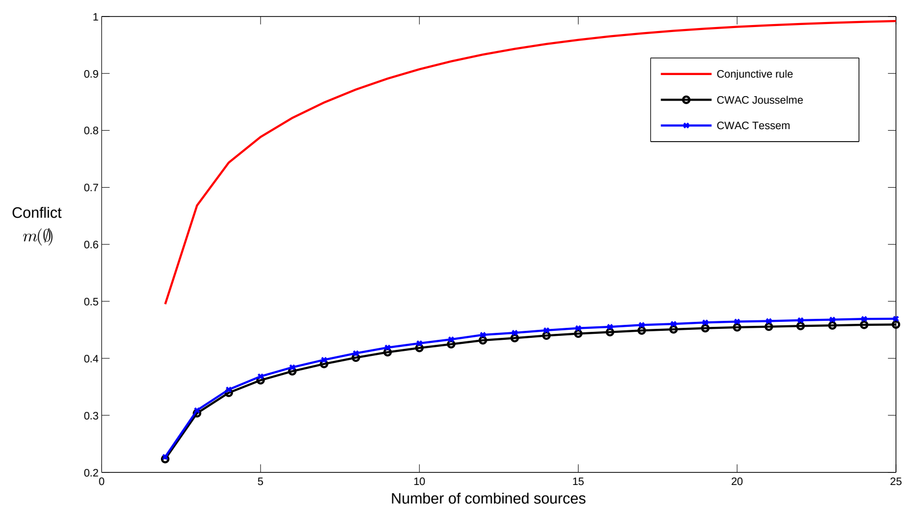

Fig. 2. Conflict evolution of the combination of $N-1$ not contradictory bbas and one in conflict.

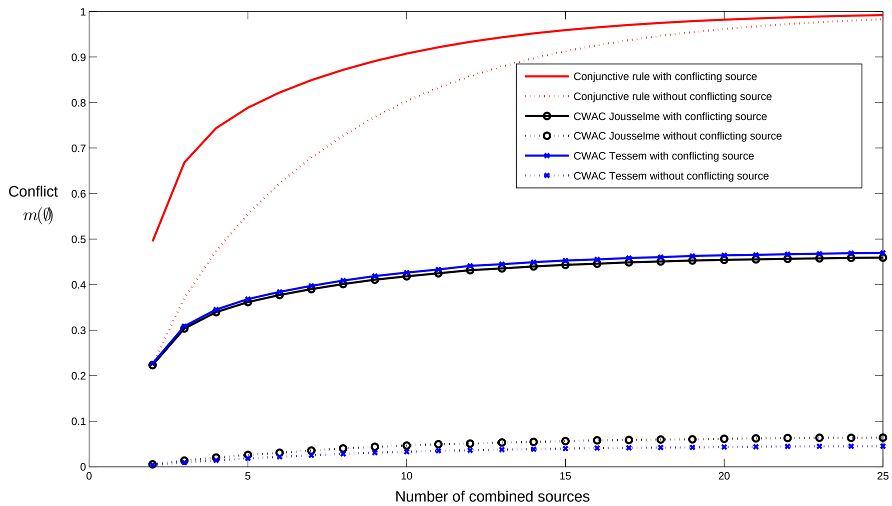

Fig. 3. Comparison between the combination of N similar functions and the combination of $N-1$ similar functions and of a contradictory function.

In this last figure 3, it can be observed that after more that 20 belief functions to combine, the value of the conflict obtained by the conjunctive combination does not allow any more the identification of a possible contradiction between bbas while it is not the case for the CWAC operator. The behaviour of the CWAC operator is equivalent with both dissimilarity measures. However, Tessem's distance allows one to have a difference between the two simulations more important. So, Tessem's distance is a better measure than Jousselme et al.'s distance for judging how contradict the two beliefs are [24].

6 Conclusion and future work

In this paper, we have proposed a combination rule with adapted conflict having the objective to better handle the conflict induced from the fusion of several bbas. Our proposed CWAC rule makes an adaptive weighting between conjunctive and Dempster's rules using Tessem's and Jousseleme et al.'s distances in order to reduce the absorbing power of the conflict and to more strengthen its initial role of alarm signal. As future work, more attention will be given to obtain the similarity measure between all belief functions involved in the combination. For each similarity measure, different properties of CWAC will be defined. Moreover, it will be interesting to study the behaviour of this operator when Jousselme et al.'s distance (or others) is approximately equal to 0.5.

References

1. Appriou, A.: Multisensor signal processing in the framework of the theory of evience. In: Application of Mathematical Signal Processing Techniques to Mission Systems. pp. (5–1)–(5–31). Research and Technology Organization, Lecture Series (1999)
2. Bar-Shalom, Y., Li, X.: Multitarget-Multisensor Tracking: Principles and Techniques. YBS Publishing, Storrs, CT (1995)
3. Bauer, M.: Approximations algorithm and decision making in the Dempster-Shafer theory of evidence - an empirical study. International Journal of Approximate Reasoning 17(2-3), 217–237 (1997)
4. Caron, F., Duflos, E., Pomorski, D., Vanheeghe, P.: GPS/IMU data fusion using multisensor kalman filtering: Introduction of contextual aspects. Information Fusion 7(2), 221–230 (2006)
5. Cholvy, L.: About Merged Information, pp. 233–263. Kluwer Academic Publishers (1998)
6. Chunli, X., Qiang, G.: Research on data fusion diagnosis systems based on neural network and evidence theory. In: Prodeedings of IEEE International Conference on Testing and Diagnosis (ICTD). pp. 1–4 (2009)
7. Cuzzolin, F.: Two new Bayesian approximation of belief functions based on convex geometry. IEEE Transactions on Systems, Man and Cybernetics - Part B 37(4), 993–1008 (2007)
8. Cuzzolin, F.: Credal semantics of bayesian transformation in terms of probability intervals. IEEE Transaction on Systems, Man and Cybernetics - Part B 40(2), 421–432 (2010)
9. Cuzzolin, F.: Geometric conditioning of belief functions. In: Proceedings of 1st Workshop on the Theory of Belief Functions (2010)
10. Daniel, M.: On transformation of belief functions to probabilites. International Journal of Intelligent Systems, special issue on Uncertainty Processing 21(3), 261–282 (2006)
11. Delmotte, F., Dubois, L., Desodt, A., Borne, P.: Using trust in uncertainty theories. Information and Systems Engineering 1, 303–314 (1995)
12. Dempster, A.: Upper and lower probabilities induced by a multivalued mapping. Annals of Mathematical Statistics 38, 325–339 (1967)
13. Denoeux, T.: Conjunctive and disjunctive combination of belief functions induced by non distinct bodies of evidence. Artificial Intelligence 172(23), 234–264 (2008)
14. Dubois, D., Prade, H.: A set theoric view of belief functions: Logical operation and approximations by sets. International Journal of General Systems 12(3), 193–226 (1986)
15. Dubois, D., Prade, H.: Representation and combination of uncertainty with belief functions and possibility measures. Computational Intelligence 4, 244–264 (1988)
16. Dubois, D., Prade, H.: Possibility theory and data fusion in poorly informed environments. Control Engineering Practice 2(5), 811–823 (1994)
17. Elouedi, Z., Mellouli, K., Smets, P.: Belief decision trees: Theorical foundations. International Journal of Approximate Reasoning 28(2-3), 91–124 (2001)
18. Elouedi, Z., Mellouli, K., Smets, P.: Assessing sensor reliability for multisensor data fusion with the Transferable Belief Model. IEEE Transaction on Systems, Man and Cybernetics, part B 34, 782–787 (2004)
19. Florea, M., Dezert, J., Valin, P., Smarandache, F., Jousselme, A.L.: Adaptative combination rule and proportional conflict redisribution rule for information fusion. In: Proceedings of COGnitive Systems with Interactive Sensors (COGIS'06) (2006)

20. Florea, M., Jousselme, A.L., Boisé, E., Grenier, D.: Robust combination rules for evidence theory. Information Fusion 10(2), 183–197 (2009)
21. Jousselme, A.L., Grenier, D., Boissé, E.: A new distance between two bodies of evidence. Information Fusion 2, 91–101 (2001)
22. Jousselme, A.L., Maupin, P.: On some properties of distance in evidence theory. In: Proceedings of 1st Workshop on the Theory of Belief Functions (2010)
23. Klein, J., Colot, O.: Automatic discounting rate computation using a dissent criterion. In: Proceedings of 1st Workshop on the Theory of the Belief Functions (2010)
24. Liu, W.: Analyzing the degree of conflict among belief functions. Artificial Intelligence 170(11), 909–924 (2006)
25. Martin, A., Jousselme, A., Osswald, C.: Conflict measure for the discountng operation on belief functions. In: Proceedings of Int. Conf. Information Fusion (FUSION'08). pp. 1003–1010 (2008)
26. Martin, A., Osswald, C.: Toward a combination rule to deal with partial conflict and specificity in belief functions theory. In: Proceedings of Int. Conf. On Information Fusion (Fusion'07) (2007)
27. Mercier, D., Cron, G., Denoeux, T., Masson, M.H.: Decision fusion for postal address recongition using belief functions. Experts Systems with Applications 36(issue 3, part1), 5643–5653 (2009)
28. Mercier, D., Lefevre, E., Delmotte, F.: Belief functions contextual discounting and cononical decomposition. International Journal of Approximate Reasoning accepted for publication (2011)
29. Mercier, D., Lefevre, E., Jolly, D.: Object association in the TBM framework, application to vehicle driving aid. In: Proceedings of 6th International Symposium on Imprecise Probability: Theories and Applications (ISIPTA). pp. 317–326. Durham, United Kingdom (2009)
30. Osswald, C., Martin, A.: Understanding the large family of Dempster-Shafer theory's fusion operators - a decsion-based measure. In: Proceedings of Int. Conf. On Information Fusion (FUSION'06) (2006)
31. Périsse, F., Mercier, D., Lefevre, E., Roger, D.: Robust diagnostics of stator insulation based on high frequency measurements. IEEE Transactions on Dielectrics and Electrical Insulation 16(5), 1496–1502 (2009)
32. Schubert, J.: Conflict management in Dempster-Shafer theory using the degree of falsity. International Journal of Approximate Reasoning 52(3), 449–460 (2011)
33. Shafer, G.: A Mathematical Theory of Evidence. Princeton University Press, Princeton, NJ (1976)
34. Smets, P.: The combination of evidence in the transferable belief model. IEEE Trans. on Pattern Analysis and Machine Intelligence 12(5), 447–458 (1990)
35. Smets, P.: Belief functions: The disjunctive rule of combination and the generalized Bayesian theorem. International Journal of Approximate Reasoning 9, 1–35 (1993)
36. Smets, P.: Analyzing the combination of conflicting belief functions. Information Fusion 8(4), 387–412 (2007)
37. Smets, P., Kennes, R.: The Transferable Belief Model. Artificial Intelligence 66, 191–234 (1994)
38. Tessem, B.: Approximations for efficient computation in the theory of evidence. Artificial Intelligence 61(2), 315–329 (1993)
39. Yager, R.: A framework for multi-source data fusion. Information Sciences 163(1-3), 175–200 (2004)