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Abstract
Proposition 4 and Theorem 1 of the article “Belief Functions Contextual Discounting and Canonical Decompositions” [International Journal of Approximate Reasoning 53 (2012) 146–158] provide an erroneous result. We give here the true result with a correct proof.

Keywords: Belief functions, Contextual Discounting.

We hereby correct Proposition 4 and Theorem 1 in [2], which contained erroneous results.

Let us first recall the problem. A source $S$ of information provides to agent $Ag$ a piece of information represented by a mass function $m^S_\Omega$ (with $\Omega = \{\omega_1, \ldots, \omega_K\}$), simply denoted by $m$ in this corrigendum. Let $A$ be a non empty set of subsets of $\Omega$ called contexts. Agent $Ag$ owns a meta-knowledge regarding the reliability of $S$ conditionally on each set $A \in A$. Formally, for all $A \in A$, we suppose that

$$
\begin{align*}
    m_{\text{Rel}}^{Ag}(A)(\{R\}) &= 1 - \alpha_A = \beta_A \\
    m_{\text{Rel}}^{Ag}(A)(\mathcal{R}) &= \alpha_A,
\end{align*}
$$

(1)

where $\alpha_A \in [0,1]$ and $\mathcal{R} = \{R, NR\}$ ($R$ meaning the source is reliable, $NR$ otherwise), and the notation $m[\cdot]$ denotes conditioning.

With the same reasoning as in [1] (where $A$ was supposed to form a partition of $\Omega$), the knowledge $m^\Omega_{Ag}$ held by agent $Ag$ on $\Omega$, based on the in-
formation \( m \) provided by \( S \) and his metaknowledge regarding \( S \) represented by (1) for all \( A \in \mathcal{A} \), can be obtained by the following computation,

\[
\left( m^{\Omega \times \mathcal{R}} \bigodot_{A \in \mathcal{A}} m^{\mathcal{R}}[A]^{\Omega \times \mathcal{R}} \right)^{\downarrow \Omega},
\]

where symbol \( \uparrow \) and \( \downarrow \) denote, respectively, the deconditioning and projection operations, and \( m^{\Omega \{\{R\}\}} = m \).

It is stated in [2] that, for \( A = 2^\Omega \) (Proposition 4) and more generally for any set \( \mathcal{A} \) of contexts (Theorem 1), Equation (2) is equivalent to

\[
m \bigodot (\bigodot_{A \in \mathcal{A}} A_{\beta_A}).
\]

This statement is incorrect. In the general case, for any non empty \( \mathcal{A} \), Equation (2) is equivalent to

\[
m \bigodot (\bigodot_{A \in \mathcal{A}} \overline{A^\mathcal{R}}),
\]

as shown by the following proof, which corrects Theorem 1 from [2]. The fact that, in general, (4) is not equivalent to (3) (and particularly when \( \mathcal{A} = 2^\Omega \)), and therefore (2) is not equivalent in general to (3), is shown below by Example 1.

**Proof 1.** Let us denote by \( A_i, i \in I = \{1, \ldots, n\} \), the contexts present in \( \mathcal{A} \), and let us write \( \beta_{A_i} \) simply by \( \beta_i \), for all \( i \in I \). For all \( A_i \in \mathcal{A} \), the deconditioning of \( m^{\mathcal{R}}[A_i] \) over \( \Omega \times \mathcal{R} \) is given by

\[
m^{\mathcal{R}}[A_i]^{\uparrow \Omega \times \mathcal{R}}(A_i \times \{R\} \cup \overline{A_i} \times \mathcal{R}) = \beta_i, \quad (5a)
\]

\[
m^{\mathcal{R}}[A_i]^{\uparrow \Omega \times \mathcal{R}}(\Omega \times \mathcal{R}) = \alpha_i. \quad (5b)
\]

Moreover, for all \( (A_i, A_j) \in \mathcal{A}^2 \), such that \( j \neq i \),

\[
(A_i \times \{R\} \cup \overline{A_i} \times \mathcal{R}) \cap (A_j \times \{R\} \cup \overline{A_j} \times \mathcal{R})
\]

\[
= (A_i \cap A_j) \times \{R\} \cup (A_i \cap \overline{A_j}) \times \{R\} \cup (\overline{A_i} \cap A_j) \times \{R\} \cup (\overline{A_i} \cup \overline{A_j}) \times \mathcal{R}
\]

\[
= (A_i \cup A_j) \times \{R\} \cup (A_i \cup A_j) \times \mathcal{R}.
\]

With \( \mathcal{A} \) composed of two elements denoted by \( A_i \) and \( A_j \), we then have

\[
\left\{ \begin{array}{ll}
(m^{\mathcal{R}}[A_i]^{\uparrow \Omega \times \mathcal{R}} \bigodot m^{\mathcal{R}}[A_j]^{\uparrow \Omega \times \mathcal{R}})((A_i \cup A_j) \times \{R\} \cup (A_i \cup \overline{A_j}) \times \mathcal{R}) = \beta_i \beta_j \\
(m^{\mathcal{R}}[A_i]^{\uparrow \Omega \times \mathcal{R}} \bigodot m^{\mathcal{R}}[A_j]^{\uparrow \Omega \times \mathcal{R}})(A_i \times \{R\} \cup \overline{A_i} \times \mathcal{R}) = \beta_i \alpha_j \\
(m^{\mathcal{R}}[A_i]^{\uparrow \Omega \times \mathcal{R}} \bigodot m^{\mathcal{R}}[A_j]^{\uparrow \Omega \times \mathcal{R}})((A_j \times \{R\} \cup \overline{A_j} \times \mathcal{R}) = \alpha_i \beta_j \\
(m^{\mathcal{R}}[A_i]^{\uparrow \Omega \times \mathcal{R}} \bigodot m^{\mathcal{R}}[A_j]^{\uparrow \Omega \times \mathcal{R}})(\Omega \times \mathcal{R}) = \alpha_i \alpha_j
\end{array} \right. .
\]
In other words, all the focal elements of \( \otimes_{A \in \mathcal{A}} m^R[A]^{\oplus \Omega \times \mathcal{R}} \) are the elements \( C \times \{R\} \cup \overline{C} \times \mathcal{R} \) with \( C \) composed of a union of elements \( A_i \) in \( \mathcal{A} \), \( I' \) being the set of indices of the \( A_i \)'s, which means with \( C = \bigcup_{i \in I' \subseteq I} A_i \). Moreover, each focal element has a mass equal to \( \prod_{i \in I'} \beta_i \prod_{j \in I' \setminus I} \alpha_j \). Let us note that this latter result is also true if \( \mathcal{A} \) is composed of one element \( A \subseteq \Omega \) (directly from Equations (5)).

By induction, we can show that this property remains true with \( \mathcal{A} \) composed of \( n \) contexts \( A_i \), \( i \in I = \{1, \ldots, n\} \). Indeed, let us suppose the property true with \( \mathcal{A} \) composed of \( n-1 \) contexts \( A_i \), \( i \in I = \{1, \ldots, n-1\} \), we then have for all focal elements \( C \times \{R\} \cup \overline{C} \times \mathcal{R} \) of \( \otimes_{i \in I} m^R[A_i]^{\oplus \Omega \times \mathcal{R}} \), with \( C = \bigcup_{i \in I' \subseteq I} A_i \),

\[
(\otimes_{i \in I} m^R[A_i]^{\oplus \Omega \times \mathcal{R}} \otimes m^R[A_n]^{\oplus \Omega \times \mathcal{R}})(C \times \{R\} \cup \overline{C} \times \mathcal{R})
= \beta_n \prod_{i \in I'} \beta_i \prod_{j \in I' \setminus I} \alpha_j = \prod_{i \in I'} \beta_i \prod_{j \in (I' \setminus I) \setminus \{n\}} \alpha_j ,
\]

and

\[
(\otimes_{i \in I} m^R[A_i]^{\oplus \Omega \times \mathcal{R}} \otimes m^R[A_n]^{\oplus \Omega \times \mathcal{R}})(C \times \{R\} \cup \overline{C} \times \mathcal{R})
= \alpha_n \prod_{i \in I'} \beta_i \prod_{j \in I' \setminus I} \alpha_j = \prod_{i \in I'} \beta_i \prod_{j \in (I' \setminus I) \setminus \{n\}} \alpha_j ,
\]

which means that focal elements of \( \otimes_{i \in \{1, \ldots, n-1\}} m^R[A_i]^{\oplus \Omega \times \mathcal{R}} \otimes m^R[A_n]^{\oplus \Omega \times \mathcal{R}} \) are also of the form \( C \times \{R\} \cup \overline{C} \times \mathcal{R} \), with \( C = \bigcup_{i \in I' \subseteq I} A_i \), \( I = \{1, \ldots, n\} \), \( A_i \in \mathcal{A} \), and have for mass: \( \prod_{i \in I'} \beta_i \prod_{j \in I' \setminus I} \alpha_j \).

Besides, for all \( B \subseteq \Omega \),

\[
m^\Omega(\{R\})^{\oplus \Omega \times \mathcal{R}}(B \times \{R\} \cup \Omega \times \{NR\}) = m(B) ,
\]

and, for all \( B \subseteq \Omega \), for all \( C = \bigcup_{i \in I' \subseteq I} A_i \),

\[
(C \times \{R\} \cup \overline{C} \times \mathcal{R}) \cap (B \times \{R\} \cup \Omega \times \{NR\}) = B \times \{R\} \cup \overline{C} \times \{NR\} .
\]

Therefore, after the projection on \( \Omega \), \((m^\Omega[\{R\}]^{\oplus \Omega \times \mathcal{R}} \otimes_{A \in \mathcal{A}} m^R[A]^{\oplus \Omega \times \mathcal{R}})^i_{\Omega} \) consists in transferring a part \( \prod_{i \in I'} \beta_i \prod_{j \in I' \setminus I} \alpha_j \) of each mass \( m(B) \), \( B \subseteq \Omega \), from \( B \) to \( B \cup \overline{C} \), for all \( C = \bigcup_{i \in I' \subseteq I} A_i \).

On the other hand, \( m^\Omega \left( \left( \otimes_{A \in \mathcal{A}} A_i^{\alpha_i} \right) \right) \) can be written as

\[
m^\Omega \left( \left( \otimes_{i \in I} A_i^{\alpha_i} \right) \right) = m^\Omega \left( \otimes_{i \in I} \left\{ \begin{array}{c} \Omega \\ A_i \end{array} \rightarrow \alpha_i \rightarrow \beta_i \right\} \right) .
\]
As for all \((i, j) \in I^2\) s.t. \(i \neq j\), \(\overline{A_i} \cap \overline{A_j} = \overline{A_i \cup A_j}\), it can be shown (with an induction for example) that the focal elements of \(\bigcap_{i \in I} \overline{A_i}\) are the elements \(\overline{C}\) with \(C = \bigcup_{i \in I' \subseteq I} A_i\) and have a mass equal to \(\prod_{i \in I'} \beta_i \prod_{j \in I \setminus I'} \alpha_j\).

Consequently, operation \(m \circ (\bigcap_{i \in I} \overline{A_i})\) also consists in transferring a part \(\prod_{i \in I} \beta_i \prod_{j \in I \setminus I'} \alpha_j\) of each mass \(m(B)\), \(B \subseteq \Omega\), from \(B\) to \(B \cup \overline{C}\), for all \(C = \bigcup_{i \in I' \subseteq I} A_i\). We can then conclude that Equations (2) and (4) are equivalent for any non empty set of contexts \(A\).

\[\square\]

**Example 1.** Let us consider \(\Omega = \{\omega_1, \omega_2\}\) and \(A = 2^\Omega\), and let us denote \(\alpha_{\{\omega_1\}}\) by \(\alpha_1\), \(\alpha_{\{\omega_2\}}\) by \(\alpha_2\), and \(\alpha_\Omega\) by \(\alpha_{12}\). Equation (4) gives

\[
m \circ \left( \bigcap_{A \in A} \overline{A^A} \right) = m \circ \left( \emptyset \circ \{\omega_1\} \cup \{\omega_2\} \cup \overline{\Omega^{12}} \right) = m \circ \left( \{\omega_2\} \cap \alpha_1 \cap \{\omega_1\} \cap \beta_1 \cap \alpha_2 \cap \Omega \cap \alpha_\Omega \cap \alpha_{12} \right) \mapsto \emptyset \mapsto \beta_1 \beta_2 \alpha_{12} + \beta_1 \alpha_2 \mapsto \{\omega_1\} \mapsto \alpha_1 \alpha_2 \alpha_{12} \mapsto \Omega \mapsto \alpha_1 \alpha_2 \alpha_{12}.
\]

In contrast, Equation (3) leads to

\[
m \circ \left( \bigcap_{A \in A} A^A_{\beta_{12}} \right) = m \circ \emptyset \circ \{\omega_1\} \circ \{\omega_2\} \circ \Omega_{\beta_{12}} = m \circ \left( \{\omega_1\} \cup \{\omega_2\} \cup \alpha_{12} \right) \mapsto \emptyset \mapsto \beta_1 \cap \alpha_1 \cap \{\omega_1\} \cap \{\omega_2\} \mapsto \beta_2 \cap \alpha_2 \cap \{\omega_1\} \cap \{\omega_2\} \mapsto \emptyset \mapsto \beta_1 \beta_2 \alpha_2 \mapsto \{\omega_1\} \mapsto \alpha_1 \beta_2 \alpha_{12} \mapsto \{\omega_2\} \mapsto \beta_1 \alpha_2 \alpha_{12} \mapsto \Omega \mapsto \alpha_1 \alpha_2 \beta_{12} \alpha_{12}.
\]

To summarize, in [1], the equivalence was shown between (2) and (3) when \(A\) forms a partition of \(\Omega\). This corrigendum shows that this equivalence does not hold for any \(A\), and that (2) is actually equivalent to (4) for any (non empty) \(A\).
References
