A First Basic Solution within the Transferable Belief Model

A model due to Philippe Smets (1938-2005)

D. Mercier

University of Artois, LGI2A, Béthune, France

Uncertainty Forum, 13^{th} International Conference on Information Fusion, Edinburgh, UK

Wednesday, July 28^{th}, 2010

UNIVERSITÉ D'ARTOIS

Outline

1. Transferable Belief Model (TBM): Fundamental Concepts
2. Example: a first basic solution within the TBM

Outline

Transferable Belief Model

A subjectivist and non-probabilistic view of the Dempster-Shafer theory of evidence

- **Belief Functions Interpretation**: **weighted opinions** held by an agent, irrespective of any underlying probabilistic model;

Transferable Belief Model

A subjectivist and non-probabilistic view of the Dempster-Shafer theory of evidence

- **Belief Functions Interpretation**: **weighted opinions** held by an agent, irrespective of any underlying probabilistic model;
- A clear separation between the **credal level**, where beliefs are entertained, and the **decision level** where standard utility theory applies, the belief functions being converted into probabilities using the pignistic transformation;

Transferable Belief Model

A subjectivist and non-probabilistic view of the Dempster-Shafer theory of evidence

- **Belief Functions Interpretation**: **weighted opinions** held by an agent, irrespective of any underlying probabilistic model;
- A clear separation between the **credal level**, where beliefs are entertained, and the **decision level** where standard utility theory applies, the belief functions being converted into probabilities using the pignistic transformation;
- The notions of **unnormalized belief function** and **unnormalized conjunctive rule of combination**: $m(\emptyset)$ may be strictly greater than 0;
- The mass on the empty set plays an alarm role in the TBM (got a problem somewhere);

Outline

LGI2A

UNIVERSITÉ D'ARTOIS

Frame of discernment
Credal Level

Frame of discernment $\boldsymbol{\Omega = \{yes, no\}}$ standing for *"yes, suspect vehicle is near building B"* and *"no, suspect vehicle is not near building B"*.

- Let $\mathbf{m_1}$ be the information provided by Analyst 1.
  - 10-year-experience Analyst 1 says: **"it is probable that individual A is near building B"**
- Let $\mathbf{m_s}$ be the information provided by the Automatic Number Plate Recognition (ANPR) system.
  - The ANPR system outputs a **30% probability that individual A is near building B**
- Let $\mathbf{m_2}$ be the information provided by Analyst 2.
  - Novice Analyst 2 says: **"it is improbable that individual A is near building B"**

Initializing mass functions and combining them

Credal Level

A solution:

$$
\begin{array}{lll}
m_1(\{yes\}) = \mu_1 & m_1(\{no\}) = 0 & m_1(\Omega) = 1 - \mu_1 \\
m_s(\{yes\}) = .3 & m_s(\{no\}) = .7 & m_s(\Omega) = 0 \\
m_2(\{yes\}) = 0 & m_2(\{no\}) = \mu_2 & m_2(\Omega) = 1 - \mu_2
\end{array} \qquad (1)
$$

Questions: How to set μ_1 and μ_2? Is m_s acceptable?

LGI2A

UNIVERSITÉ D'ARTOIS

Initializing mass functions and combining them
Credal Level

A solution:

$$\begin{array}{lll} m_1(\{yes\}) = \mu_1 & m_1(\{no\}) = 0 & m_1(\Omega) = 1 - \mu_1 \\ m_s(\{yes\}) = .3 & m_s(\{no\}) = .7 & m_s(\Omega) = 0 \\ m_2(\{yes\}) = 0 & m_2(\{no\}) = \mu_2 & m_2(\Omega) = 1 - \mu_2 \end{array} \quad (1)$$

Questions: How to set μ_1 and μ_2? Is m_s acceptable?

Conjunctive Rule of Combination, under the hypothesis that all the sources are reliable and distinct:

$$m = m_1 \textcircled{\cap} m_s \textcircled{\cap} m_2 \ . \quad (2)$$

Definition of the Conjunctive Rule of Combination (CRC):

$$m_1 \textcircled{\cap} m_2(A) = \sum_{B \cap C = A} m_1(B) \times m_2(C), \ \forall A \subseteq \Omega \ . \quad (3)$$

Making a decision 1/3

Decision Level

- We have to bet on yes or no building B should be evacuated.

Making a decision 1/3

Decision Level

- We have to bet on yes or no building B should be evacuated.
- Let us define the following set of decisions $\mathcal{D} = \{yesEvac, noEvac\}$.

Making a decision 1/3

Decision Level

- We have to bet on yes or no building B should be evacuated.
- Let us define the following set of decisions $\mathcal{D} = \{yesEvac, noEvac\}$.
- The states of nature considered are the elements of Ω.

Making a decision 1/3

Decision Level

- We have to bet on yes or no building B should be evacuated.
- Let us define the following set of decisions $\mathcal{D} = \{yesEvac, noEvac\}$.
- The states of nature considered are the elements of Ω.
- The chosen decision $d \in \mathcal{D}$ is then the one which minimizes the *expected risk* defined by:

$$R(d) = \sum_{\omega \in \Omega} c(d,\omega) P^{\Omega}(\{\omega\}), \qquad (4)$$

where $P^{\Omega} : 2^{\Omega} \to [0,1]$ is a probability measure and $c : \mathcal{D} \times \Omega \to \mathbb{R}$ a cost function, $c(d,\omega)$ representing the cost to decide d while the truth is ω.

LGI2A
UNIVERSITÉ D'ARTOIS

Making a decision 2/3

Decision Level

To define P^{Ω} we choose the *pignistic probability* defined by:

$$BetP^{\Omega}(\{\omega\}) = \sum_{\{A\subseteq\Omega,\omega\in A\}} \frac{m(A)}{|A|\,(1-m(\emptyset))}, \quad \forall\omega\in\Omega. \qquad (5)$$

Making a decision 2/3

Decision Level

To define P^{Ω} we choose the *pignistic probability* defined by:

$$BetP^{\Omega}(\{\omega\}) = \sum_{\{A \subseteq \Omega, \omega \in A\}} \frac{m(A)}{|A|\,(1 - m(\emptyset))}, \quad \forall \omega \in \Omega. \tag{5}$$

To define the cost function c, we have to fix a value for:

- $c(yesEvac, yes)$: the cost of deciding the evacuation of building B while the individual A is near the building B,
- $c(yesEvac, no)$: the cost of deciding the evacuation of building B while the individual A is not near the building B,
- $c(noEvac, yes)$: the cost of deciding to not evacuate while the individual A is near the building B,
- and $c(noEvac, no)$: the cost of deciding to not evacuate while the individual A is not near the building B,

Making a decision 3/3

Decision Level

- When the decision is appropriated, a zero-cost is naturally chosen:

$$c(yesEvac, yes) = c(noEvac, no) = 0.$$

Making a decision 3/3

Decision Level

- When the decision is appropriated, a zero-cost is naturally chosen:

$$c(yesEvac, yes) = c(noEvac, no) = 0.$$

- When the decision is not correct we can distinguished different degrees of errors.
  - Cost $c(noEvac, yes)$ is more important that the cost $c(yesEvac, no)$.

Making a decision 3/3

Decision Level

- When the decision is appropriated, a zero-cost is naturally chosen:

$$c(yesEvac, yes) = c(noEvac, no) = 0.$$

- When the decision is not correct we can distinguished different degrees of errors.
  - Cost $c(noEvac, yes)$ is more important that the cost $c(yesEvac, no)$.
- Then, we have the following order relation:

$$0 = c(yesEvac, yes) = c(noEvac, no) < c(yesEvac, no) < c(noEvac, yes)$$

Making a decision 3/3

Decision Level

- When the decision is appropriated, a zero-cost is naturally chosen:

$$c(yesEvac, yes) = c(noEvac, no) = 0.$$

- When the decision is not correct we can distinguished different degrees of errors.
  - Cost $c(noEvac, yes)$ is more important that the cost $c(yesEvac, no)$.
- Then, we have the following order relation:

$$0 = c(yesEvac, yes) = c(noEvac, no) < c(yesEvac, no) < c(noEvac, yes)$$

- Finally, **chosen decision**: the one which minimizes:

$$R_{Bet}(d) = c(d, yes)BetP^{\Omega}(\{yes\}) + c(d, no)BetP^{\Omega}(\{no\})$$

LGI2A

UNIVERSITÉ D'ARTOIS

Thank you for your attention.